An Interactive Tool for Experimenting with Bounded-Degree Plane Geometric Spanners

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Abstract

The construction of bounded-degree plane geometric spanners has been a focus of interest in the field of geometric spanners for a long time. To date, several algorithms have been designed with various trade-offs in degree and stretch factor. Using JSXGraph, a state-of-the-art JavaScript library for geometry, we have implemented seven of these sophisticated algorithms so that they can be used for further research and teaching computational geometry. We believe that our interactive tool can be used by researchers from related fields to understand and apply the algorithms in their research. Our tool can be run in any modern browser. The tool will be permanently maintained by the second author at `https://ghoshanirban.github.io/bounded-degree-plane-spanners/index.html`

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Supplementary Material *Software (Tool)*:
`https://ghoshanirban.github.io/bounded-degree-plane-spanners/index.html`

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1 Introduction

Given a set P of n points in the Euclidean plane, a *geometric t-spanner* on P is a geometric graph $G := (P, E)$, such that for every pair of points $u, v \in P$, the distance between them in G (the length of a shortest path between u, v in G) is at most t times their Euclidean distance $|uv|$, for some $t \geq 1$. The complete geometric graph on P is a 1-spanner with $\Theta(n^2)$ edges. The quantity t is referred to as the *stretch factor* of G. A geometric spanner G is *plane* if it is crossing-free. If there is no necessity to specify t, we simply use the term *geometric spanner*.

Bose, Gudmundsson, and Smid [7] were the first to show that there always exists a plane 8.3-spanner of degree at most 27 on any point set. This result was subsequently improved in a series of papers [8, 3, 10, 6, 25, 22] in terms of degree and stretch factor. Bonichon et al. [5] reduced the degree to 4 with $t \approx 156.8$. Soon after this, Kanj et al. improved this stretch factor upper bound to 20 in [19]. A summary of these results is presented in Table 1.

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Table 1 A summary of results on constructions of plane geometric spanners, sorted by the degree (Δ) they guarantee. The implemented algorithms are marked in bold. The best known upper bound of 1.998 for the stretch factor of the L_2-Delaunay triangulation [27] is used in this table for expressing the stretch factors. These algorithms run in time that is polynomial in n.

Reference	Δ	Upper bound on stretch factor
Bose, Gudmundsson, and Smid [7]	27	$1.998(\pi+1) \approx 8.3$
Li and Wang [22]	23	$1.998(1+\frac{\pi}{\sqrt{2}}) \approx 6.4$
Bose, Smid, and Xu [10]	17	$1.998(2+2\sqrt{3}+\frac{3\pi}{2}+2\pi\sin\frac{\pi}{12}) \approx 23.6$
Kanj, Perković, and Xia [20]	14	$1.998(1+\frac{2\pi}{14\cos(\pi/14)}) \approx 2.9$
Bose, Hill, and Smid [8]	8	$1.998\left(1+\frac{2\pi}{6\cos(\pi/6)}\right) \approx 4.4$
Bose, Carmi, and Chaitman-Yerushalmi [6]	7	$1.998(1+\sqrt{2})^2 \approx 11.6$
Bose, Carmi, and Chaitman-Yerushalmi [6]	6	$1.998\left(\frac{1}{1-\tan(\pi/7)(1+1/\cos(\pi/14))}\right) \approx 81.7$
Bonichon et al. [3]	6	6
Bonichon et al. [5]	4	$\sqrt{4+2\sqrt{2}}(19+29\sqrt{2}) \approx 156.8$
Kanj, Perković, and Türkoğlu [19]	4	20

The question whether the degree can be reduced to 3 keeping t bounded, still remains open at this time; refer to [9, Problem 14] and [26, Chapter 32]. If one does not insist on constructing a plane geometric spanner, Das and Heffernan [14] showed that degree 3 is always achievable. It is shown in the book [24, Section 20.1] by Narasimhan and Smid that no degree-2 plane spanner of the infinite integer lattice can have constant stretch factor. Thus, a minimum degree of 3 is necessary to achieve a constant stretch factor. Biniaz et al. [2] showed that if P is convex, then it is always possible to construct a plane 5.2-spanner having degree 3. From the other direction, lower bounds on the stretch factors of plane spanners for finite point sets have been investigated in [16, 15, 21, 23]. Plane geometric spanners find their applications in robotics and wireless networks where edge crossings may cause interference. An advantage of using plane spanners is that they have $O(n)$ edges and consequently take less storage space. In related works, the construction of plane hop spanners (where the number of hops in shortest paths is of interest) for unit disk graphs has been considered in [11, 1, 17].

Every algorithm designed so far that can construct bounded-degree plane spanners relies on some variant of Delaunay triangulation as the starting point. The rationale behind this is that these triangulations are geometric spanners [27, 12, 13, 4] and are plane by definition. As such, this family of plane spanner construction algorithms has turned out to be a fascinating application of Delaunay triangulation. In this work, we have implemented a set of seven novel algorithms that rely only on the L_2-Delaunay triangulation; refer to Table 1. The algorithms in Table 1 that are not implemented, use other kinds of Delaunay triangulations and are not considered in this work due to their inherent implementation complexities. To our knowledge, this is the first time that these novel algorithms have been implemented. We urge the readers to refer to the source papers to gain an understanding of the implemented algorithms.

Our implementations have two-fold contributions. First, they will help in the research of geometric spanners where tools are rarely available for experiments. Second, they can be used in teaching geometric spanners in computational geometry courses. This tool will be permanently maintained by the second author in his GitHub[1]. In-browser implementations of path-greedy, gap-greedy, Θ-graph, and Yao-graph algorithms were considered by Farshi and Hosseini in [18].

2 Implementation and usage

We have implemented the spanner algorithms using the JSXGraph library for attractive visualization and easy interaction. A technical description and documentation of this tool is included in the tool itself; click on the `ReadMe` button in the tool to launch this. We encourage users to read this read-me before using the tool.

After the tool is launched in a browser, the user can enter points manually either by clicking on the canvas or by entering coordinates explicitly in a text-box. The tool also comes with several built-in point sets which can also be used for experiments along with the manually entered points. Random point generation is also supported by the tool. Once the point set is finalized, an algorithm needs to be selected for spanner construction. Some of these algorithms accepts an extra parameter from the user for spanner construction which can be easily specified using a slider.

The tool draws the generated spanner and outputs the following: $|V|$, $|E|$, the exact stretch factor of G, degree of G, average degree (taken over all points in P), lightness (the ratio of the weight of G to that of the Euclidean Minimum Spanning Tree on P), and the spanner edges. It also shows the point pair that achieves the exact stretch factor for G in red, along with a shortest path between them in G. The built-in screenshot support allows the user to export the current board to a `png` or `svg` image for future uses.

3 Conclusions

We believe that our tool can bring new insights to the research of geometric spanners. In particular, we hope that this tool will aid researchers to solve the fascinating open problem posed in [9, Problem 14] and [26, Chapter 32] that asks whether degree-3 plane geometric spanners having bounded stretch factor are always possible. This tool can also be used in teaching computational geometry courses where geometric spanners hold special importance. Furthermore, researchers from related fields such as robotics and networking can use our tool in their research.

References

1 Ahmad Biniaz. Plane hop spanners for unit disk graphs: Simpler and better. *Comput. Geom.*, 89:101622, 2020. doi:10.1016/j.comgeo.2020.101622.

2 Ahmad Biniaz, Prosenjit Bose, Jean-Lou De Carufel, Cyril Gavoille, Anil Maheshwari, and Michiel Smid. Towards plane spanners of degree 3. *Journal of Computational Geometry*, 8(1):11–31, 2017.

3 Nicolas Bonichon, Cyril Gavoille, Nicolas Hanusse, and Ljubomir Perković. Plane spanners of maximum degree six. In *International Colloquium on Automata, Languages, and Programming*, pages 19–30. Springer, 2010.

4 Nicolas Bonichon, Cyril Gavoille, Nicolas Hanusse, and Ljubomir Perković. The stretch factor of l_1-and l_∞-delaunay triangulations. In *European Symposium on Algorithms*, pages 205–216. Springer, 2012.

5 Nicolas Bonichon, Iyad Kanj, Ljubomir Perković, and Ge Xia. There are plane spanners of degree 4 and moderate stretch factor. *Discrete & Computational Geometry*, 53(3):514–546, 2015.

[1] URL to the tool: https://ghoshanirban.github.io/bounded-degree-plane-spanners/index.html

6 Prosenjit Bose, Paz Carmi, and Lilach Chaitman-Yerushalmi. On bounded degree plane strong geometric spanners. *Journal of Discrete Algorithms*, 15:16–31, 2012.

7 Prosenjit Bose, Joachim Gudmundsson, and Michiel Smid. Constructing plane spanners of bounded degree and low weight. *Algorithmica*, 42(3-4):249–264, 2005.

8 Prosenjit Bose, Darryl Hill, and Michiel Smid. Improved spanning ratio for low degree plane spanners. *Algorithmica*, 80(3):935–976, 2018.

9 Prosenjit Bose and Michiel Smid. On plane geometric spanners: A survey and open problems. *Computational Geometry*, 46(7):818–830, 2013.

10 Prosenjit Bose, Michiel Smid, and Daming Xu. Delaunay and diamond triangulations contain spanners of bounded degree. *International Journal of Computational Geometry & Applications*, 19(02):119–140, 2009.

11 Nicolas Catusse, Victor Chepoi, and Yann Vaxès. Planar hop spanners for unit disk graphs. In *International Symposium on Algorithms and Experiments for Sensor Systems, Wireless Networks and Distributed Robotics*, pages 16–30. Springer, 2010.

12 L Paul Chew. There are planar graphs almost as good as the complete graph. *Journal of Computer and System Sciences*, 39(2):205–219, 1989.

13 LP Chew. There is a planar graph almost as good as the complete graph, 2nd symp. on computational geometry, 1986.

14 Gautam Das and Paul J Heffernan. Constructing degree-3 spanners with other sparseness properties. *International Journal of Foundations of Computer Science*, 7(02):121–135, 1996.

15 Adrian Dumitrescu and Anirban Ghosh. Lattice spanners of low degree. *Discrete Mathematics, Algorithms and Applications*, 8(03):1650051, 2016.

16 Adrian Dumitrescu and Anirban Ghosh. Lower bounds on the dilation of plane spanners. *International Journal of Computational Geometry & Applications*, 26(02):89–110, 2016.

17 Adrian Dumitrescu, Anirban Ghosh, and Csaba D Tóth. Sparse hop spanners for unit disk graphs. In *31st International Symposium on Algorithms and Computation (ISAAC 2020)*. Schloss Dagstuhl-Leibniz-Zentrum für Informatik, 2020.

18 Mohammad Farshi and Seyed Hossein Hosseini. Visualization of geometric spanner algorithms. In *32nd International Symposium on Computational Geometry (SoCG 2016)*. Schloss Dagstuhl-Leibniz-Zentrum fuer Informatik, 2016.

19 Iyad Kanj, Ljubomir Perkovic, and Duru Türkoğlu. Degree four plane spanners: Simpler and better. *Journal of Computational Geometry*, 8(2):3–31, 2017.

20 Iyad A Kanj, Ljubomir Perković, and Ge Xia. On spanners and lightweight spanners of geometric graphs. *SIAM Journal on Computing*, 39(6):2132–2161, 2010.

21 Rolf Klein, Martin Kutz, and Rainer Penninger. Most finite point sets in the plane have dilation > 1. *Discrete & Computational Geometry*, 53(1):80–106, 2015.

22 Xiang-Yang Li and Yu Wang. Efficient construction of low weighted bounded degree planar spanner. *International Journal of Computational Geometry & Applications*, 14(01n02):69–84, 2004.

23 Wolfgang Mulzer. Minimum dilation triangulations for the regular n-gon. *Master's thesis, Freie Universität Berlin, Germany*, 2004.

24 Giri Narasimhan and Michiel Smid. *Geometric spanner networks.* Cambridge University Press, 2007.

25 Ljubomir Perkovic and Iyad A Kanj. On geometric spanners of euclidean and unit disk graphs. In *25th International Symposium on Theoretical Aspects of Computer Science*. Schloss Dagstuhl-Leibniz-Zentrum für Informatik, 2008.

26 Csaba D Toth, Joseph O'Rourke, and Jacob E Goodman. *Handbook of discrete and computational geometry.* Chapman and Hall/CRC, 2017.

27 Ge Xia. The stretch factor of the delaunay triangulation is less than 1.998. *SIAM Journal on Computing*, 42(4):1620–1659, 2013.