Holonomic Techniques, Periods, and Decision Problems

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⸻ Abstract ⸻

Holonomic techniques have deep roots going back to Wallis, Euler, and Gauss, and have evolved in modern times as an important subfield of computer algebra, thanks in large part to the work of Zeilberger and others over the past three decades (see, e.g., [3, 2]). In this talk, I give an overview of the area, and in particular present a select survey of known and original results on decision problems for holonomic sequences and functions. I also discuss some surprising connections to the theory of periods and exponential periods, which are classical objects of study in algebraic geometry and number theory; in particular, I relate the decidability of certain decision problems for holonomic sequences to deep conjectures about periods and exponential periods, notably those due to Kontsevich and Zagier.

Parts of this exposition draws upon [1].

⸻ References ⸻

1 George Kenison, Oliver Klurman, Engel Lefaucheux, Florian Luca, Pieter Moree, Joël Ouaknine, Markus A. Whiteland, and James Worrell. On positivity and minimality for second-order holonomic sequences. In Filippo Bonchi and Simon J. Puglisi, editors, *46th International Symposium on Mathematical Foundations of Computer Science (MFCS 2021), Tallinn, Estonia*, volume 202 of *LIPIcs*. Schloss Dagstuhl – Leibniz-Zentrum für Informatik, 2021. `doi:10.4230/LIPIcs.MFCS.2021.67`.

2 Marko Petkovšek, Herbert Wilf, and Doron Zeilberger. *A=B*. A. K. Peters, 1997.

3 Doron Zeilberger. A holonomic systems approach to special functions identities. *Journal of Computational and Applied Mathematics*, 32(3):321–368, 1990.

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