An Integrated Model for Rapid and Slow Transit Network Design

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Abstract

Usually, when a rapid transit line is planned a less efficient system already partially covers the demand of the new line. Thus, when the rapid transit starts its regular services, the slow mode (e.g. bus lines) have to be cancelled or their routes modified. Usually this process is planned according to a sequential way. Firstly, the rapid transit line is designed taking into account private and public flows, and possibly surveys on mobility in order to predict the future utilization of the new infrastructure and/or other criteria. Then, in a second stage, the bus route network is redesigned. However, this sequential process can lead to a suboptimal solution, for which reason in this paper a cooperative model for rapid and slow transit network design is studied. The aim is to design simultaneously both networks and the objective is to maximize the number of passengers captured by both public modes against the private mode. We present a mathematical programming formulation and solve the problem by an improved Benders decomposition approach.

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1 Location of a rapid transit line along with improving the feeder bus system: definitions

In this section we assume that changes in the bus routes can be done. Rerouting bus lines is very common when a rapid transit line starts functioning. During the last five years more than 80 new metro lines have been added to metro networks around the world, and 27 new metro systems have been inaugurated. Therefore, more than one hundred new lines have become operating. Many other existing lines have been extended or upgraded. Moreover, new modern trams, train-trams, and commuter lines have also started their operation. In almost all the cases, bus lines were (partially) doing the service before, and when a rapid transit line is put in service bus routes could become totally or partially useless. One typical example is the adaptation of the Bus Rapid Transit TranSantiago when Metro Line 7 will start operation. Another example is bus line 3 of TUSSAM (Urban Transport of Seville) with planned Line 3 of Metro de Sevilla. Usually, the metro planning projects do not take into account the bus system because often they depend on a different agency, and after the introduction of the rapid transit service the bus system is reorganized. However, this procedure could lead to suboptimal solutions. The feeder buses planning problem has been researched to some extent ([3]) and models and algorithms for the Rapid transit network design problem have been recently revised ([4]), but as far as the authors are aware for the cooperative slow and rapid transit network design problem, no research has been done. For this purpose, in this section, an integer mathematical programming program is presented.

1.1 Data

In order to describe the problem we need to define the following elements.

1. We consider the network $\mathcal{N} = (N, E)$ used by the private mode, where N and E is the set of nodes and edges. For homogeneity purposes the rapid transit line R and the slow line S will be selected from this network.
2. The network (N_R, E_R) is the subgraph of $\mathcal{N}$ where the rapid transit line can be selected, thus $N_R \subset N$ and $E_R \subset E$.
3. The network (N_S, E_S) is the subgraph of $\mathcal{N}$ where the slow line S can be selected, thus $N_S \subset N$ and $E_S \subset E$.
4. For the rapid transit line R, there exists a maximum number of edges E^R_{max} to build. For the slow line S, bounds E^S_{max} and E^S_{id} are given to limit the number of edges to build and the minimum number of edges that must be coincident between the old and the modified line S (i.e. the number of edges not relocated). For that, vector v^S_e, $e \in E_S$ denotes the current path of the slow line S.
5. For each $e = \{k, l\} \in E$, we define two arcs: $a = (k, l)$ and $\hat{a} = (k, l)$. The resulting set of arcs is denoted by A. With respect to each mode of transport we refer the set of arcs by A_R and A_S, respectively. We use notation $\delta^+_w(k)$ ($\delta^-_w(k)$ respectively) to denote the set of arcs going out (in respectively) of node $k \in N_R$. In the same way, we use notation $\gamma^+_w(k)$ ($\gamma^-_w(k)$ respectively) to denote the set of arcs going out (in respectively) of node $k \in N_S$.
6. For each mode of transport, we assume that there is a set of possible starting points, O_R and O_S, of the lines. In the same way, sets containing possible end points D_R and D_S.
7. The set of demands W is a subset of $N \times N$. The mobility pattern is given by a matrix $G = (g^w)$, where $g^w, w = (w^s, w^t)$, denotes the number of passengers going from w^s to w^t, $(w^s, w^t) \in W$. The fixed cost of going from node w^s to node w^t using the private network is denoted by u^w_{priv}.
8. The set of possible transfer nodes is denoted by $N_{trans} = N_R \cap N_S$.
9. Other costs are those of traversing arc a in the rapid and slow mode, t^R_a and t^S_a, respectively. The transfer cost at station k from S to R and from R to S are t^{SR}_k and t^{RS}_k, respectively. The dwell time costs (stops) are t^R_{stop} are t^S_{stop}, which will be assumed independent from nodes. The waiting time at stations/stops, t_{wait}, is usually set as a half of the headway.

1.2 Variables

1. $x^R_e = 1$ if edge $e = \{k, l\} \in E_R$ is included in the rapid public line R; 0 otherwise. Analogously, $x^S_e = 1$ if edge $e = \{k, l\} \in E_S$ is included in the slow public line S; 0 otherwise.
2. $y^R_i = 1$ if node $i \in N$ is included in the alignment of the rapid system R, but it does not stop on it; 0 otherwise.

3. $z_i^R = 1$ if R stops at i; 0 otherwise. Analogously, $z_k^S = 1$ if k is a stop of mode S; 0 otherwise.
4. $f_a^{wR} = 1$ if demand w traverses arc $a \in A_R$, 0 otherwise.
5. $f_a^{wS} = 1$ if demand w traverses arc $a \in A_S$; 0 otherwise.
6. $f_k^{wSR} = 1$ if demand w transfers from S to R at node $k \in N_{trans}$; 0 if there is no transfer of w from S to R at k.
7. $f_k^{wRS} = 1$ if demand w transfers from R to S at node $k \in N_{trans}$; 0 if there is no transfer of w from S to R at k.
8. $f^w = 1$, if demand w uses S, R, or the combined modes RS and SR.

1.3 Objective and constraints

The aim of the problem in to design line R and to re-design line S so that the trip coverage of both public modes would be maximized, thus minimizing the private traffic:

$$\max_{\boldsymbol{x},\boldsymbol{y},\boldsymbol{z},\boldsymbol{f}} \sum_{w \in W} g^w f^w \tag{1}$$

- Budget constraints: Impose upper bounds on the budget and/or on the number of edges for both modes of transport.

$$\sum_{e \in E_R} x_e^R \leq E_{max}^R, \tag{2}$$

$$\sum_{e \in E_S} x_e^S \leq E_{max}^S. \tag{3}$$

- Design constraints: Among them are the following: If an edge is constructed for the rapid system its endpoints are either a station or a non-stop node. At least one node has to be selected from the sets of origins and destinations of the rapid and slow lines. The lines must be chain graphs. If an edge is selected to be in the rapid or slow line its endpoints are nodes of the line.

$$x_e^R \leq z_i^R + y_i^R, \quad e \in E_R, i \in e, \tag{4}$$

$$\sum_{o \in O_R} \sum_{e \in \delta(o)} x_e^R = 1, \tag{5}$$

$$\sum_{d \in D_R} \sum_{e \in \delta(d)} x_e^R = 1, \tag{6}$$

$$\sum_{o \in O_R} z_o^R = 1, \tag{7}$$

$$\sum_{d \in D_R} z_d^R = 1, \tag{8}$$

$$z_i^R + y_i^R \leq 1, \quad i \in N_R, \tag{9}$$

$$\sum_{e \in E_R} x_e^R + 1 = \sum_{i \in N_R} (y_i^R + z_i^R), \tag{10}$$

$$\sum_{e \in \delta(k)} x_e^R \leq 2(z_k^R + y_k^R), \quad k \in N_R \setminus (O_R \cup D_R), \tag{11}$$

$$x_e^S \leq z_i^S, \quad e \in E_S, i \in e, \tag{12}$$

$$\sum_{o \in O_S} z_o^S = 1, \tag{13}$$

$$\sum_{d \in D_S} z_d^S = 1, \tag{14}$$

$$\sum_{e \in E_S} v_e^S x_e^S \geq E_{id}^S, \tag{15}$$

$$\sum_{e \in E_S} x_e^S + 1 = \sum_{i \in N_S} z_i^S, \tag{16}$$

$$\sum_{e \in \gamma(k)} x_e^S \leq 2 z_k^S, \quad k \in N_S \setminus (O_S \cup D_S), \tag{17}$$

$$f^w \leq 1 - y_{w^s}^R, \quad \text{if } w^s \in N_R, \tag{18}$$

$$f^w \leq 1 - y_{w^t}^R, \quad \text{if } w^t \in N_R, \tag{19}$$

- Relation between variables f^w, z_k^R and z_k^S.

$$f^w \leq \begin{cases} z_k^R + z_k^S, & \text{if } k \in N_R \cap N_S, \\ z_k^R, & \text{if } k \in N_R \text{ and } k \notin N_S, \\ z_k^S, & \text{if } k \in N_S \text{ and } k \notin N_R, \end{cases} \quad w \in W,\ k \in \{w^s, w^t\}, \tag{20}$$

- Flow conservation constraints. Flows have to be maintained either by slow or rapid modes.

$$\sum_{a \in \delta_w^+(k)} f_a^{wR} + \sum_{a \in \gamma_w^+(k)} f_a^{wS} - \left(\sum_{a \in \delta_w^-(k)} f_a^{wR} + \sum_{a \in \gamma_w^-(k)} f_a^{wS} \right) =$$
$$\begin{cases} f^w, \text{ if } k = w^s, \\ -f^w, \text{ if } k = w^t, \\ 0, \text{ otherwise} \end{cases} \quad w \in W, k \in N_R \cup N_S, \tag{21}$$

- Transfer constraints. Only one transfer from slow to rapid mode and from rapid to slow is allowed.

$$\sum_{k \in N_{trans} \setminus \{w^s, w^t\}} f_k^{wSR} \leq 1, \quad w \in W, \tag{22}$$

$$\sum_{k \in N_{trans} \setminus \{w^s, w^t\}} f_k^{wRS} \leq 1, \quad w \in W, \tag{23}$$

$$\sum_{a \in \delta_w^-(k)} f_a^{wR} + f_k^{wSR} - \left(\sum_{a \in \delta_w^+(k)} f_a^{wR} + f_k^{wRS} \right) = 0, \quad w \in W, k \in N_{trans} \setminus \{w^s, w^t\}, \tag{24}$$

$$\sum_{a \in \gamma_w^-(k)} f_a^{wS} + f_k^{wRS} - \left(\sum_{a \in \gamma_w^+(k)} f_a^{wS} + f_k^{wSR} \right) = 0, \quad w \in W, k \in N_{trans} \setminus \{w^s, w^t\}, \tag{25}$$

- Location-allocation constraints. Link design and flow variables.

$$f_a^{wR} + f_{\hat{a}}^{wR} \leq x_e^R, \quad w \in W, e = \{i, j\} \in E_R : a = (i, j), \hat{a} = (j, i), \tag{26}$$

$$f_a^{wS} + f_{\hat{a}}^{wS} \leq x_e^S, \quad w \in W, e = \{i, j\} \in E_S : a = (i, j), \hat{a} = (j, i), \tag{27}$$

- Alignment stop constraints. Stated conditions on the construction of a node regarding in- or out-flows.

$$f_k^{wSR} + f_k^{wRS} \le z_k^R, \quad w \in W, k \in N_{trans} \setminus \{w^s, w^t\}, \tag{28}$$

$$f_k^{wSR} + f_k^{wRS} \le z_k^S, \quad w \in W, k \in N_{trans} \setminus \{w^s, w^t\}, \tag{29}$$

$$\sum_{a \in \delta^+(w^s)} f_a^{wR} \le z_{w^s}^R, \quad w = (w^s, w^t) \in W, \text{ if } w^s \in N_R, \tag{30}$$

$$\sum_{a \in \delta^-(w^t)} f_a^{wR} \le z_{w^t}^R, \quad w = (w^s, w^t) \in W, \text{ if } w^t \in N_R, \tag{31}$$

- Mode choice. Assign the demand either to the public modes or to the private one depending of the total time of the trip.

$$\begin{aligned}&\sum_{a \in A_R} t_a^R f_a^{wR} + \sum_{a \in A_S} t_a^S f_a^{wS} + \sum_{k \in N_{trans}} t_k^{RS} f_k^{wRS} + \sum_{k \in N_{trans}} t_k^{SR} f_k^{wSR} + \\ &+ t_{stop}^R \sum_{k \in N^R} z_k^R \sum_{a \in \delta^+(k)} f_a^{wR} + t_{stop}^S \sum_{k \in N^R} z_k^S \sum_{a \in \gamma^+(k)} f_a^{wS} + f^w \left(t_{wait}^R - \frac{1}{2} t_{stop}^R \right) \le u_{priv}^w,\end{aligned} \tag{32}$$

- Binary constraints. All the variables are assumed to be in $\{0, 1\}$.

$$x_e^R,\ x_e^S,\ y_k^R,\ z_k^R,\ z_k^S,\ f_a^{wR},\ f_a^{wS},\ f_k^{wRS},\ f_k^{wSR},\ f^w \in \{0, 1\}. \tag{33}$$

2 Solving the problem

Since the problem is NP-hard, we use a Benders decomposition approach to exactly solve it (see [1]). With this exact procedure we pretend to improve the computational time. Actually, our Benders implementation is used as a sub-routine in a Branch-and-Benders-cut scheme. We use the ideas exposed in [2] in order to get stronger cuts than the standard ones.

Our computational experiments were performed on a computer equipped with a Intel Core i5-7300 CPU processor, with 2.50 gigahertz 4-core, and 16 gigabytes of RAM memory. The operating system is 64-bit Windows 10. Codes were implemented in Python 3.8. These experiments have been carried out through `CPLEX 12.10` solver, named `CPLEX`, using its Python interface. `CPLEX` parameters were set to their default values and the model was optimized in a single threaded mode.

The tested instance is composed by 64 nodes and 128 edges. The W set is formed by all possible O/D pairs. The new slow line S must coincide with the old one on at least 3 edges and can consist of a maximum of 6 edges in total. With respect to the rapid transit line, it must be composed by 9 nodes and 9 edges at most.

After two hours, the optimal solution is obtained using the implemented routine Branch-and-Benders-cut scheme ad-hoc to the problem, an hour and a half earlier than if we directly solve the MIP formulation with `CPLEX`. It should be noted that the Benders decomposition algorithm existing in `CPLEX` is not competitive with the two named methods.

This integrated model results in an optimum design with respect to the maximization of the coverage for the whole public transport (composed by the rapid and slow modes). Locating each line independently without taking into account the influence that may exist between them, or even locate them in a sequential way can result in suboptimal solutions. The sequential design method is the one used in practice. That is, currently the rapid transit line R is located first and then the slow line S is relocated. For example, considering the tested instance, the optimal objective value for the integrated model is 35.8% bigger than that of the independently localization.

References

1 J Benders. Partitioning procedures for solving mixed-variables programming problems. *Numerische mathematik*, 4(1):238–252, 1962.

2 M Conforti and L. A Wolsey. “Facet” separation with one linear program. *Mathematical Programming*, 178(1):361–380, 2019.

3 Lian-bo Deng, Wei Gao, Wen-liang Zhou, and Tian-zhen Lai. Optimal design of feeder-bus network related to urban rail line based on transfer system. *Procedia-Social and Behavioral Sciences*, 96:2383–2394, 2013.

4 Gilbert Laporte and Juan A Mesa. The design of rapid transit networks. In G. Laporte, S. Nickel, and F. Saldanha da Gama, editors, *Location science*, chapter 24, pages 685–701. Springer, 2020.