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DAGSTUHL WORKSHOP
ON
EFFICIENT INTERPOLATION ALGORITHMS

December 02 – 06, 1991

Organizers:

Andreas Dress (Bielefeld)
Marek Karpinski (Bonn)
Michael Singer (Raleigh)

O V E R V I E W

The main interest of this Workshop was in the design and analysis of efficient sequential and parallel interpolation algorithms for a number of boolean, rational, group theoretic and algebraic problems. A special emphasis was on the relatively new classes of problems in the so called *sparse* (or *arithmetic-circuit*) representations. The various applications in combinatorial optimization, computational geometry, learning theory, computer algebra, and in algebraic complexity theory have been also discussed.

The 27 participants of this workshop came from 5 countries. Besides the formal program there has been an ample time for free discussions and informal meetings between participants. The nice setup of the Dagstuhl Institute made this workshop a very enjoyable experience.

The organizers would like to thank everyone who contributed to the success of this meeting.

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Interpolation of Sparse Rational Functions

DIMA GRIGORIEV, MAREK KARPINSKI, MICHAEL SINGER

A t-sparse n-variable rational function f is a rational function that can be written as a quotient of polynomials (with rational coefficients) each of which has at most t terms. We discuss an algorithm which, given t and a black box allowing one to evaluate f at arbitrary integer points, allows one to find the exponents and coefficients of t-sparse polynomial f_1 and f_2 such that $f = f_1/f_2$. This algorithm uses $t^{O(tn)} \log^{O(1)} d$ arithmetic operations and has $(tn)^{O(1)} \log^{O(1)} d$ depth, where d is the maximum of the exponents appearing in f_1 and f_2.

Computational Complexity of Interpolating Sparse Real Algebraic Functions

DIMA GRIGORIEV, MAREK KARPINSKI, MICHAEL SINGER

An algebraic function $Y(X_1, \dots, X_n)$ is called t-sparse if it satisfies a fractional-power polynomial

$$f = \sum_{1 \le i \le t} c_i X_1^{a_1^{(i)}} \cdots X_n^{a_n^{(i)}} Y^{b^{(i)}} = 0$$

where $a_1^{(i)}, \dots, a_n^{(i)}, b^{(i)} \in \mathbb{Q}, c_i \in \mathbb{R}$. We suppose that Y could be multivalued, defined on a positive orthant $(\mathbb{R}_+)^n$ and a black-box is given which allows for any point from $(\mathbb{R}_+)^n$ to get the value of Y at this point together with the partial derivatives of Y up to order $t-1$.

An algorithm is designed which finds all minimal sparse representations of Y using $2^{t^{O(nt)}}$ arithmetic operations. It is proved also that the number of minimal sparse representations does not exceed $t^{O(nt)}$.

Verification Complexity of Linear Prime Ideals

PETER BÜRGISSER, THOMAS LICKTEIG

The topic of the talk is the complexity of algebraic decision trees deciding membership in an algebraic subset $X \subset R^m$ where R is a real or algebraically closed field. We define a notion of verification complexity of a (real) prime ideal (in a prime cone) which gives a lower bound on the decision complexity. We determine the verification complexity of some prime ideals of linear type generalizing a result by Winograd. As an application we show uniform optimality with respect to the number of multiplications and divisions needed for two algorithms:

- For deciding whether a number is a zero of several polynomials – if this number and the coefficients of these polynomials are given as input data – evaluation of each polynomial with Horner's rule and then testing the values for zero is optimal.
- For verifying that a vector satisfies a system of linear equations – given the vector and the coefficients of the system as input data – the natural algorithm is optimal.

A Note on a $P \neq NP$ Result for a Restricted Class of Real Machines

KLAUS MEER

The following modification of the computation-model introduced by Blum, Shub, and Smale is considered: the machines in view are unlimited register machines over the reals with three kinds of operations:

i) linear evaluations,

ii) copy-instruction,

iii) equality-branches (i.e. "$x = 0$?" for a real x).

In this setting it is shown that the complexity classes P and NP (introduced as in the BSS model) don't coincide.

Modular Interpolation of Sparse Polynomials in Polynomial Time

ALEXANDR L. CHISTOV

The problem of interpolation of sparse polynomials is considered in the case when sizes of all coefficients are taken into account. The modular oracle is used. This oracle computes reductions modulo differents primes of values of the given polynomial in prescribed points. The following result is proved.

Theorem. Let the modular oracle for a t-sparse polynomial f of degree $\leq d$ with rational coefficient of the bitwise length $\leq l$ in n variable be given. Then using this oracle one can reconstruct f in time polynomial in t, n, l and $\log d$.

Polynomial Time Interpolation of Sparse Polynomials Using Real Value Oracle

SERGEI EVDOKIMOV

Let $f(x_1, \ldots, x_n) = \sum_{i=1}^{t} a_i x_1^{m_1^{(i)}} \cdots x_n^{m_n^{(i)}}$ be a sparse polynomial with real coefficients. There is an oracle which gives us values of the polynomial f at real points up to a prescribed accuracy. The bit model of computation is used.

Theorem. An interpolating algorithm for f is constructed. If $\log m_j^{(i)}$, size (a_i),t, n are less than a bound L then the running time of the algorithm is polynomial in L. The number of calls for the oracle is also polynomial in L.

Interpolation of Sparse Character Sums

Andreas Dress

For an abelian monoid A (with $1 \in A$), a field K, and a set $X \subseteq Hom(A,K) := \{(\chi : A \to K \mid \chi(1) = 1, \chi(ab) = \chi(a)\chi(b)\}$ put $X_k := \{\sum_{\chi \in X} c_\chi \chi \mid c_\chi \in K, \#\{\chi \mid c_\chi \neq 0\} \leq k\}$. A basic problem in interpolation theory is then to find a subset $T = T_k \subseteq A$ such that $f|_{T_k} = 0$ for some $f \in X_k$ implies $f \equiv 0$. If there exists some $a \in A$ with $\chi(a) \neq \chi'(a)$ for all $\chi, X' \in X$ with $\chi \neq \chi'$, this is easy: $T_k(a) := \{1, a, \ldots, a^{k-1}\}$ will do, – so, in consequence, if $C \subseteq A$ is such that for any two $\chi, X' \in X$ one has $\#\{a \in C \mid \chi(a) = \chi'(a)\} \leq n$, while $\#C \geq \binom{k}{2}n+1$, then $T_k(C) := \bigcup_{a \in C} T_k(a)$ will do. In general, if $A = A_1 \times \cdots \times A_n, X^i \subseteq Hom(A_i, K)$ $(i = 1, \ldots, n), X := X^1 \times \cdots \times X^n$, and if $T_k^i \subseteq A_i$ distinguishes functions in X_k^i, then $T_k = \bigcup_{k_1 \cdots k_n \leq k} T_{k_1}^1 \times \cdots \times T_{k_n}^n$ distinguishes function in X_k. In some cases, e.g. $A = I\!F_2^n$, this gives optimal sets, in other cases, e.g. $A = \{\pm 1\}^n$, it is not known, how much better one might be able to do.

Modeling and Visualization Based on Piecewise Algebraic Interpolation[1]

Wolfgang Dahmen

We show that, for a given finite set of points in affine 3-space, there exists a tangent plane continuous piecewise algebraic surface of degree two or three which interpolates the given points according to any prescribed topology and attains given (compatible) normal directions at these points. We briefly discuss the role of shape parameters which can be used for instance to control curvature or to reproduce quadric surfaces exactly. Since the construction allows for accurate surface modeling as well as for high quality rendering based on ray tracing techniques, using significantly fewer patches than piecewise planar representations, and since the construction with third degree surfaces is completely local, both tasks, modeling and rendering, can be performed interactively using the same implicit surface representation, which results in a significant trade off from storage requirements toward floating point activity. In particular, this is expected to favorably affect animation.

[1]Software development and implementation by Mental Images, Berlin

Codes and Interpolation

Johannes Grabmeier

I reported on a joint work with Arne Dür, University of Innsbruck. We explained close relation between coding theory and black-box interpolations of k-sparse multivariate polynomials or – more generally – k-sparse sums of characters of a monoid A to a field K.

If a test set $T_{2k} \subseteq A$ for this situation is given (i.e. all k-sparse sums of characters can be distinguished from their values on T_{2k}), then $\mathcal{H}_{T_{2k}} = (\chi(a))_{a\in T, \chi\in X}$, where X is the set of length $\#X$, dimension $\geq \#X - \#T_{sk}$ and minimum distance $\geq 2k+1$. Interpolations now means, that the 0-code word is sent over a channel and the syndrome is received directly. Hence, every decoding method, starting from the syndrome gives an interpolation algorithm.

In the situation of k-sparse boolean polynomials, where the unique, minimal test set was found to be the set of elements of $\{0,1\}^n$ having at most $1 + \lfloor \log_2 k \rfloor$ zeros by Clausen, Dress, Grabmeier and Karpinski 1988, was identified to be the Reed-Muller code of length 2^n and order $n - 2 - \lfloor \log_2 k \rfloor$. If $\{0,1\}^n$ is ordered properly, it is a subcode of an extended BCH code of designed distance $2^{2+\lfloor \log_2 k \rfloor} - 1$, so the efficient decoding algorithms for BCH codes can be used, provided that we can compute the syndrome in this case from the black-box evaluation. We have derived a formula by using the 2 different parity-check matrices.

In a second application of coding theory we give conditions of free linear codes over $\mathbb{Z}/e\mathbb{Z}$ to construct test sets for the case $A = U^n$, U cyclic groups of order e.

If such a code has length r, dimension n and minimum distance $\geq d$ then we can construct a test set, if for every collection of k distinct code words, there is a component where all codewords differ. This is the case, if the condition $\frac{d}{r} > 1 - \frac{1}{\binom{k}{2}}$ holds. If $e = p \geq \binom{k}{2}(n-1) - 1$, then the Reed-Solomon code over $\mathrm{GF}(p)$ of length $r = \binom{k}{2}(n-1)$ and minimum distance $d = r + 1 - n$ can be used. If e is square-free such that the smallest prime factor satisfies $p > \binom{k}{2}$, then the Gilbert-Varshamov bound guarantees the existence of such a code.

Interpolation and Approximation Techniques in Finite Fields and of Boolean Functions

KAI WERTHER

In this talk we consider functions over finite fields that can be written as a sparse sum of monomials as well as Boolean functions that can be written as a t-term DNF. We give an improvement of the lower bound on the size of a minimal test set for sparse multivariate polynomials over fields of odd characteristic. We also present an algorithm for the interpolation problem when queries are restricted to the ground field. This algorithm runs in time linear in the size of a minimal test set and the number of variables and polynomial in the number of terms if the field is fixed.

Next we give an overview over approximation techniques and pose the open problem whether t-term DNF formulae can be learned in polynomial time.

Wavelets

WOLFGANG DAHMEN

Wavelets offer powerful tools for linking discrete and continuous structures. We highlight some fundamental construction principles and techniques based on the notions of multiresolution analysis, stability, and refinability. In particular, we discuss some algebraic aspects in connection with multivariate constructions.

Computing Triangular Sets from U-resultants Using Sparse Interpolation

YAGATI N. LAKSHMAN

In this talk, we describe algorithms for computing triangular set representations for radicals of zero-dimensional ideals from specializations of the u-resultant (or the trailing coefficient of a generalized characteristic polynomial). The algorithm uses the D5 model for dealing with algebraic extensions and interpolation.

Fast Fourier Transforms on Supersolvable Groups

Ulrich Baum

The linear complexity $L_K(A)$ of matrix A over a field K is defined as the minimal number of additions, subtractions and scalar multiplications sufficient to evaluate A at a generic input vector. If G is a finite group and K a field containing a primitive $\exp(G)$-th root of unity, $L_K(G) := min\{L_K(A) \mid A$ a Fourier transform for $KG\}$ is called the K-linear complexity of G. We show that every supersolvable group G has a *monomial* Fourier transform adapted to a chief series of G. The proof is constructive and gives rise to an efficient algorithm with running time $O(|G|^2 \log |G|$. Moreover, we prove that these Fourier transforms are efficient to evaluate: $L_K(G) \leq 8.5|G| \log |G|$ for any supersolvable group G and $L_K(G) \leq 1.5|G| \log |G|$ for any 2-group G.

Processor-Efficient Parallel Solution of Linear Systems

Erich Kaltofen, Victor Pan

An algorithm of poly-logarithmic parallel running time is said to be processor-efficient if the number of processors required is within a poly-logarithmic factor of the best-known sequential running time. Computing the solution manifold of a possibly singular linear system can be accomplished sequentially asymptotically as fast as multiplying two matrices of dimension as is the system. We present a randomized circuit of poly-logarithmic depth that computes the inverse of a non-singular matrix and uses asymptotically within a factor log of the dimension as many processors as matrix multiplication. Our circuit is valid over any field.

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