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Complexity and Realization of Boolean Functions

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Summary

This was the first time that a conference on the complexity of Boolean functions took place at the IBFI. For scientists in the former DDR and the Soviet-Union there has been a long tradition to have a meeting once or twice a year called "Arbeitstagung Diskrete Mathematik und ihre Anwendungen in der mathematischen Kybernetik" (workshop on Discrete Mathematics and its Applications in Mathematical Cybernetics). The location of this conference has alternated between both countries, typically it was close to a University or the Academy of Science. After encountering difficulties to continue this workshop in Mittweida last year the possibility to meet this time in Dagstuhl was very helpful.

The organizers took the chance to broaden the circle of participants and to bring research groups from different parts of the world together. Scientists from Eastern and Western European countries and from the United States were invited and came to Dagstuhl.

The following topics were the main subjects of the given talks: computational models like Boolean circuits, contact networks, circuits with bounded Fan-in, neural Nets and Boolean decision trees. Furthermore asymptotic behaviour of the complexity of Boolean functions, fault-tolerance of Boolean circuits and communication complexity turned out to be of general interest. In addition, several talks dealt with other aspects of Boolean functions or related subjects. The conference focussed on basic research in this area, but also practical applications were considered in some contributions.

The participants are very gratefull for this opportunity to meet in such an excellent atmosphere and like to thank the staff of the IBFI for their help and the German Science Foundation, DFG, for providing travel support for some of the participants, which, otherwise, would not have been able to attend.

The organizers: R. Reischuk and D. Uhlig

On Communication Complexity

Rudolf Alhswede
Universität Bielefeld

Contributions to deterministic 2–way communication complexity, which we obtained together with N. Chai and Z. Zhang, were presented.

In [1] maximal monochromatic rectangles in the sense of Yao are determined for the Hamming distance in case of alphabet sizes α equal to 2, 4, and 5. The case $\alpha = 3$ is complicated but doable and we have a conjecture for all $\alpha \geq 6$ (500 DM for first solver).

Sum–type functions are a natural generalization for distances in sequence spaces. For them we introduced a 4–words property for rectangles, which is much more general than the property "monochromatic" and allows an easier analysis. A general inequality of [2] for the area of such rectangles gives excellent, often very exact bounds for the 2–way complexity of special functions such as Hamming distance for arbitrary alphabets, Lee distance, Manhattan distance, etc.

The recent paper [3] contains new upper and lower bounds on the 2–way complexity for abstract functions $g : \mathcal{X} \times \mathcal{Y} \to \mathcal{Z}$. They give tight bounds, when applied to vector valued functions $f^n = (f_1, ..., f_n) : \mathcal{X}^n \times \mathcal{Y}^n \to \mathcal{Z}^n$, if the alphabets are small. For the set–intersection function an optimal protocol is presented. The 2–way communication complexities of all other Boolean functions are also determined. The results are extended to meets in abstracts lattices and to a probabilistic model.

Finally, a seemingly important step was taken in [4]. After we have understood that for sum–type functions logrank of its function table is in general a poor bound for 2–way communication complexity, we introduce an exponential transform of this table over the complex numbers and obtain excellent, often even exact, bounds in terms of logrank of this transform. The role of this 4–words inequality and the decomposition lemma of [2] become also transparent.

Refernces

[1] R. Ahlswede, On code pairs with specified Hamming distances, Colloquia Mathematica Societatis János Bolyai 52. Combinatorics, Eger, 9 - 47, 1987.

[2] R. Ahlswede, N. Cai, and Z. Zhang, A general 4–words inequality with consequences for 2–way communication complexity, Advances in Applied Mathematics 10, 75 - 94, 1989.

[3] R. Ahlswede and N. Cai, On communication complexity of vector–valued functions, Preprint 91 – 041 SFB 343 'Diskrete Strukturen in der Mathematik', to appear in IEEE Trans. on Information Theory.

[4] R. Ahlswede and N. Cai, 2–way communication complexity of sum–type functions for one processor to be informed, Preprint 91 – 053 SFB 343 'Diskrete Strukturen in der Mathematik', to appear in IEEE Trans. on Information Theory.

On the Number of Functions in Some Closed Classes of Partial Boolean and Many-Valued Functions

Veleri Alekseev
Lomonossow University Moscow

Let $E_k = \{0, 1, ...k-1\}$, $P_k = \{f : E_k^n \to E_k \mid n = 1, 2...\}$, $P_k^* = \{f : E_k^n \to (E_k \cup \{*\}) \mid n = 1, 2, ...\}$ and let $R : E_k^m \to \{true, false\}$ be a predicate. A function $f(x_1, ..., x_n) \in P_k^*$ is said to preserve given predicate $R(y_1, ..., y_m)$ if for any $\tilde{\alpha}^1, ..., \tilde{\alpha}^m$ (where $\tilde{\alpha}^j = (\alpha_1^j, ..., \alpha_n^j)$) the following implication holds:
$\forall i R(\alpha_i^1, ..., \alpha_i^m) \Rightarrow [R(f(\tilde{\alpha}^1), ..., f(\tilde{\alpha}^m)) \vee \exists j f(\tilde{\alpha}^j) = *]$. Many closed classes in P_k or P_k^* are classes of all functions preserving a predicate R. The problem is: given a predicate R how to estimate the number $\varphi_R(n)$ of functions of n variables preserving R in P_k or P_k^*. A general method for finding the asymptotics for $\log_2 \varphi_R(n)$ when $n \to \infty$ is developed and by this method such asymptotics are established for all maximal closed (precomplete) classes in P_k, P_2^* and for those maximal closed classes in P_k^* that are defined by binary predicates $R(y_1, y_2)$.

Complexity of Greedy Gating Circuits

Alexander E. Andreev
University of Wolgograd

We consider gating circuits of depth 2. In this case the problem of construction of minimal size circuits for a given Boolean matrix is equivalent to the following covering problem:

to cover all units of the matrix by submatrices without zeros with minimal sum of number of rows and columns of all submatrices.

For this covering problem we apply a greedy algorithm. Let $L_\nabla(T)$ denote the complexity of greedy circuits obtained for matrix T, and let $F(p, q, m_0, m_1)$ be the set of all (p, q) matrices having m_0 zeros, m_1 units, and $pq - m_0 - m_1$ undefined elements. Moreover, let $L_\nabla(p, q, m_0, m_1) = max_{T \in F(p,q,m_0,m_1)} L_\nabla(T)$. The main results can be stated as follows:

$L_\nabla(p, q, m_0, m_1) \sim \frac{\log \binom{m_0+m_1}{m_1}}{\log \frac{m_0+m_1}{q}}$ (if $p \geq q$ and supposed some natural restriction for p, q, m_0, m_1).

Multiparity Communication Complexity

Pavol Duris
Slovak Academy of Sciences

Suppose a coordinator wishes to exchange a Booelan function $f(x_1, ..., x_n)$. The input vector $x = (x_1, ..., x_n)$ is distributed among n parties so that the i–th party can see only the input $x_i \in \{0,1\}^m$. The coordinator is connected by a communication link to each party.

The communication complexity of f is the maximum number of bits exchanged through all links that are needed to compute f by the best protocol. We estimate lower and upper bounds on deterministic and nondeterministic communication complexity of f and $1 - f$.

Complexity of Self–Correcting Circuits

Regina Fischer
Hochschule für Technik und Wirtschaft Mittweida

We consider a special cellular array model. The model is a modification of Kravcov schemes. The elements (cells) used in our model are computation elements labelled with NAND, NOR, NOT and wiring elements. This model can be considered as a special VLSI–model to support circuit design.

A scheme is called r–self–correcting if it has $2r + 1$ outputs and if it outputs the correct value for each input vector on most outputs, even if arbitrary elements, at most r, failed.

It can be shown that for $r = 2^{\frac{n}{6} - \psi(n) - \frac{3}{4}\log_2 n}$, where $\psi(n)$ is an arbitrary slowly to ∞ tending function, if $n \to \infty$, there exist Boolean functions for which the complexity of r–self–correcting modified Kravzov schemes is almost the same as the complexity of minimal non–self–correcting ones. The greatest possible number of failures is here the best in comparison with analogous results in different approaches of r–self–correcting realizations for Boolean functions (e.g. $r = 2^{o(n)}$ for circuits).

Complexity of Probabilistic Decision Trees for Boolean Operators

Rusins Freivalds
University of Latvia, Riga

The complexity of a decision tree (the length of the longest path) is investigated

for deterministic, nondeterministic and probabilistic decision trees which compute a Boolean operator defined as a map $\{0,1\}^n \to \{0,1,...,k-1\}$, i.e., a k–valued function. We study the dependence of the complexity on k (instead of the more usual dependence on n). Some of our results remind their counterparts in the complexity dependence on n. However, the minimal complexity (i.e., minimum of the complexity of an essentially n–ary k–valued function) of the nondeterministic decision tree does not exceed $\frac{1}{2}\log_2 n + o(\log_2 n)$ but it is precisely $\lfloor \log_2 n \rfloor + 1$ for deterministic decision tree. We prove it to be no less than $\lceil \log_2 k \rceil$.

For a probabilistic decision tree computing a Boolean operator with probability of correctness exceeding $1/2$ the complexity can be $\log_2 k - 1$ and is never below this bound. However, if we allow the probability of the correct result to be only 2^{-s} (provided that the probability of an arbitrary incorrect result is strictly less than this value) then the complexity of the decision tree is no less than $\log_2 k - s$. Finally, for non–minimal complexities the advantage of probabilistic decision trees can be more considerable: there is a Boolean operator for which a linear amount of questions can be saved.
(joint work with Masahiro Miyakawa, Tsukuba, Japan)

On the Redundancy of Fault–Tolerant (Robust) Computation

Péter Gács
Boston University, Massachusetts

The talk gave an overview of the theory of fault-tolerant (we will say, robust) computation in models where there is an unbounded number of elementary components and each can make an error with probability bounded by a constant. The errors are transient since only the momentary state of the computing component is changed.

We reviewed Von Neumann's robust Boolean circuits along with the lower bound on their redundancy (given by Dobrushin-Ortyukov and corrected later by others). Then Kuznietsov's constant-redundancy information-storage scheme was mentioned. There followed a review of several constructions for robust cellular automata. The 3-dimensional automaton of Gacs-Reif has no time-redundancy and logarithmic space-redundancy. A 2-dimensional automaton of the author has constant space-redundancy and logarithmic time redundancy. The open problem of constructing robust Turing machines was mentioned.

On the basis of the above results, an argument is made that the product of the space- and time-redundancies of robust computation is at least a logarithm of the size (space×time). The above-mentioned lower bound is not really a step in this direction since it only shows the necessity of redundancy in the input. The rest of the talk was devoted to the formulation of two lower-bound conjectures which claim the necessity of redundancy within the computation itself, even if the input is in the form of an error-correcting code. A heuristic argument was given in support of the conjecture: its essence is that it is probably not possible to perform all information-processing

operations without decoding the information from its redundant form (especially if the redundancy of the code is only a constant factor). The decoding-processing-encoding cycle will then probably require repetition—giving rise to nonconstant redundancy.

On the Networks of Cyclic Commutators

Michail Grintschuk
Lomonossow University, Moscow

Let $x \in Z$ be a control variable. Let the *cyclic commutator* modulo n be an object having $2n$ nodes $a_0, ..., a_{n-1}$ and $b_0, ..., b_{n-1}$ where a_i is connected with b_j iff $i - j = x \ (mod\ n)$. We can construct networks with such elements. These networks realize some periodical function $F : Z \to \{0,1\}$. Let the complexity of cyclic commutators modulo n be (by definition) n. We can define the complexity $C(\Sigma)$ of a network Σ and the complexity $C(F)$ of a periodical function F in the standard way. Let, as usually, the Shannon function $C(T)$ be $max_F C(F)$, where F have period T. The following bounds for $C(F)$ and $C(T)$ are proved.

a) if F has *minimal* period T of form $T = p_1^{\alpha_1} ... p_s^{\alpha_s}$ ($\alpha_i \in N$, p_i are primes), then $C(F) \geq p_1^{\alpha_1} + ... + p_s^{\alpha_s}$.

b) $C(F) \leq T$ for each F of period T.

c) $C(T) \geq T/(2\log_2 T)$.

d) if $T = ab$, where a and b have no common divisors, then

   i) $C(F) \leq a + b2^{b-1}$

   ii) $C(F) \leq T/\log_2 a$ for $b \geq \log_2 T - 3\log_2\log_2 T$ (in particular, $C(F) \leq 2T/\log_2 T$ for $a > b$ and $C(F) \leq T/\log_2 T$ for $b = T^{o(1)}$).

The complexity of networks of cyclic commutators is closely related to the complexity of contact networks for periodical Boolean functions. Let $F : Z \to \{0,1\}$ be a periodical function and let $f_i : \{0,1\}^i \to \{0,1\}$ be defined by the expression $f_i(x_1, ..., x_i) = F(x_1 + ... + x_i)$ (here $+$ denotes integer addition). Let $C(f_i)$ be the complexity of realization of f_i by contact networks. Then the relation $C(f_i) = 2iC(F) + O(1)$ holds.

1. *Problem 1:* Can the bounds b) and c) be made asymptotically equal?

2. *Problem 2:* Does the periodical function F with the property $C(\overline{F}) > 2C(F)$ exist? (I propose to prove that $C(\overline{F_i})/C(F_i) \to \infty$ for the following F_i. Let p_i be the i–th prime number. Let $F_i(x) = 1$ iff $0 \leq (x \ mod\ p_i) = (x \ mod\ p_{i+1}) < p_i$ or $p_i \leq (x \ mod\ p_{i+1}) < p_{i+1}$.

Constructions of Feebly–One–Way Families of Permutations

Alain Hiltgen
ETH, Zürich

The unrestricted circuit complexity $C()$ over the computable basis of all logic two input/output gates is considered. It is proved that certain explicitly defined families of permutations $\{f_n\}$ are feebly–one–way of order 2, i.e., the functions f_n satisfy the property that for increasing n, $C(f_n^{-1})$ approaches $2C(f_n)$ while $C(f_n)$ tends to infinity. Both these functions and their corresponding complexities are derived by a method that exploits certain graphs called $(n-1,s)$–stars.

Some Remarks on Read k–times Switching Networks

Stasys Jukna
University of Vilnius, and Dortmund Univeristy

A syntactic read k–times switching network has the restriction that no variable occurs more than k times on any path (whether or not consistent) of the network. In the case of branching programs (the deterministic version of switching networks) Okolnishnikova (1991) exibited a Boolean function requiring read k–times branching programs of size $exp(\Omega(n^{\frac{1-\epsilon}{2}}))$ for $k = \epsilon\frac{ln\, n}{lnln\, n}$. At the same time Borodin, Razborow and Smolensky (1991) exibited a function f_n such that both f_n and $\neg f_n$ require syntactic read–k times switching networks of size $exp(\Omega(\frac{n}{4^k k^3}))$.

Here we slightly extend these results by exhibiting an explicit Boolean function f_n requiring syntactic read–k times switching networks of size $exp(\Omega(\frac{\sqrt{n}}{k^3}))$ and such that $\neg f_n$ has syntactic read–once network of size $O(n^2)$.

Next we introduce non–syntactic read–k times networks in which only consistent paths are required to test each variable at most k times. We exhibit a function f_n which has a non–syntactic read–once network of size $O(n)$ but requires syntactic read–once switching networks of size $exp(\Omega(\sqrt{n}))$.

Computational Complexity of (XOR, AND) Counting Problem

Marek Karpinski
Universität Bonn

We present some new techniques for proving lower and upper bounds on the Boolean (XOR,AND)-Counting Problem, both for the exact and approximate counting schemes.

We display some surprising hardness results for the restricted classes of this problem, and give efficient randomized approximation algorithms for them. We discuss also the problem of derandomization of these algorithms, and some applications for approximating constant depth Boolean circuits.

On Some Complexity Measures for Boolean Functions

Otkay Kassim–Zade
Keldysh Institute Moscow

Let S be a circuit of functional elements over a finite complete basis $\mathcal{A}$ and let S realize a Boolean function f of n variables. If values $\tilde{\alpha} = (\alpha_1, ..., \alpha_n) \in \{0,1\}^n$ are put to the inputs of S, then each element e of S must output either 0 or 1. We denote by $E(S, \tilde{\alpha})$ the overall number of 1's on the elements outputs and call this number *the circuit power* of S at the input $\tilde{\alpha}$. We will consider two complexity measures for circuits, *maximal circuit power* $E^{max}(S) = max_{\tilde{\alpha}} E(S, \tilde{\alpha})$, and *expected circuit power* $\overline{E}(S) = \frac{1}{2^n} \sum_{\tilde{\alpha}} E(S, \tilde{\alpha})$. Both measures have very natural interpretation in terms of heat ratiation and energy consumption, if we use the general setting of circuits of functional elements to model real electronics. Note that both complexity measures have their duals analogues for which instead of 1's at elements outputs 0's are counted. Fortunately enough, dual problems could be solved by duality principle. For any finite complete basis $\mathcal{A}$ and for any Boolean function f we put $E^{max}_{\mathcal{A}}(f) = minE^{max}(s)$, and $\overline{E}_{\mathcal{A}}(f) = min\overline{E}(S)$, both minima extend to all possible circuits S over $\mathcal{A}$ realizing f.

We will consider circuit power complexity of two individual functions, $K_n = x_1 \& ... \& x_n$ and $D_n = x_1 \vee ... \vee x_n$, and one individual basis $\mathcal{A}_0 = \{\&, \vee, -\}$. We show for this basis $E^{max}_{\mathcal{A}_0}(K_n) \sim n/2$, when $n \to \infty$, while $\overline{E}_{\mathcal{A}_0}(K_n) = 1/2 - \frac{1}{2^n}$ for all $n \geq 1$. Also $E^{max}_{\mathcal{A}_0}(D_n) \asymp \sqrt{n}$, and $3(1/2 - \frac{1}{2^n}) \leq \overline{E}_{\mathcal{A}_0}(D_n) \leq 5 \log^*_2 n$, where $\log^* n$ denotes well–known superlogarithm function to base 2. For slightly more general case of expected circuit power with respect to probability $p \in [0,1]$ of 1 on circuit input (all inputs beeing set to 1 and 0 independently at random with probability p and $1-p$ respectively). We show that for all $p \in [0, (\sqrt{5}-1)/2]$ $\overline{E}^p_{\mathcal{A}_0}(K_n) = \frac{p^2}{1-p}(1 - p^{n-1})$, but for the values of p closer to 1 , if $n \geq 3$, better realizations of K_n do exist, for which $\overline{E}^p(K_n) < \frac{p^2}{1-p}(1 - p^{n-1})$ (it is certainly so, if $p \geq 0.93$).

Depth and Delay in a Network

Valeri Khrapchenko
Keldysh Institute Moscow

It is a widespread point of view that the delay of a combinatorial logical network is equal to its depth, i.e., to the maximum of the delays of its paths from input to output. It is clear that for any combinatorial network the delay is no larger than the

depth. But it would be a mistake to assume that it cannot be smaller. This is trivial for redundant networks. On the base of the new definition of delay that corresponds to its physical meaning, it is shown that even a minimal network can have delay less than depth. Moreover, it is proved that for every natural l there exists a Boolean function f_l such that an arbitrary minimal network for f_l has delay $l+8$ and depth $2l+8$.

Some Aspects on Bounded Depth Circuits over Symmetric Gates

Matthias Krause
Universität Dortmund

Realizing Boolean functions by constant depth circuits over the symmetric gate operations AND, OR, MOD_m, m integers, and over Threshold functions has become an extensively studied field in Theoretical Computer Science. Let us denote by $\tilde{AC}_0$, $\tilde{AC}_0[m]$, m integer, $\tilde{TC}_0$ the sets of languages computable by constant depth and quasipolynomial, i.e., size $\exp(\log^{O(1)})$, circuits over the basis AND, OR, NOT, the basis AND, OR, MOD_m, over Threshold operations, respectively. Further let $\tilde{AC}_{0,k}[m]$ and $\tilde{TC}_{0,k}$ denote the class of languages induce by depth k quasipolynomial size circuits over the corresponding basis.

Using a matrix method approach to the complexity of depth two circuits over symmetric gates we are able to establish some new results concerning relations between the above defined complexity classes. In particular, we are able to prove that for all integers m $\tilde{AC}_0$ is not a subset of $\tilde{AC}_{0,2}[m]$ and $\tilde{AC}_0[m]$ is not a subset of $\tilde{TC}_{0,2}$. These results give counterparts to the important lower bound results of *Yao* (1985), *Razborov* (1987), *Smolensky* (1987) that for all integers m $\tilde{AC}_0$ is a proper subset of $\tilde{AC}_0[m]$ and for all prime powers r $\tilde{AC}_0[r]$ is a proper subset of $\tilde{TC}_0$ and to the important upper bound result of *Yao* (1990) that for all integers m $\tilde{AC}_0[m]$ is a subset of $\tilde{TC}_{0,3}$.

Our technique yields the only known exponential lower bound on the size of depth two MOD_m–circuits for arbitrarily fixed integers m and a new lower bound method for proving exponential lower bounds for Threshold circuits of depth 2.

Complexity of Boolean Functions on PRAMs

Miroslaw Kutylowski
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(report on joint work with F. Fich, R. Impagliazzo, B. Kapron, V. King, K. Loryś, M. Kowaluk and P. Ragde)

The ROBUST PRAM is a concurrent-read concurrent-write (CRCW) parallel random access machine in which any value might appear in a memory cell as a result of a write conflict. We address the question of whether a PRAM with such a weak

form of write conflict resolution can compute functions faster than the concurrent-read exclusive-write (CREW) PRAM.

We prove a lower bound on the time required by the ROBUST PRAM to compute Boolean functions in terms of the number of different values each memory cell of the PRAM can contain and the degree of the function when expressed as a polynomial over a finite field. In the case of 1-bit memory cells, our lower bound for the problem of computing the OR of n Boolean variables exactly matches Cook, Dwork, and Reischuk's upper bound on the CREW PRAM.

On the other hand, if we assume that the input strings may contain at most k ones, then it is possible to compute any Boolean function of n arguments in time $O(\log k)$ on a ROBUST PRAM using n processors. The algorithm presented uses a new kind of hashing technique.

Separating the Lower Levels of the Sublogarithmic Space Hierarchy

Maciej Liskiewicz
University of Wroclaw and TH Darmstadt

For $S(n) \geq \log n$ it is known that the complexity classes $NSPACE(S)$ are closed under complementation. Furthermore, the corresponding alternating space hierarchy collapses to the first level. Till now, it is an open problem if these results hold for space complexity bounds between $\log \log n$ and $\log n$, too. We give some partial answer to this question. We show that for each S between $\log \log n$ and $\log n$, $\Sigma_2 SPACE(S)$ and $\Sigma_3 SPACE(S)$ are not closed under complement. This implies the hierarchy

$$\Sigma_1 SPACE(S) \subset \Sigma_2 SPACE(S) \subset \Sigma_3 SPACE(S) \subset \Sigma_4 SPACE(S)$$

(joint work with Rüdiger Reischuk, TH Darmstadt)

On the Complexity and Structure of Contact Circuits of Different Types Realizing Some Specific Boolean Function

Sergej Loshkin
Lomonossow University Moscow

The new general methods for obtaining the linear lower bounds for the contact circuits (abbr. CC) complexity are described. Using this methods new non–trivial examples of optimal CC of different types realizing some specific Boolean functions (BF) are given. The optimal CC for monotone symmetrical BF with threshold 2 and the optimal p–stuck at 0 CC for parity BF are among them. The first structural descriptions of all optimal CC for part of these examples have been done. In particular

such descriptions have been obtained:

(1) for 1–stuck at 0 CC realizing parity BF,

(2) for CC realizing prime symmetrical BF with working number 1,

(3) for CC realizing parity BF.

In the first of these cases there exists only one optimal CC while in the other two cases the number of non–isomorphic optimal CC is the linear function depending on the number of variables. There are examples when this number is more than some exponential function depending on number of variables.

On the Realization Complexity of Boolean Operators Degree

Oleg B. Lupanow
Lomonossow University Moscow

Let $\mathcal{M} = \{\sigma_1, ..., \sigma_m\}$ be a set of binary strings of length n, let $\mathcal{S}_\mathcal{M}$ be the set of all one–to–one mappings $\mathcal{M} \leftrightarrow \mathcal{M}$ (Boolean (n,n)–operators). For any F from $\mathcal{S}_\mathcal{M}$ let $\mathcal{A}_F(\tilde{x}, \tilde{y})$ be the following function:

$$\mathcal{A}_F(\tilde{\sigma}, \tilde{\tau}) = \underbrace{F(F(...F(\tilde{\sigma})...)}_{|\tilde{\tau}| \text{ times}}$$

($|\tilde{\tau}|$ denotes the number, the binary notation of which is $\tilde{\tau}$.)

Let us consider all possible continuations of F and $\mathcal{A}_F$ of the outside of $\mathcal{M}$. The complexity of a function f is defined to be equal to the minimal number of elements which is sufficient for realization of f by a circuit of functional elements over the basis $\{\&, \vee, \neg\}$. Let $L^*(F)$ denote the complexity of the simplest continuation of $\mathcal{A}_F$ to the outside of $\mathcal{M}$ and let $L^*(n, M) = max_{F \in \mathcal{S}_\mathcal{M}} L^*(F)$.

Theorem. *If* $\frac{M}{\log_2 n} \to \infty$ *then* $L^*(n, M) \sim \frac{Mn}{\log_2(Mn)}$.

Moreover, for any function $F \in \mathcal{S}_\mathcal{M}$ there exists a circuit S for $\mathcal{A}_F$ such that

$$L(S) \leq \frac{Mn}{\log_2(Mn)}, \text{ and} T(S) = O(\log M).$$

($T(S)$ denotes the depth of S.)
The proof of the theorem is based on the principle of local coding of the author, along with certain version of the result of D. Uhlig on the simultaneous realization of a function on several strings (mass–production), some modification of certain theorems on the complexity of partial functions (E.I. Nechiporuk, N.P. Redkin, A.E. Andreev) and the result of V.M. Khrapchenko on relationship between formula complexity and depth; there is also a certain amount of "programming" in the terms of circuits.

Which Boolean Functions Can be Computed on Analog Neural Nets

Wolfgang Maass
Universität Graz

We consider circuits of constant depth and polynomial size over the following basis $f(\sum_{i=1}^{m} w_i x_i - t)$ for various functions f. The weights w_i and the threshold t may be arbitrary parameters. For the case where f is the step–function

$$H(x) = \begin{cases} 1 & : \quad x \geq 0 \\ 0 & : \quad otherwise \end{cases}$$

these circuits are well–known (under the name of threshold circuits), and the class of Booelan functions which they compute is denoted TC^0. However, it has remained open, which Boolean functions can be computed by such circuits if f is a continous function, and the weights w_i and the threshold t of each gate are arbitrary numbers. We show for the case where f is piecewise polynomial (with polynomials of bounded degree, and only polynomially many "pieces") that these circuits compute exactly the Boolean functions that lie in TC^0. We also prove the first nontrivial upper bound for the Vapnik–Chervonenkis dimension (VC-dimension) of neural networks with continious gate functions and arbitrary real weights. This parameter is of great importance for the analysis of learning on neural networks.

On the side we show that for neural networks with gates H (see above) the well-known upper bound $O(e \log e)$ for the VC–dimension of a neural network with e edges (due to Baum and Haussler) is in fact asymptotically optimal. This follows from classical circuit construction due to Nechiporuk and Lupanow.

About Irredundant Tests for Linear Tables

Hickar Madatjan
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In solving practical problems of pattern recognition one uses so–called test algorithms. The most important element of these algorithms is the construction of irredundancy tests for tables. Subset of the columns H of the table T is called test in T, if for any rows, belonging to T in H we can find column which in intersection which this row gives 1. A test is said to be irredundant if any its proper subsets is not a test. Our problem is to construct all irredundant tests in T.

Let's consider so–called full table Π_k there are all different columns of k–length in Π_k, except zero, that is $2^k - 1$ columns. If all irredundant tests of full table Π_k are constructed it is easy to get irredundant tests of any table which k rows.

Let $L(\Pi_k)$ be the number of irredundant tests of Π_k table, then

$$L(\Pi_k) = \sum_{t=1}^{k} \sum_{j=t}^{k} \frac{(2^t - t - 1)^{k-j}}{t!} \binom{k}{k-j} \sum_{i=0}^{t} (-1)^t \binom{t}{i} (t-1)^i$$

Modified Ranks of Tensors and Communication Networks

Pavel Pudlak
Academy of Sciences Prag

(joint work with V. Rödl, Emry Univ., Atlanta)
We consider a concept of the rank of a tensor (rigidity rank) related to the *contact rank* introduced by Razborow. We prove some upper and lower bounds for the tensor of multiplication of two polynomials $mod(x^n - 1)$. This tensor appears also in connection with circuits computing all cyclic permutations. Our upper bound disproves a conjecture of Razborow and sets some limits on the way in which lower bounds to such circuits can be proved. We also give some bounds on the size of circuits of depth 2 computing cyclic permutations. As a corollary we get: Any Boolean circuit of depth 2 for multiplication has size $\Omega(n(\log n)^{3/2})$. (We allow arbitrary unbounded fan–in Boolean gates; the size is the number of edges.)

Realibility and Diagnostic of Schemes of Functional Elements

Nikolai Redkin
Lomonossow University Moscow

The possibility of constructing easily testable schemes of functional elements in the basis $\{\&, \vee, \neg\}$ in the presence of one type constant errors on the outputs of elements is studied. It is established constructively that any Boolean function of n variables can be realized by a scheme permitting a single diagnostic test whose length does not exceed $2n + 1$.
Further the schemes of functional elements over the basis $\{\overline{x} \vee \overline{y}\}$ are considered. It is supposed that each correct element of the scheme realizes the Boolean function $\overline{x} \vee \overline{y}$ and each error element realizes Boolean constant $p \in \{0, 1\}$. The functional elements of the scheme pass into the error states independently one from the other and each element passes into error state with probability γ ($\gamma \leq 1/50$). For "almost all" Boolean functions the following fact is established. In case $\gamma \to 0$ a Boolean function can be realized by a certain scheme S for which a highest possible probability of the error on the output asymptotically is equal to $(2 - p)\gamma$. Such scheme S is asymptotically most reliable.
(joint work with M.A. Alehina, Moscow)

Can Unbounded Fan–in Circuits be Made Fault–tolerant?

Rüdiger Reischuk
Technische Hochschule Darmstadt

We consider circuits of small depth built form either AND and OR-gates or from threshold gates, with unbounded fan–in that may have faults. More specific, a wire w may switch a Boolean value that runs from its orign to its destination with a certain probability ϵ_w, independently of other wires. In the weaker model the error probabilities ϵ_w are all identical to a fixed known value $\epsilon < 1/2$, while in the stronger model each ϵ_w is an unknown value in the interval $[0, \epsilon]$. A circuit C computes a function f in a fault-tolerant way if for some $\delta < 1/2$ for all inputs X the probability that $C(X) \neq f(X)$ is bounded by δ.

For AND-OR circuits it is shown that any unbounded fan–in circuit of depth d can be simulated by a circuit of the same depth and fan–in bounded by $O(d \log d)$ with an arbitrarily small increase in the reliability parameter δ. This implies even for the weak error model that fault tolerant constant depth circuits can only compute functions that depend on a constant number of inputs.

Thus fault tolerant computation is not possible in constant depth with AND-OR gates.

This impossibility result can also be shown for threshold circuits in the strong error model assuming that an adversary can select the ϵ_w. It suffices to set ϵ_w either to the minimal value 0 or to the maximal value ϵ. On the other hand, for the weak error model we prove that an arbitrary constant depth circuit can be made fault-tolerant without increasing its depth or the number of gates, only the fan–in gets enlarged moderately.

(joint work with B.Schmelz)

On Searching of Maximal Uper Zeros of Monotone Boolean Functions

Alexander Saposhenko
Lomonossow University Moscow

An upper zero of monotone Booelean function f (abbr. MBF) is defined as a vertex v of the n–cube with $f(v) = 0$ and $f(u) = 1$ for all u such that $u > v$. A maximal (upper) zero of a MBF f is defined as a zero of f with the maximal sum of coordinates. Two problems are discussed. *The first:* To find some maximal zero of an arbitrary MBF f which specified by an oracle O_f with minimal number of appeals to the oracle O_f. *The second:* To find all upper zeros of an arbitrary MBF f (i.e. to decipher f with minimal

number of appeals to oracle O_f). The first problem was solved by Katerinochkina who proves that in the worst case it takes $\binom{n}{\lfloor n/2 \rfloor} + 1$ of appeals to the oracle. The scond problem was solved by V.K Korobkov and G. Hansel. They showed that it takes $\binom{n}{\lfloor n/2 \rfloor} + \binom{n}{\lfloor n/2 \rfloor + 1}$ of appeals to the oracle. The talk is devoted to solve these problems for almost all MBF. We show that everywhere it takes no more than $\omega_n 2^{n/2}$ of appeals to the oracle to find a maximal zero of MBF f, where ω_n is an arbitrary function with $\lim_{n \to \infty} \omega_n = \infty$. Note that increasing function ω_n cannot be replaced by a constant. Further, almost everywhere it takes $\binom{n}{\lfloor n/2 \rfloor}(1 + O(n2^{-n/2}))$ of appeals to the oracle for deciphering MBF. Some extentions on the monotone functions on partially ordered sets are considered.

Parallelizing Graph Embedding Heuristics for VLSI*

John E. Savage
Brown University, Providence

Graph embedding is a key step in the realization of VLSI layouts from circuit descriptions. It is known as the placement problem, the first step in assigning rough positions to circuit elements on a VLSI chip. In this talk we describe three variants of the graph embedding problem, graph partitioning, in which the destination graph has two vertices and one edge, and grid and hypercube embedding.

These problems are NP-complete. Consequently, many good serial heuristics have been invented for them. Now that parallel computers are available, parallel algorithms for them are needed. We sketch proofs that some of the best serial heuristics are P-complete and therefore not likely to be parallelizable in the worst case. We also present the Mob Heuristic, a highly parallelizable heuristic that we introduced recently and which we have implemented on the Connection Machine. It gives results as good as the best serial heuristics but handles problems up to 1,000 times larger.

*This is joint work with Markus Wloka of Motorola.

Complexity Problems with Algorithms for Determining the Probability of a Boolean Function Being 1

Winfried Schneeweiss
FernUniversität Hagen

A number of problems are discussed which practioners encounter when trying to determine the probability of a Boolean function being 1, a task of some importance e.g. in reliability theory, where a Boolean function – typically that of a fault tree – describes the system's redundancy structure. Details:

a) The inclusion/exclusion principle is well known for its exponential complexity,

which makes is useless except for small "professor" examples.

b) With the J. Abraham approach (IEEE-Trans. R, 1979) characterized by $A \vee B \vee C = A + \overline{A}B + \overline{AB}C$, the future will show more and more its limitations due to non–parallelizability.

c) With the Shannon decomposition $\psi = x_i\psi_{/x_i=1} + \overline{x_i}\psi_{/x_i=0}$ of the DNF $\psi = x_i\psi_i' \vee \overline{x_i}\psi_i'' \vee \psi_i'''$ the following question arises: Is it wise to choose i such that a long

   - ψ_i' results, since many terms of $\psi_{/x_i=1}$ are by 1 literal shorter than the corresponding terms of ψ,
   - ψ_i'' results, since then $\psi_{/x_i=0}$ becomes short,
   - ψ_i''' results, since then there will be more possibilities for absorptions of the type $A \vee AB = A$ in order to shorten $\psi_{/x_i=0}$ and / or $\psi_{x_i=1}$?

d) The similarity and subtle differences between the Shannon decomposition and the decision tree approach were mentioned.

Analog and Digital Computations for Unbounded Fan–in Circuits

Georg Schnitger
Pennstate University, Pennsylvania

We compare different gate functions in terms of the approximation power of their circuits (when computing real–valued functions). Evaluation criteria are circuit size s, circuit depth d and the approximation error $e(s,d)$. We consider two different error models, namely $e(s,d) = 2^{-s}$ and $e(s,d) = s^{-d}$. Our goal is to determine those gate functions that are equivalent to the standard signoid $\sigma(x) = 1/(1+e^{-x})$ under these two error models.

Equivalent functions (for $e(s,d) = 2^{-s}$) include (non–polynomial) rational functions, (non–polynomial) roots and splines of proper degree. Polynomials and the sine function are inequivalent.

The last two become equivalent for the more relaxed error $e(s,d) = s^{-d}$.

Finally, Boolean functions are considered. We show that the standard sigmoid computes a certain language family in constant size, wheras the binary threshold requires size $\Omega(\log_2 n)$.

(This is joint work with Bhaskar Das Gupta.)

Proof Checking and Intractability of Approximation Problems

Mario Szegedy
AT & T Bell Labs, Murray Hill

The class $PCP(f(n), g(n))$ consists of all languages L for which there exists a polynomial–time probabilistic oracle machine that uses $O(f(n))$ random bits, queries $O(g(n))$ bits of its oracle and behaves as follows: If $x \in L$ then there exists an oracle y such that the machine accepts for all random choices but if $x \in L$ then for every oracle y the machine rejects with high probability. Arora and Safra recently characterized $\mathcal{NP}$ as $PCP(\log n, \log\log n)$. We improve on their result by showing that $\mathcal{NP} = PCP(\log n, 1)$. Our result has the following consequences:

1. *MAX SNP*-hard problems (e.g. metric TSP, MAX-SAT, MAX-CUT) do not have polynomial time approximation schemes unless $\mathcal{P} = \mathcal{NP}$.
2. For some $c > 0$ the size of the maximal clique in a graph cannot be approximated within a factor of n^c unless $\mathcal{P} = \mathcal{NP}$.

(This is joint work with S. Arora, C. Lund, R. Motwani and M. Sudan)

A Lower Bound for Unrestricted Threshold Circuits and Related Problems

Gyoergy Turan
University of Szeged, Ungarn

We consider the computational power of threshold circuits without any restriction on the depth, or the size of the weights. It is shown that every circuit computing the mod 2 inner product of two n–bit vectors contains at least $n/2$ gates. Considering the related model of linear decision trees, it is shown that the depth of every tree computing the same function is at least $n/2$. If randomization is also allowed, the lower bound obtained is $0.21n$. If furthermore we allow an error at most $\epsilon < 1/4$ for every input vector, then a lower bound of $0.157(1 - 4\epsilon)(n/2 + \log(1 - 4\epsilon))$ can be proved.

Circuits Simulataneously Computing Values of Monotone Boolean Functions

Dietmar Uhlig
Hochschule für Technik und Wirtschaft Mittweida

Let us assume we have simulataneously to compute the values $f(\underline{a}^1, ..., \underline{a}^r)$ of a Boolean Function f on arbitrary Boolean vectors $\underline{a}^1, ..., \underline{a}^r$. Then we can do it by r

copies of a circuit computing f. But in most cases it can be done more efficiently (with a smaller complexity) by a network having nr inputs and r outputs (as already shown for the hardest Boolean functions in 1976). We prove that for $r = 2^{o(\frac{n}{\log n})}$ an arbitrary monotone Boolean function can be computed simultaneously on r Boolean vectors with $\sqrt{2/\pi}\frac{2^n}{n^{3/2}}(1+\alpha_n)$ binary elements, where $\alpha_n \to 0$ if $n \to \infty$.

Graph Driven BDDs - a New Data Structure for Boolean Functions

Ingo Wegener
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Graph driven BDD's - a new data structure for Boolean functions (ordered) binary decision diagrams (OBDD's) are used as data structure for Boolean functions in the logical synthesis process, for verification and test pattern generation, and as part of CAD tools. For several important functions like arithmetical and logical units with quite different functions, the indirect storage access function or the hidden weighted bit function OBDD's have exponential size for any ordering of variables. Since an ordering of variables may be stored as a list, ordered binary decision diagrams may be called also list driven BDD's. Two new generalized models of graph driven BDD's are presented. The above mentioned and many other functions can be represented in small polynomial size in this model and the usual operations on OBDD's can be performed efficiently also for graph driven BDD's.

Taking Discrete Roots in Parallel

Thomas Zeugmann
Technische Hochschule Darmstadt

Highly parallel algorithms computing the modular inverse, discrete roots, or a large polynomial power modulo a number that has only small prime factors are elaborated. The designed uniform families of Boolean circuits simultaneously achieve depth $O(\log n)$ and size $O(n^{O(1)})$ for P-uniformity and depth $O(\log n \log\log n)$ and polynomial size for log-space uniformity. The results are achieved in using high order Newton-like iteration techniques that are based on the introduction of generic root coefficients.

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