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Algorithms and Complexity for Continuous Problems

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Dagstuhl-Seminar

Algorithms and Complexity for Continuous Problems

Organized by:

Erich Novak (Universität Erlangen-Nürnberg)
Steve Smale (University of California, Berkeley)
Joseph F. Traub (Columbia University, New York)

October 12–16, 1991

Overview

The Dagstuhl-Seminar on Algorithms and Complexity of Continuous Problems was attended by 39 computer scientists and mathematicians from 12 countries. We express our gratitude to the staff of Schloß Dagstuhl for providing a great atmosphere.
Our Seminar was devoted to the study of continuous problems such as computation over the reals, decision problems, problems with noisy data, numerical integration, optimal recovery, n-widths, partial differential equations, integral equations, zero finding, linear programming and image reconstruction.
This list contains problems on finite dimensional as well as on infinite dimensional spaces. In the finite dimensional case the information is usually complete and the computational cost is crucial. In the infinite dimensional case the information is usually partial and in most of the research done so far the information cost is crucial.
This Seminar-Report contains the abstracts of 32 lectures in alphabetical order. We also had a plenary session on new research directions and open questions.

During the Dagstuhl-Seminar we had also a pleasure to celebrate the sixtieth birthday of Joseph F. Traub who is one of the founders of our field which is now called continuous computational complexity.
On Wednesday afternoon, we had a traditional walk and we stopped in a local café for coffee and cakes. Two beautiful birthday cakes with candles were served with best wishes to Joe.
We gathered to express our warm feelings for Joe on Wednesday evening. Zvi Galil, Stefan Heinrich, Roberto Tempo, Niko Vakhania and Henryk Woźniakowski spoke about the role of Joe Traub as our teacher, collaborator, dear friend and organizer of Computer Science in the United States.
We had then a pleasure to listen to a beautiful guitar concert of Kerstin Eisenbarth. After the concert, Joe received many presents. And finally there was a wine and cheese party with many warm wishes for Joe to continue his vigorous activities for many years to come.

Henryk Woźniakowski

Abstracts

Speeding up the Construction of Convex Hulls by Exploitation of Additional Information

Karl Heinz Borgwardt
Universität Augsburg

Our task is to construct the convex hull of m (randomly generated) given points $a_1, \ldots, a_m$. We possess the additional information that the points are distributed on $\mathbb{R}^n$ independently, identically, and symmetrically under rotations. Then almost surely our configuration is nondegenerate and this implies that every facet of $\mathrm{CH}(a_1, \ldots, a_m)$ is a convex hull of n of these points, that each such facet is bounded by n so-called side-simplices (convex hulls of $n-1$ such points), and that every side-simplex is adjacent to exactly two facets. Our strategy of exploring all facets is now to walk around the surface of $\mathrm{CH}(a_1, \ldots, a_m)$ from facet to facet in a Simplex-Method-like manner. Each time we discover a side-simplex, we store it in a file. As soon as such a side-simplex is stored twice, both neighbours are known and the side-simplex is saturated. Only nonsaturated side-simplices are admitted for pivot steps from facet to facet (only these can be travesed). If our walk gets stuck, we jump back to a facet which still has nonsaturated side-simplices. Complexity considerations show that as many iterations as facets must be performed. In each iteration one has to consider all points with euclidean length greater than the height of the facet to be constructed as candidates to enter the facet-basis. Also the effort for checking the file lies in that order. Now for a parametrized family of distributions on the unit ball we are able to give the precise order of growth for the expected value of the total effort in this method. It is shown that even expected linearity can be achieved as long as the radii $< 1/2$ have more weight than the radii $> 1/2$. In the extremal case (uniform distribution on the unit sphere) the expected effort is $O(m^2)$.

Information Constraints in Image Reconstruction

Terry Boult
Columbia University, New York

This talk looks at the use of integral information in the problem of image reconstruction. We discuss models wherein a camera returns an image containing pixels, each of which are the integral of the underlying function over a small patch. We discuss different mathematical representations and assumptions about the underlying class of functions, rejecting the model of Nyquist sampled band-limited functions. We introduce the idea of image-consistent reconstruction, i.e., algorithms which are interpolatory in the sense of information-based complexity. Error measures for image quality are introduced and used to compare the new algorithms with such standard algorithms as linear interpolation, cubic convolution, and cubic splines. The new algorithms have almost the same complexity as cubic convolution but perform at the quality level of global cubic splines. We end with a short discussion of the use of optimal algorithms to help refine mathematical models.

'Universal' Approximations of Functionals in the Space of Periodic Functions

Helmut Braß
Universität Braunschweig

Let I be a fixed functional on the space C^* of 2π-periodic continuous real valued functions, let $x_1, \ldots, x_n \in [0, 2\pi[$ be given points. We are interested in an estimation of $I(f)$ based on the function values $f(x_1), \ldots, f(x_n)$ as an information on f. If we would have the further information $f \in K$ with a specified $K \subset C^*$, then we could define the (worst case) error $\rho(Q, K)$ of a linear algorithm Q in K, the optimal error $\rho^{\text{opt}}(K)$, and the quality $\text{qual}(Q, K) = \rho(Q, K)/\rho^{\text{opt}}(K)$. The next step is to eliminate (partially) the dependence on K by considering only algorithms Q with a good quality for all $K \in W$, where W denotes a large set of classes.
Some special choices of W are studied. We mention as an example of the results: Let $W = \{K_s : s = 1, 2, \ldots\}$, $K_s = \{f \in C^* : \|f^{(s)}\|_\infty \leq 1\}$, then there is one and only one rule Q such that $\lim_{s\to\infty} \text{qual}(Q, K_s) = 1$ holds.

On Randomized Semi-Algebraic Test Complexity

Peter Bürgisser
Universität Bonn
(joint work with Marek Karpinski and Thomas Lickteig)

We investigate the impact of randomization on the complexity of deciding membership in a (semi-)algebraic subset $X \subset \mathbb{R}$. Examples are exhibited where allowing for a certain error probability ε in the answer of the algorithms the complexity of decision problems decreases. A randomized $(\Omega^K, \{=, \leq\})$-decison tree ($K \subset \mathbb{R}$ a subfield) over m will be defined as a pair (T, μ), where μ is a Borel probability measure on some $\mathbb{R}^n$ and T is a $(\Omega^K, \{=, \leq\})$-decison tree over $m + n$. We prove a general lower bound on the randomized decision complexity for testing membership in a irreducible algebraic subset $X \subset \mathbb{R}^m$ and apply it to K-generic complete intersections of polynomials of the same degree. We also give applications to nongeneric cases, such as graphs of elementary symmetric functions, $\text{SL}(m, \mathbb{R})$, and determinant varieties, extending results of Lickteig.

Parallel Complexity Classes over the Reals

Felipe Cucker
Universidad Politécnica de Cataluna

We introduce a uniformity condition for families of algebraic circuits that allow us to define the classes $\mathrm{NC}^k_{\mathbb{R}}$ ($k \geq 1$) in a natural way. Then we show the following result. If $t, t' : \mathbb{N} \to \mathbb{N}$ such that t' is time constructible and $\lim t/t' = 0$, then there exists a set $L \subseteq \mathbb{R}^\infty$ with $L \in \mathrm{DTIME}(t')$ but $L \notin \mathrm{PARALLEL{-}TIME}(t)$. We deduce that the inclusions $\mathrm{NP}_{\mathbb{R}} \subset \mathrm{EXP}_{\mathbb{R}}$ and $\mathrm{NC}_{\mathbb{R}} \subset \mathrm{P}_{\mathbb{R}}$ are strict. Finally we show a lower bound for PARALLEL–TIME (Montaña-Pardo). Let $L \subseteq \mathbb{R}^\infty$ and $L_n = \{x \in L \mid \text{size of } x \text{ is } n\}$. If the circuit family $\{b_n\}_{n \in \mathbb{N}}$ recongnizes L, then

$$\mathrm{depth}(b_n) = \Omega\left((\log(\#\mathrm{conn.comp.}(L_n))/n)^{1/2}\right).$$

Multilevel Techniques for Operator Equations

Wolfgang Dahmen
Technische Hochschule Aachen

We describe some basic ideas of using multilevel techniques for the approximate solution of a class of pseudodifferential equations covering elliptic partial differential equations, certain integral equations, as well as operators arising in boundary element methods. This concerns partly joint work with A. Kunoth, S. Prößdorf, and R. Schneider. The role of wavelet type expansions in this context is pointed out. We focus on two basic issues, namely preconditioning and compression of stiffness matrices in case those are, due to the nature of the underlying operator, not sparse. We indicate that these techniques allow us to solve the discritized problem in $O(N)$ (or $O(N(\log N)^\gamma)$) floating point operations, where N is the order of the linear system needed to approximate the exact solution within a desired tolerance.

Nonlinear Algorithms for Compression and Noise Removal in Image Processing

Ron DeVore
University of South Carolina

We consider digitized images represented by a wavelet expansion. We first consider how to choose n terms of this expansion to approximate the image in a given metric. We present a near optimal (optimal up to numerical constants) algorithm for this problem. Our solution of course depends on the metric and leads to different quantization strategies. We also classify the images which can be compressed well by this algorithm in terms of smoothness of the image in Besov spaces (the image should possess smoothness in an L_p space with typically $p < 1$). We also discuss nonlinear strategies for removing noise from pixel data. Such algorithms arise from extremal problems in Besov spaces and agree with those posed by Donoho and Johnstone. We estimate the expected error of this algorithm for a given class of functions with fixed smoothness in a Besov space. We then show that this algorithm is optimal in the class of all stable non-linear algorithms. This represents joint work with Bread Lucier.

Hunting Lions in the Desert or A Constant Time Optimal Parallel String Matching Algorithm

Zvi Galil
Columbia University, New York

Given a pattern string x, we describe a way to preprocess it and give a parallel algorithm that finds all occurences of x in any input text y in constant time using $O(|x| + |y|)$ processors on a CRCW (concurrent read, concurrent write) PRAM.

Complexity of Integral Equations: Computing Functionals of the Solution

Stefan Heinrich
Universität Kaiserslautern

The complexity of solving Fredholm integral equations of the second kind is studied. We consider two situations: computing an approximation to the full solution or computing the value of the solution at a given point. For standard information both problems have the same ε-complexities, while for arbitrary linear information the approximation of the value in a point can be accomplished much faster than the full solution. Concrete numerical algorithms are developed, and an open problem in n-widths is formulated, the solution of which would produce a matching lower bound for functional computation.

A General Theory of the $\delta \to$ o Limit in Probabilistic Complexity

Mark A. Kon
Boston University

In probabilistic analytic complexity we measure the complexity of solving a problem within error $\varepsilon > 0$, allowing a (small) set of problems (of probability $\delta > 0$) to have error greater than ε. Thus we seek to reduce the required complexity of a problem by requiring that it be solved to accuracy ε only with probability $1 - \delta$ instead of probability 1. In worst case complexity, we require that all problems be solved within error ε, and measure problem complexity under this requirement. An open question has been whether, as $\delta \to 0$, probabilistc ε-complexity approaches worst case ε-complexity. Specifically, if we encode our analytic problem as approximating a map S from F to G, with F and G Banach spaces, and assume a probability measure μ on some bounded, convex, balanced subset F_0 of F, what happens as $\delta \to 0$ to approximating S within ε. The answer is that if the space G is uniformly convex and the measure μ is non-vanishing, the $\delta \to 0$ limit of probabilistic complexity is indeed the same as worst case complexity. However, this is false if G being uniformly convex is not assumed, as is shown with an example where G is the Wiener space of Brownian paths. It is also shown that the theorem does not hold if μ is not non-vanishing.

On Passive and Active Algorithms of Recovery of Functions

Nikolay P. Korneichuk
Ukrainian Academy of Sciences

We consider the problem of recovery of the function $f(t)$ with the help of information about values of the funtion f in separate points $t_1, \ldots t_N$. We can produce the set of points all the same time (passive algorithm) or choose points t_k, $k = 1, \ldots, N$, successively, using information about $f(t_1), \ldots, f(t_{k-1})$ (active or adaptive algorithm). Let $H^\alpha = H^\alpha[a,b]$, $0 < \alpha \le 1$, be the class of functions, which satisfy Hölder's condition on $[a,b]$, and let H^α_m be the set of monotone functions from H^α. It is known that in the case of $\alpha = 1$ passive and active algorithms give the same order $O(N^{-1})$ of error in the metric C on H^1 and on H^1_m. In the case $0 < \alpha < 1$ the best order for passive algorithms on H^α and H^α_m and for active algorithms on H^α is $O(N^{-\alpha})$.
Main result: The algorithm of bisection guarantees for every function $f \in H^\alpha_m$, $0 < \alpha \le 1$, the following error

$$|f(a) - f(b)|/2 \cdot (1/\alpha - 1) \cdot N^{-1} \log_2 N + O(N^{-1})$$

in the metric C, and this order is optimal for all active algorithms.
We have a reason to suppose that the algorithm of bisection is optimal in the sense of exact constants.

Recovering Linear Operators from Inaccurate Data

Marek A. Kowalski
University of Warsaw
(joint work with Bolesław Z. Kacewicz)

The presentation deals with approximating linear operators from data contaminated with bounded noise. Let F and G be linear spaces (over $\mathbb{C}$ or $\mathbb{R}$) endowed with a seminorm $\|\cdot\|_F$ and a norm $\|\cdot\|_G$, respectively. Let $S : F \to G$ be a linear operator. We wish to approximate $S(f)$ for $f \in B = \{x \in F : \|x\|_F \leq 1\}$ assuming that f is unknown to us, and instead inaccurate information about f is given. Namely, for every $f \in B$ we have a vector $\vec{y} \in \mathbb{C}^n$ such that $|\vec{y} - N(f)| \leq \delta$, where $N : F \to \mathbb{C}^n$ is a linear operator (corresponding to exact information), $|\cdot|$ is a fixed norm on $\mathbb{C}^n$, and δ is a given nonnegative number (noise bound). Our goal is to compare the diameter of inaccurate information $D(S, N, \delta) = 2\sup\{\|S(f)\|_G : f \in B, \ |N(f)| \leq \delta\}$ to the diameter of exact information $D(S, N, 0)$. We present upper and lower bounds on $D(S, N, \delta)$ and indicate practical examples when $D(S, N, \delta) - D(S, N, 0)$ can be explicitly expressed in terms of δ.

n-Widths and Smoothness

Alex K. Kushpel
Ukrainian Academy of Sciences

The n-widths problem is very connected with the problem of optimal recovery, because n-widths (in the sense of Kolmogorov) often coincide with the respective Gelfand widths. Many equations have solutions with different kind of smoothness, for instance: finite, fractional, infinte, or analytical, but many methods can be effectively used only in finite smoothness situations. We study n-widths of convolution classes $K * U_p$, where $K \sim \sum_{k=1}^{\infty} \alpha_k \cos(kt - \beta\pi/2)$ and U_p is the unit ball in L_p $(1 \leq p \leq \infty)$. It is clear that if we take $\alpha_k = k^{-\alpha} \ln^{-\beta} k$, $\alpha > 0$, $\beta \geq 0$, then we can receive sets of functions with finite smoothness. If we take $\alpha_k = \exp(-\alpha k^r)$, $\alpha > 0$, $0 < r < 1$, we can receive sets of infinitely smooth functions. In the case $\alpha_k = \exp(-\alpha k^r)$, $\alpha \geq 1$, we can receive sets of analytical functions. The exact order of decreasing n-widths $d_n(K * U_p, L_q)$ $(n \to \infty)$ has been obtained for different kind of smoothness.

On Optimal Approximation of Wiener Paths

Peter Mathé
Institut für Angewandte Analysis und Stochastik, Berlin

This talk is devoted to the average approximation problem with respect to Wiener measures (in one dimension). It reports results by V. Maiorov who has recently computed the asymptotic in most cases and has given probabilistic estimates.
In the case of optimal linear approximation (allowing arbitrary linear information) it is shown that the original approximation problem can be reduced to a specific bilinear approximation problem, which is presented, and its solution is discussed. This way we get geometric insight on the optimal approximating methods, telling that standard information with equally spaced points is optimal (in order).

Real Number Models under Various Sets of Operations

Klaus Meer
Technische Hochschule Aachen

We consider the computational model of Blum, Shub, and Smale together with its generalization given by Mediggo: the computations are done over an abstract domain D; as allowed operations on D there are two finite sets O_1 (binary operations) and O_2 (unary operations). Finally, for a fixed subset $T \subseteq D$ one can decide "is $x \in D$ in T".
The following two specifications are examined: 1) $D = \mathbb{R}$, $O_1 = \{+, -, \cdot\}$, $O_2 = \{\cdot, x \mapsto \sin x\}$, $T = [0, \infty[$, and 2) $D = \mathbb{R}$, $O_1 = \{+, -\}$, $O_2 = \{\cdot\}$, $T = \{0\}$. For both a P $\neq$ NP result is proven.

Continuous Verification Problems

Erich Novak
Universität Erlangen-Nürnberg
(joint work with Henryk Woźniakowski)

We analyze the complexity of verifying whether a given element is within ε of the solution element. For problems with incomplete information and $\varepsilon > 0$ verification can be easier or harder than computation. In the worst case, the complexity of verification is often infinite, therefore we change the problem:
a) One can study the probabilistic setting where we allow a probability of failure $\leq \delta$. Here we present results of Woźniakowski for linear functionals and Gaussian measures.
b) We study a relaxed verification problem where we allow both answers if $\varepsilon < \|S(f) - g\| < (1 + \alpha) \cdot \varepsilon$. We give precise results for the case of diagonal operators $S : l_p \to l_p$. We study two cases. We prove that adaption helps a lot in the general case $g \in l_p$. If we only consider g from the solution set, i.e., $g \in S(l_p)$, then we obtain different results and adaption does not help in this case.

Average Case Complexity of Multivariate Integration for Smooth Functions

Spassimir Paskov
Columbia University, New York

We study the average case complexity of multivariate integration for the class of smooth functions equipped with the folded Wiener sheet measure. The complexity is derived by reducing this problem to multivariate integration in the worst case setting but for a different space of functions. Fully constructive optimal information and an optimal algorithm are presented. Next, fully constructive almost optimal information and an almost optimal algorithm are also presented which have some advantages for practical implementation.

The Complexity of Solving Integral Equations by Direct Methods

Sergei V. Pereverzev
Ukrainian Academy of Sciences

In a Hilbert space X we consider some class $[\mathfrak{H}, \Phi]$ of operator equations of the second kind

$$z = Hz + f, \tag{1}$$

where $H \in \mathfrak{H} \subset \mathrm{L}(X, X)$ and $f \in \Phi \subset X$. Following S. L. Sobolev we regard as direct methods those in which finding an approximate solution of (1) reduces to solving a finite system of linear algebraic equations. For fixed N we denote by $\mathfrak{D}_N$ the set of all direct methods in which we obtain a system of N equations. Since it is known that in general no fewer than N^2 arithmetic operations must be performed for the solution of a system of N equations, we assume that the computational complexity $\mathrm{comp}(D, [\mathfrak{H}, \Phi])$ of any direct method $D \in \mathfrak{D}_n$ is at least N^2, too. Our assumption is true for all known direct methods of solving operator equations from rather broad classes $[\mathfrak{H}, \Phi]$. Although the direct methods are very simple in the sense of their realization, it is possible that within the framework of our assumption these methods are not optimal in the sense of ε-complexity for some natural classes of equations (1).

Let $\Theta_N([\mathfrak{H}, \Phi])$ be the optimal accuracy of direct methods from $\mathfrak{D}_N$ on the class $[\mathfrak{H}, \Phi]$. We consider the following classes of equations (1). The class V_S^r of Volterra integral equations with free term $f \in L_2^r$ and kernels from the Sobolev space W_2^r. The class $V_W^{r,r}$ of Volterra integral equations with $f \in L_2^r$ and kernels from the space $W_2^{r,r}$ of functions having predominat mixed partial derivatives $\frac{\partial^{2r}}{\partial t^r \partial \tau^r}$. Moreover, let A_2^h be a normed space of periodic analytic functions which admit the analytic extension in the stripe $\{z : z = x + iy,\ |y| \leq h\}$ of the complex plane. We consider also the class F_h^A of Fredholm integral equations (1) such that $H, H^* \in \mathrm{L}(L_2, A_2^h)$ and $f \in A_2^h$. We have the following results

class $[\mathfrak{H}, \Phi]$	exact order Θ_N	$\mathrm{comp}(D, [\mathfrak{H}, \Phi])$	best order of lower bound of ε-complexity
V_S^r	N^{-r-1}	$\geq \varepsilon^{-2/(r+1)}$	$\varepsilon^{-1/r}$
$V_W^{r,r}$	N^{-r-1}	$\geq \varepsilon^{-2/(r+1)}$	$\varepsilon^{-1/r}$
F_h^A	e^{-Nh}	$\geq \log^2 \varepsilon^{-1/h}$	$\log \varepsilon^{-1/h}$

Thus, for the above the classes within the framework of the natural assumption that $\mathrm{comp}(D, [\mathfrak{H}, \Phi]) \geq N^2$, $D \in \mathfrak{D}_N$, the direct methods do not help to reach the order of the best lower bound of ε-complexity.

Average Case Complexity for Linear Problems in a Model with Varying Noise of Information

Leszek Plaskota
University of Warsaw

We study the problem of approximating values $S(f)$ of a linear continuous operator $S : F \rightarrow G$, where F is a Banach space and G is a Hilbert space, based on noisy observations of linear functionals. The noise coming from each observation is Gaussian and its (known) variance σ^2 influences the cost via a nonincreasing cost function $c(\sigma^2)$. Adaptive choice of successive functionals L_i to observe as well as variances σ_i^2 are allowed. The error of observation is equal to the average squared distance between the exact and the approximate solutions, with repect to the noise and a Gaussian measure μ on F.
We show sufficient conditions under which adaptive methods are not (much) better than nonadaptive methods. Main complexity results are obtained in the case where the functionals L_i are in the ball $\int_F L(f)\,\mu(df) \leq 1$. In particular, we give formulas for ε-complexity of multivariate function approximation in the case where μ is the folded Wiener sheet measure and the cost function is given by $c(\sigma^2) = (1 + \sigma^{-2})^\alpha$ for some $\alpha \geq 0$.

Probabilistic Complexity of Interior Point Methods

Florian A. Potra
University of Iowa

Recent advances in the theory of interior point methods for linear programming such as the finite termination scheme of Ye and the introduction of the homogeneous self-dual artificial linear program of Ye, Mizuno, and Todd have allowed a probabilistic analysis of the number of iterations required for an interior point method to find the exact solution of a linear program both in the integer and the real model. On a probabilistic model proposed by Todd which is arbitrarily degenerate and possesses no feasible starting points, the expected value of this number is $O(n^{1/2} \log n)$. Because each iterate requires $O(n^3)$ iterations at most, this implies strongly polynomial complexity. This result was obtained in a joint work with Kurt Anstreicher, Yinyn Ye, and Jun Ji.

Incorporating Condition Measures into the Complexity Theory of Linear Programming

James Renegar
Cornell University, Ithaka

This work is an attempt, among other things, to begin developing a complexity theory in which problem instance data is allowed to consist of real numbers, even irrational ones, yet computations are of finite precision.

Complexity theory generally assumes that the exact data specifying a problem instance is used by algorithms. The efficiency of an algorithms is judged relative to the "size" of the input.
We replace customary measures of size by "condition measures". These measures reflect the amount of data accuracy necessary to achieve the desired computational goal; the measures are dependent on the goal. The measures are similar in spirit, and closely related, to condition numbers.

Varying Cardinality for Finding Roots of Smooth Functions

Klaus Ritter
Universität Erlangen-Nürnberg

We give an average case analysis for finding roots of C^r-functions with $r \geq 2$ and a change of sign on $[0, 1]$. The functions are assumed to be distributed according to a conditional r-fold Wiener measure. We compare methods which use an a priori fixed number of knots (fixed cardinality) with methods which determine the number of evaluations by means of a stopping rule (varying cardinality). Unlike for many linear problems, it turns out that varying cardinality is very powerful for root finding. In the case of fixed cardinality we need $\Theta(\log 1/\varepsilon)$ knots to get an average error ε. However, using varying cardinality $O(\log \log 1/\varepsilon)$ knots on the average are sufficient to get a maximal error ε. Contains joint work with Erich Novak and Henryk Woźniakowski.

Complexity and Bezout Theorem II

Mike Shub
IBM Research Center, Yorktown Heights

In joint work with Steve Smale we prove that the average number of real roots of a system of n complex polynomial equations in n variables is the square root of the Bezout number and estimate the volume of complex systems with a root of condition bigger than k by $k^{-4}n^3N^2D$, where N is the dimension of the space $\mathfrak{H}_{(d)}$ of such systems and $D = \prod d_i$ is the Bezout number.

Complexity of Fixed Points

Christopher Sikorski
University of Utah

We address the problem of approximating fixed points of Lipschitz continuous functions with the factor q under the absolute and the residual error criteria. For the contractive ($q < 1$) univariate case we show that the bisection-envelope algorithm enjoys minimal complexity equal to $\approx \log(1/\varepsilon)/\log((1+q)/q)$. For the contractive

multivariate problems the Banach's simple iteration algorithm is optimal whenever the dimension d is at least $\log(1/\varepsilon)/\log(1/q)$. For small moderate dimension d we present an ellipsoid algorithm with cost $d^3(\log(1/\varepsilon)+\log(1/(1-q))+\log d)$. We conjecture that the complexity in this case has the form $c_d(\log(1/\varepsilon)+p(q))$ where $p(q)$ slowly approaches $+\infty$ as $q \to 1^-$. For the noncontractive case $q \geq 1$ we show that the complexity with absolute error criterion is infinte even for $q = 1$ and $d = 2$. The same problem under the residual error criterion has finite but exponential complexity in the form $(q/\varepsilon)^k$ where $d-2 \leq k \leq d$.

Complexity and Bezout Theorem

Steve Smale
University of California, Berkeley

An introduction to 3 papers with above title, joint with Mike Shub, is given. Emphasis is given to the condition number.

Average Error Bounds of Best Approximation of Continuous Functions on the Wiener Space

Sun Yongsheng
Beijing Normal University
(joint work with Wang Chengyong)

On the classical Wiener space of continuous functions we consider the p-average value of the L_q-best polynomial approximation of order n as well as the p-average value of the L_q-best linear approximation by Jackson-type linear operators of rank n, and obtain some upper bounds for these p-average values. Besides these, we also give some theorems of Bernstein type (i.e., converse theorems of approximation theory) in the average case setting.

Estimates of Singular Numbers of Integral Operators and Bilinear Approximation

Volodya N. Temlyakov
Steklov Institute, Moscow, and University of South Carolina

In this talk we obtain orders of upper bounds for best approximation of functions in anisotropic Sobolev and Nikol'skii classes in various mixed (vector) norms by means of arbitrary M-term linear combinations of products of functions of the first variable by functions of the second variable. Such approximations are called bilinear. These results are applied to estimate the singular numbers of integral operators with kernels in the classes mentioned above.

One problem concerned with estimating approximation numbers of integral operators is solved.

Discontinuity Properties of the Robustness Margin

Roberto Tempo
Politecnico di Torino

We have shown that for polynomials with real coefficients the robustness margin for stability can be a discontinuous function of the problem data.

On Boundary Value Problems for the Hyperbolic Case

Nikholas N. Vakhania
Georgian Academy of Sciences

The survey of results on ill-posed boundary value problems for the hyperbolic case is given. The following three problems are particularly discussed: (i) the Dirichlet problem for the vibrating string equation, (ii) the Soblev ill-posed problem (equivalent to the boundary value problem with presrcibed skew derivative for the vibrating string equation, (iii) the Dirichlet problem for the mixed (elliptic-hyperbolic) equation.

On Average case Complexity of Problems that are Intractable in the Worst Case Setting

Greg W. Wasilkowski
University of Kentucky

A number of important problems have very high complexity in the worst case setting. These include multivariate integration and approximation, singularity detection, etc. In the first part of the talk, we discuss multivariate integration and approximation in the average case setting. When the corresponding probability measure has a tensor-product form, the complexity depends only mildly on the number d of variables. When the measure is isotropic, the complexity significantly increases with d.
In the second part of the talk, we discuss (probabilistically) efficient algorithms for problems defined over piecewise smooth functions. In particular, we discuss singularity (edge) detection, and integration of piecewise smooth integrands.

Asymptotically Optimal Estimates of the n-Widths of Bounded Analytic Functions

Klaus Wilderotter
Universität Bonn

Let Δ be the unit disc in the complex plane and let E be a compact, simply connected subset of Δ, whose boundary is assumed to belong to the class $C^{1,\alpha}$. Let A be the unit ball in the Hardy space $H^\infty(\Delta)$. The n-widths d_n of A in $C(E)$, the space of continuous functions on E, are asymptotically optimal estimated. It is shown that

$$\lim_{n\to\infty} \frac{d_n(A, C(E))}{\exp(-n/\mathrm{cap}(E,\Delta))} = 1.$$

Here $\mathrm{cap}(E,\Delta)$ denotes the Green capacity of E with respect to Δ.

Tractability of Multivariate Linear Problems

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We consider how the complexity of a multivariate linear problem depends on the error tolerance ε and on the number d of variables. This is done in worst case, average case, probabilistic, and randomized settings. Information about multivariate problems is given by oracles which consist of functions values or linear functionals. It is known that the complexity is proportional to the number of oracles needed to compute an approximate solution.
The multivariate problem is said to be tractable, if the number of oracles needed for its solution depends polynomially on $1/\varepsilon$ and d. It is called strongly tractable if the degree of the polynomial in d is zero.
Under some assumptions, we prove that tractability in the class Λ^{std} of function values is equivalent to tractability in the class Λ^{all} of linear functionals. We provide necessary and sufficient conditions on tractability in the class Λ^{all}. In particular, we show that if the domain of a multivariate linear problem is a reproducing kernel Hilbert space, then we have strong tractability and the exponent in $1/\varepsilon$ is at most 2 in class Λ^{all} and at most 4 in the class Λ^{std}. We also check strong tractability for the class of once differentiable functions for each variable and prove that the exponent in the worst case setting is $1.41\ldots$. This is done by using Tauberian estimates in number theory due to Andrew Odlyzko [1992].

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