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Report

of the Third Dagstuhl Seminar on

Computational Geometry

March 22–26, 1993

The Third Dagstuhl Seminar on Computational Geometry was organized by Helmut Alt (FU Berlin), Bernard Chazelle (Princeton University), and Emo Welzl (FU Berlin). The 32 participants came from 10 countries, among them 13 who came from North America and Israel.

This report contains abstracts of the 30 talks (in chronological order) given at the meeting as well as Micha Sharir's report of the open problem session which took place on Tuesday afternoon and was chaired by Kurt Mehlhorn.

Berichterstatter: Frank Hoffmann

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Emo Welzl, Freie Universität Berlin
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Abstracts

On Fat Partitioning, Fat Covering and the Union Size of Polygons

by MARC VAN KREVELD

The complexity of the contour of simple polygons with $O(n)$ vertices in total can be $\Theta(n^2)$ in general. In this paper, a necessary and sufficient condition is given which guarantees smaller union size. A δ-corridor in a simple polygon is a passage between two edges in a polygon with width/length ratio δ. If a set of polygons with $O(n)$ vertices in total has no δ-corridors than the union size is $O\left((n \log\log n)/\delta\right)$, which is close to optimal in the worst case. The result has many applications to basic problems in computational geometry, such as hidden surface removal, motion planning, ray shooting, injection moulding, etc.

The result is based on a new method to partition a simple polygon P with n vertices into $O(n)$ convex quadrilaterals, without introducing angles smaller than $\pi/12$ radiant, or narrow corridors. Furthermore, a convex quadrilateral with no δ-corridor can be covered (but not partitioned!) by $O(1/\delta)$ fat triangles. The maximum overlap of the triangles at any point is 2. The bound on the maximum union size now follows from a result of Matoušek et al, FOCS 1991. They prove that the maximum union size of n fat triangles is $O(n \log\log n)$.

The partitioning and covering algorithms take $O(n \log^2 n)$ and $O(n \log^2 n + n/\delta)$ time, respectively.

Coloring Rectilinear Polygons and the Prison Yard Problem

by FRANK HOFFMANN (joint work with Klaus Kriegel)

One of the main open Art-Gallery-type problems is to determine the minimal number of vertex guards sufficient to watch simultaneously both the interior and the exterior of any n-sided simple rectilinear polygon. First, we present a class of pyramids that need asymptotically $5n/16$ guards, which is a new lower bound for the problem, and we show that $\lfloor \frac{5n}{16} \rfloor + 2$ guards suffice for any orthoconvex rectilinear polygon. The proof uses a new generalized coloring argument that can be applied to a convex partition of the plane derived from the quadrilateralized polygon. Then we prove a purely graphtheoretic result: To an embedded planar graph with all inner faces being 4-cycles one can add one diagonal per 4-cycle in such a way that the resulting graph is 3-colorable. As immediate consequences we get the following new upper bounds:

(1) $\lfloor \frac{n}{3} \rfloor$ vertex guards are sufficient to solve the Art Gallery Problem for rectilinear polygons with holes.

(2) $\lfloor \frac{5n}{12} \rfloor + 3$ vertex guards (resp. $\lfloor \frac{n+4}{3} \rfloor$ point guards) are sufficient to solve the Prison Yard Problem for rectilinear polygons.

Generalized Hidden Surface Removal

by MARK DE BERG

One of the basic problems in computer graphics is the hidden surface removal problem: Given a set of objects in 3-space and a view point, compute which parts of the objects are visible. To obtain realistic images, however, one should also take lighting considerations into account. We start the investigation of this problem from the computational geometry point of view by considering the generalized hidden surface removal problem: Given a set of objects in 3-space, a view point, and a (point) light source, compute which parts of the objects are visible, subdivided into parts which are lit and parts which are not lit. We prove tight bounds on the maximum combinatorial complexity of such views for three different settings, namely where the objects are triangles, where the objects are axis-parallel horizontal rectangles and where the objects are the faces of a polygonal terrain. We also give output-sensitive algorithms for all three cases.

Distribution of Distances

by JÁNOS PACH

Extremal graph theory has proved to be a powerful tool in bounding the combinatorial complexity of arrangements arising in various fields of computational geometry. Unfortunately, in many cases it fails to provide the best possible answers. The following result (joint with P. Erdös and E. Makai) can be established by using Szemeredi's Regularity Lemma. For any positive integer k and $\epsilon > 0$, there exist $n_{k,\epsilon}, c_{k,\epsilon} > 0$ with the following property. Given any system of $n > n_{k,\epsilon}$ points in the plane with minimal distance 1 and any $t_1, \ldots, t_k > 0$, the number of those pairs whose distance is between t_i and $t_i + c_{k,\epsilon}\sqrt{n}$ for some $1 \leq i \leq k$, is at most $\frac{n^2}{2}\left(1 - \frac{1}{k+1} + \epsilon\right)$. This bound is asymptotically tight (in more than one sense).

Exact and Approximate Solutions to the Embedding Problem

by PETER SCHORN

We solve one of the simplest, non-trivial, instances of the embedding problem, the fundamental problem of Distance Geometry. The exact solution with the Computer Algebra System Maple generates for the x-coordinate of one of the vertices a 12th-degree minimal-polynomial which has 15-digit coefficients. Alternatively we propose a simple iterative technique for the solution of the embedding problem. An implementation on the XYZ GeoBench, our workbench for geometric computation, can quickly find all solutions to small instances of the embedding problem.

Optimal Parallel Algorithms for Rectilinear Link Distance Problems

by JÖRG RÜDIGER SACK (joint work with: A. Lingas, A. Maheshwari// T. Hagerup)

We provide parallel solutions to several link distance problems set in trapezoided rectilinear polygons. All algorithms are deterministic and designed to run on the EREW PRAM.

Let P be a trapezoided rectilinear single polygon with n vertices. In $O(n \log n)$ time using $O(\log n)$ processors we can optimally compute

- minimum rectilinear link paths, or shortest path in the L_1 metric from any point in P to all vertices of P,
- minimum rectilinear link paths from any segment to all vertices of P,
- rectilinear window (histogram) partition of P,
- both covering radii and vertex intervals for any diagonal of P,
- a data structure to support rectilinear link distance queries between any two points in P.

This improves on the preciously best known sequential algorithm for this problem which used $O(n \log n)$ time and space.

We employ the parallel technique for example to optimally compute the link diameter, link radius and link center of a rectilinear polygon.

Fast Euclidean Lattice Reduction

by Chee Yap

Reduction of lattices can be viewed as a generalization of the Euclidean algorithm for integer GCD. In 2-dimensions, Gauss has a generalization of Euclid's algorithm. We review the technique of Lehmer-Knuth-Schönhage for fast GCD computation and show how this can be extended to the 2-dimensional case. The result is that if $u, v \in \mathbb{Z}^2$ are given, then the smallest basis for the lattice $\Lambda(u, v)$ can be computed in

$$O(M(n) \log n)$$

bit complexity, where $n = \log(|u| + |v|)$. To achieve this, we introduce the concept of coherent remainder sequences and analyze some basic properties.

We then consider how to extend the reduction method to 3-dimensions. Here we must define the "reduction" operation "$u \bmod(v, w)$" where $u, v, w \in \mathbb{R}^3$. We may also assume that v, w are already reduced, using the 2-dimension algorithm. We want to use this operation to repeatedly reduce a vector by the other 2 until no more reduction occurs. A simple example shows that the definition

$$u \bmod(v, w) := (u \bmod v) \bmod w$$

does not work. We introduce the concept of the "minimal fundamental region" of (v, w) and give a suitable definition. This gives rise to a simple lattice reduction algorithm to find the shortest basis in 3-dimensions. The complexity is $O(M(n) n \log n)$.

A Pathological Shortest Path

by JEAN-DANIEL BOISSONNAT (joint work with André Ceregzo and Juliette Leblond)

We consider the class of C^2 piecewice regular paths joining two given configurations (i.e. position, orientation and curvature) x_0 and x_f in the plane, along which the derivative of the curvature (with respect to the arc length) remains bounded. Regular means here that the path consists of an at most countable number of C^3 arcs of finite length and that the set of endpoints of such arcs admits at most a finite number of points of accumulation. We prove that there is no path of minimal length in this class for generic x_0 and x_f. This means that the optimal control of a car-like robot with a bounded wheel-turning speed is extremely irregular.

This problem is a natural generalization of a problem studied by Dubins who gave a complete characterization of the C^1 shortest paths of bounded curvature joining two points with prescribed tangents.

Some Thoughts on Perturbations

by RAIMUND SEIDEL (joint work with Yiannis Emiris and John Canny)

A sequence $S = (q_1, \ldots, q_n)$ of points in $\mathbf{R}^d$ is said to be *non-degenerate* with respect to a function $g : \mathbf{R}^{ds} \to \mathbf{R}$, iff for every s-tuple of distinct $i_1, \ldots, i_s$ we have $g(q_{i_1}, \ldots, q_{i_s}) \neq 0$. We call S non-degenerate with respect to a set G of functions if it is so for every $g \in G$. A sequence $\Pi = (\pi_1, \ldots, \pi_n)$ of continuous, smooth functions $\pi_i : [0, 1] \to \mathbf{R}^d$ is said to be a *valid perturbation* for S with respect to G iff for all i $\pi_i(0) = \vec{0}$, and the sequence $(q_i + \pi_i(\epsilon))_{1 \leq i \leq n}$ is non-degenerate with respect to G for all sufficiently small $\epsilon > 0$.

Valid perturbations are a useful tool for making geometric algorithms that work only for non-degenerate inputs applicable to all inputs. Crucial problems for this approach are the ability to determine a valid perturbation and the ability to compute the sign of $\overline{g}(\epsilon) = g(q_{i_1} + \pi_{i_1}(\epsilon), \ldots, q_{i_s} + \pi_{i_s}(\epsilon))$ for all sufficiently small $\epsilon > 0$ efficiently. We point out that for many commonly used geometric predicate functions g such as *"orientation"*, *"sidedness"*, *"in-sphere"* and many others there is a surprisingly simple solution to these problems.

1. If G consists of multivariate homogeneous polynomials and $P = (p_1, \ldots, p_n)$ is non-degenerate for G, then for *any* sequence $S = (q_1, \ldots, q_n)$ a valid perturbation is given by $(\epsilon p_i)_{1 \leq i \leq n}$.

2. If g is a multivariate polynomial of degree Δ, then determining the sign of $\overline{g}(\epsilon) = g(g_{i_1} + \epsilon p_{i_1}, \ldots, g_{i_s} + \epsilon p_{i_s})$ requires only $\Delta+1$ evaluations of g (say for $\epsilon = 0, 1, \ldots, \Delta$) and computation of the coefficients of the polynomial $\overline{g}$ by interpolation.

Convex Distance Functions in 3D

by ROLF KLEIN (joint work with Christian Icking, Minh Lê, and Lihong Ma)

We show that given any number n one can construct a smooth convex body S in 3D and four points a_1, a_2, a_3, a_4 in $\mathbf{R}^3$, such that for sufficiently small $\epsilon > 0$ and each choice of points a_i' in $U_\epsilon(a_i)$ there are n or $n+1$ homothetic copies of S that contain the points a_i' on its surface. Consequently, there is no upper bound for the complexity of the Voronoi diagram of four points in 3D that is based on a convex distance funtion.

In particular, we showed that in the L_4-metric three spheres can pass through four points in general position. However, there exists an upper bound independent of p for the number of L_p-spheres that can pass through four points in general position.

Exact Algorithms for an NP-Complete Packing Problem

by KURT MEHLHORN (joint work with L. Kucera, B. Preis und E. Schwarzenecker)

We consider the following NP-complete geometric packing problem: For a given set S of n points $p_1, ..., p_n$ in the plane and a positive real r decide whether there are n axis-parallel squares $Q_1, ..., Q_n$ of sidelength r such that Q_i has one of its corners incident to point p_i for all $i, 1 \le i \le n$ and such that Q_i and Q_j are disjoint for i different from j. The problem arose in the context of map lettering.

Forman and Wagner have shown this problem to be NP-complete, that there is an approximation algorithm for the associated optimization problem (find the largest r such that ...) which comes within a factor two of optimum, and that no better approximation algorithm exists provided that $P \neq NP$.

We discuss exact algorithms for the optimization problem. A naive algorithm runs in time $O(4^n poly(n))$ and can be used to solve problems with less than 20 points. We describe two better algorithms: one with running time $4^{O(\sqrt{n})}$ and one with running time $4^{O(\sqrt{n \ln n})}$. The second algorithm can be used to solve problems with 100 points. It's analysis is related to an old problem in measure theory, namely Kakeya's problem which asks for the smallest measure of any set in which a unit length segment can be turned by 180 degrees without leaving the set. The second algorithms also allows us to show experimentally that the approximation algorithm is within 5 percent of optimum for practical problems.

Can Visibility Graphs be Represented Compactly

by PANKAJ K. AGARWAL (joint work with N. Alon, B. Aronov, and S. Suri)

We consider the problem of representing the visibility graph of segments as a union of cliques and bipartite cliques. Given a graph G, a family $\mathcal{G} = \{G_1, \ldots, G_m\}$ is called a 'clique cover' of G if each G_i is a clique or a bipartite clique, and the union of G_i's is G. The size of $\mathcal{G}$ is defined as $\sum_{i=1}^m n_i$, where n_i is the number of vertices in G_i. Our main result is that there exist visibility graphs of n nonintersecting segments in the plane whose smallest clique cover has size $\Omega\left(n^2/\log^2 n\right)$. On the other hand, we show that the visibility graph of a simple polygon always admits a clique cover of size $O(n \log^3 n)$, and that there are simple polygons whose visibility graphs require a clique cover of size $\Omega(n \log n)$.

Objects That Cannot be Taken Apart with Two Hands

JACK SNOEYINK (joint work with Jorge Stolfi)

It has been conjectured that every configuration C of convex objects in 3-space with disjoint interior can be *taken apart by translation with two hands*: that is, some proper subset of C can be translated to infinity without disturbing its complement. We show that this conjecture holds for five or fewer objects but give a counterexample with six objects that is built on the symmetry group of a tetrahedron. We extend the counterexample to a configuration of 30 objects that cannot be taken apart with two hands using arbitrary isometries (rigid motions).

Safe & Effective Determinant Evaluation

by KENNETH CLARKSON

Many geometric algorithms use as a primitive operation the computation of the sign of the determinant of a small matrix. This operation is done many times, so it should be fast, but it is vital that it is correct. Given an $n \times n$ matrix A of b-bit integers, I show that the determinant of A can be computed with low relative error using a combination of approximate & exact integer arithmetic. The running time of the algorithm depends on the orthogonality defect $OD(A)$, where $OD(A)|\det A|$ is the product of the lengths of the columns of A. Here $OD(A) \leq \left(\sqrt{n}\, 2^b\right)^n$. If $\beta + 1.5n + b + 2\lg n + 2$ bits of precision are available, the algorithm needs $\mathcal{O}(n^3) + \mathcal{O}(n^2)(\lg OD(A))/b$ time. Thus the algorithm is fast even with small β, if $\lg OD(A)$ is small, as commonly holds.

On the Union of Convex Polyhedra

by BORIS ARONOV (joint work with Micha Sharir)

We almost settle a conjecture involving a set of k convex polyhedra in $\mathbf{R}^3$, with a total of n faces. We consider the union of such a family and estimate the maximum possible number of edges, vertices, and faces on its boundary. We show that this quantity if $O\left(k^3 + nk^{1+\epsilon}\right)$ for any $\epsilon > 0$ with the constant of proportionality in the second term depending on ϵ. This almost matches a lower bound of $\Omega\left(k^3 + nk\right)$.

Almost Tight Upper Bounds for Lower Envelopes in Higher Dimensions

by MICHA SHARIR (partly joint work with Dan Halperin)

Let $\sum = \{\sigma_1, \ldots, \sigma_n\}$ be a collection of n algebraic surfaces or surface patches, of constant maximum degree, in $\mathbf{R}^d$. The lower envelope E_Σ is the map that associates with each $\underline{x} = (x_1, \ldots, x_{d-1})$ the lowest point of intersection, if any, of the vertical line through $\underline{x}$ with the surfaces of Σ. When E_Σ is projected onto $H : x_d = 0$, we get a decomposition of this hyperplane into cells of various dimensions over each of which the envelope is attained by a fixed subset of surfaces. The combinatorial complexity of $E_{\sum}$ is the number of cells in this decomposition. Our main result is: The combinatorial complexity of $E_{\sum}$ is $O\left(n^{d-1+\epsilon}\right)$, for any $\epsilon > 0$, where the constant of proportionality depends on ϵ, d, and the maximum degree of the surfaces. The bound is almost tight in the worst case. The result has many applications to the complexity of generalized or dynamic Voronoi diagrams, visibility in terrains, spaces of common transversals of convex sets, spaces of lines missing a star-shaped set in 3-space, and many more. The proof is relatively simple, and its main tool is the probabilistic analysis technique of Clarkson and Shor (1989). We also give an $O(n^{2+\epsilon})$ algorithm for computing lower envelopes in 3-space, but a technical difficulty prevents the extension of the algorithm to higher dimensions.

Advances in Exact Motion Planning with Three Degrees of Freedom

by DAN HALPERIN (joint work with Micha Sharir and Chee Yap)

We present two recent results in the study of exact (non-heuristic) motion planning of rigid bodies with three degrees of freedom:

1. We consider the problem of planning the motion of an arbitrary k-sided polygonal robot B, free to translate and rotate in a polygonal environment bounded by n edges. We show that the complexity of a single component of the free configuration space of B is $O\left(k^3 n^{2+\epsilon}\right)$, for any $\epsilon > 0$, where the constant of poportionality depends on ϵ. This is a significant improvement of the naive bound $O\left((kn)^3\right)$.

2. We study the space of free translations of a box amidst polyhedral obstacles with n features in 3-space. We show that the maximum combinatorial complexity of this space is $O\left(n^2\alpha(n)\right)$ where $\alpha(n)$ is the inverse Ackermann function. Our bound is within an $\alpha(n)$ factor off the lower bound, and it constitutes an improvement of almost an order of magnitude over the naive bound for this problem, $O(n^3)$.

Straight Skeletons of Simple Polygons

by FRANZ AURENHAMMER

Let P be a simple polygon with n edges. The medial axis of P is the set of all points in P whose nearest neighbor on the boundary of P is not unique. If P is convex then the medial axix consists of portions of angle bisectors only. Reflex polygon vertices, however, cause the appearance of parabolic arcs. In contrast, the straight skeleton of P is made up of angle bisectors only. We introduce this structure and investigate some of its properties. Care has to be taken on the definition of a straight skeleton as there are many ways of partitioning P by angle bisectors in a consistent way. Given the skeleton, a triangulation of P can be obtained very easily in $O(n)$ time. The structure is also useful for planning a motion within P that keeps away from sharp polygon angles, and for constructing a 45 degree roof of minimal volume above polygonal ground walls. In spite of its nice properties, the algorithmic construction of a straight skeleton turns out to be surprisingly difficult.

On the Number of Cells Defined by a Set of Polynomials

by M. F. ROY (joint work with R. Pollack)

Let $c(n, d, D)$ be the total number of non-empty sign conditions defined by a set of s polynomials of degree d in n space.

Theorem $c(n, d, s) = [O(sd/n)]^n$.

The formerly known bound based on Bezout-Heintz inequality is the algebraically closed case together with Thom-Milnor's Theorem gave $(sd)^{O(n)}$. Here, rather than Thom-Milnor's classical results, we use a result from Warren, which includes a more combinatorial approach (adding one equation after the other rather than making directly the product of the equations) and bounds the number of connected components of the complementary of an algebraic set union of the zero set of a polynomial of degree d in n variables by $[O(sd/n)]^n$. Our proofs use as a technical tool Puiseux series and infinitesimal elements and is an extension of Alon's result on the number of non-empty sign conditions (also in $[O(sd/n)]^n$). As a corollary we obtain a bound on the number of isotopy classes of configurations of n points in d space as $[O(n^d)]^{nd}$ which matches the already known lower bound.

Weak ϵ-nets

by LEONIDAS GUIBAS

Range spaces of infinite VC-dimension do not have (regular, strong) ϵ-nets whose size is a function of ϵ only. The most well-known example is that of a space of points in $\mathbf{E}^d$ (Euclidean d-space) in *convex ranges*. For such situations we generalize the concept of ϵ-net to allow in the set points not necessarily in the original set – these are called *weak ϵ-nets*. In this talk we summarize what is known about weak ϵ-nets for convex ranges. Specifically, we show a construction of a weak net for convex sets in $\mathbf{E}^d$ whose size is

$$O\left(\frac{1}{\epsilon^d}\log^x\frac{1}{\epsilon}\right),$$

where x is a large function of d. We also show the many connections between weak nets to other classical concepts in Combinatorial Geometry, such as centerpoints, k-sets, selection lemmas, Tverborg points, Helly type theorems, etc.

Foundations of a Convexity Theory on the Affine Grassmannian $\mathbf{G}'_{k,d}$

by RICHARD POLLACK (joint work with JACOB E. GOODMAN)

$\mathbf{G}'_{k,d}$ is the space of k-flats in $\mathbf{R}^d$ with its natural topology. We give several equivalent definitions of the convex hull of subsets of $\mathbf{G}'_{k,d}$ in terms of "convex transversals" and "surrounding" among others, which in the case $k = 0$ specialize to the standard notion of convexity of point sets in $\mathbf{R}^d$. E.g. $F' \in \text{conv}(\mathcal{F})$ iff F' meets every convex set of points which meets every k-flat in $\mathcal{F}$. Equivalently, $F' \in \text{conv}(\mathcal{F})$ iff $\exists$ an ℓ-flat G containing F', $k \leq \ell \leq d$, s.t. for every $\ell - 1$ flat $H \subset G$ which contains F', both open half spaces of G defined by H contain a k-flat of $\mathcal{F}$.

This definition satisfies,

idempotence:	$\mathrm{conv}(\mathcal{F}) = \mathrm{conv}(\mathrm{conv}(\mathcal{F}))$
monotonicity :	$\mathcal{F}_1 \subset \mathcal{F}_2 \Rightarrow \mathrm{conv}(\mathcal{F}_1) \subset \mathrm{conv}(\mathcal{F}_2)$
anti-exchange:	$F_1, F_2 \notin \mathrm{conv}(\mathcal{F}),\ F_1 \in \mathrm{conv}(\mathcal{F} \cup \{F_2\}),\ F_2 \in \mathrm{conv}(\mathcal{F} \cup \{F_1\})$
	$\Rightarrow F_1 = F_2$

It is also invariant under nonsingular affine transformations and behaves well with respect to restriction to subspaces. Moreover, the Krein-Milman Theorem (every compact convex set in $\mathbf{G}'_{k,d}$ is the convex hull of its extreme k-flats) is true. We give several examples exhibiting some unusual behavior. Finally we give a simple characterization of "parallel-closed" convex sets. E.g. a parallel-closed set $\mathcal{F} \subset \mathbf{G}'_{1,3}$ is convex if and only if the corresponding set of directions (in $\mathbf{P}^2$) is the complement of a union of lines.

Dog Bites Postmen – Point Location in Moving Voronoi Diagrams

by OLIVIER DEVILLERS (joint work with Mordicai Golin)

We present a new application of general framework of incremental randomized algorithms to the problem of moving Voronoi diagrams.

The moving Voronoi diagram is a subdivision of the 3D-space (x, y, t) where the objects are t-monotone curves describing the trajectories of sites in the plane. The structure can be computed in nearly expected output sensitive time. Queries can be answered in $\log^2(n)$ time, where a query is: "What is the nearest among the n sites at time t of a given point?"

The structure can also handle another kind of queries with the same complexities: Let us call the sites postmen, and assume they have constant speed. Now a dog wakes up at time t and point (x, y), it can run at speed v: Who is the postman it can reach and bite as soon as possible? Notice that the query is now 4D: (x, y, t, v) but we have to assume that the dog runs quicker than any postman. The structure can be dynamized (postmen can be inserted or deleted). All results are randomized.

Maintaining the Approximate Width of a Point Set

by GÜNTER ROTE

The width of a set of points in the plane is the smallest distance between two parallel lines that enclose the set. We want to maintain the set of points under insertions (and possibly also deletions) of points and be able to report an approximation of the width of this dynamic point set. R. Janardan gave a solution which reports the width of an n-point set with some specified relative error bound epsilon in time $O(b \log^2 n)$ using a data structure with space $O(bn)$ in which points can be inserted and deleted in time $O(b \log^2 n)$, where $b = \sqrt{1/\epsilon}$ is a parameter that depends on the desired precision.

The algorithm is based on finding the enclosing two parallel lines for the point set whose distance in a given direction is minimal. This is repeated for b evenly spaced directions among the angular range (π) of all possible directions.

By some easy modifications of this algorithm one can reduce the query time to $O(b \log n)$, the storage to $O(n)$, and the update time to $O(\log^2 n)$. One part of this simplification results from replacing a two-level nested binary search $\log^2 n$ by the prune-and-search strategy $\log n$-time. The insertion time results from maintaining the convex hull. Thus, in the semi-dynamic case (only insertions or only deletions), the update time is reduced to $O(\log n)$ (amortized).

Then we propose a different method: We place tangents to the points set in b evenly spaced directions. The smallest distance between two parallel tangents gives us an upper estimate of the width. Then we start from these two tangents and search in a neighborhood of this direction for a local minimum of the width. This is guaranteed to differ from the (global) minimum width by a relative error of at most $O(1/b^2)$, similarly as above. For fully dynamic point sets this leads to an update time of $O(\log^2 n)$ and $O(n)$ storage. However, the query time is $O(b \log n + \log^2 n)$. The term $\log^2 n$ could be reduced if we knew a faster way of looking for a local minimum of the width, given a hierarchical representation of a convex polygon (in the form of a balanced search tree of the vertices and edges). Only for the case of insertions only can we improve the bounds of Janardan: We get $O(\log n + \log b)$ amortized update time, $O(n+b)$ storage, and an amortized query time of $O(\log n \log\log n)$. This results by applying an idea of C. Schwarz which allows to reduce the time for finding a local minimum in this case.

A Simple Proof Technique for Geometric Discrepancy

by BERNARD CHAZELLE

It is possible to place n points in d-space so that given any 2-coloring of the points, there exists a halfspace within which one color dominates the other by as much as $c\, n^{1/2-1/2d}$, for some constant $c > 0$. This result was proven in a slightly weaker from by Beck and the bound was tightened by Alexander. It was shown to be quasi-optimal by Matoušek, Welzl and Wernisch. The lower bound proofs are highly technical and do not provide much intuitive insight into the "large discrepancy" phenomenon. We develop a proof technique which allows us to rederive the same lower bound in a much simpler fashion. We give a probabilistic interpretation of the result and we discuss the connection of our method to Beck's Fourier transform approach. We also provide a quasi-optimal lower bound on the discrepancy of fixed size rotated boxes, which significantly improves the previous bound.

A Subexponential Algorithm for Abstract Optimization Problems

by BERND GÄRTNER

Let H be a finite set, $<$ a linear order on 2^H and Φ a function that for given $F \subseteq G \subseteq H$, decides whether $F = \min_{<}(2^G)$ and if not, returns $F' \subseteq G, F' < F$. How many calls to Φ does it take to find $\min_{<}(2^H)$?

For deterministic algorithms there is a lower bound of $2^{|H|} - 1$; we give a randomized algorithm that takes expected $e^{O(\sqrt{|H|})}$ calls. The bound can be applied to yield first subexponential bounds for some geometric problems: finding the minimum spanning ball of a point set if $\mathbb{R}^d$, determining the distance between polytopes in $\mathbb{R}^d$.

Computing the Hausdorff-distance between Geometric Objects in Higher Dimension

by HELMUT ALT (joint work with Michael Godau)

Computing the Hausdorff-distance between geometric objects is motivated by questions from pattern and shape analysis. We generalize the problems considered so far to curves and surfaces in higher dimensions and finally to sets of k-dimensional simplices in d-dimensional space for arbitrary $k \leq d$. It turns out that for fixed k, d the problem can be solved in polynomial time, where the degree of the polynomial depends only on k, not on d. In fact, for two sets of n and m line segments in d-dimensional space the Hausdorff-distance can be computed, using parametric search, in time $O\left(nm \log^3(nm)\right)$. For sets of triangles ($k = 2$) we give an algorithm of runtime $O\left((n^2m + m^2n)\log^3(mn)\right)$. For arbitrary k-dimensional simplices we obtain $(n+m)^{O(k)}$.

Piecewise Linear Paths Among Convex Obstacles (I)

by OTFRIED SCHWARZKOPF (joint work with Jiří Matoušek and Mark de Berg)

Leb B be a set of n convex obstacles in the plane, and p, q two points in the same connected component of $\mathbb{R}^2 \setminus \bigcup B$. We show that there is a path connecting p and q with at most $O(n^2)$ links, and this bound is the best possible if the obstacles are allowed to intersect arbitrarily. If they are only allowed to touch, or are obtained as the Minkowski-sum of disjoint obstacles with a convex robot, the bound can be improved to $\Theta(n)$. We can compute such a path in time $O(n^2Q + n^2\alpha(n)\log n)$ for the general, $O(n \log n + nQ)$ for the touching case, where Q is the time necessary for an oracle describing the obstacles. For the pseudo disc-case, the time bound is $O(n \log Q^* + nQ)$, where Q and Q^* are oracles for the expanded/original obstacles.

Piecewise Linear Paths Among Convex Obstacles (II)

by JIŘÍ MATOUŠEK (joint work with Otfried Schwarzkopf and Mark de Berg)

We consider the problem discussed in the previous talk generalized to higher dimensions. For n convex, open and bounded obstacles in $\mathbb{R}^d$, we show that the link diameter of any connected component of their complement is $O\left(n^{(d-1)\lfloor d/2+1\rfloor}\right)$ and $\Omega(n^d)$ in the worst case. Special cases of disjoint, resp. touching obstacles are also considered, obtaining significantly better bounds. Our main tool is an "expansion lemma", saying that if we enlarge each obstacle in such a way that no new d-wise intersection is created, points connected by a path avoiding the original obstacles can still be connected among the enlarged obstacles.

Approximate Tightness-dependent Motion Planning

by STEFAN SCHIRRA

We extend the notion of the "tightness" of a motion planning problem, introduced by Alt et al. for a rectangle, to geometric objects with rotational and sliding joints. Then the technique of slicing is applied, analogously to the rectangle case, i.e. restricted motions are defined for such objects. If the tightness of a motion planning problem is large, it is sufficient to consider restricted motions. This observation leads to "tightness-dependent" algorithms, which are much more efficient than known tightness-independent motion planning algorithms for large tightness. E.g., we obtain an $O\left(n^2(f(t)+1)\right)$ time bound for moving two rectangles joint via a common corner, where $f(t)$ is a function depending only on the tightness. For large tightness, $f(t)$ is a constant. The best tightness independent bound is $O(n^4 \log n)$. Here n is the number of polygon corners in the polygonal environment.

A Lower Bound for the "3-Points on a Line" and Related Problems

by JEFF ERICKSON (joint work with Raimund Seidel)

We show that the problem of deciding whether in a set S of n points in the plane there are three that lie on a common line requires $\Omega(n^2)$ time on the decision tree model of computation, where the only test allowed is the predicate that checks whether a point $p \in S$ lies to the left of, on, or to the right of the directed line through points $q, r \in S$.

The proof consists of a simple adversary argument:

1. Any "collapsible triple $p, q, r \in S$" (i.e. p, q, r can be moved s.t. they become collinear, but no other collinearities are created during the movement) needs to be tested explicitely;

2. for every $n > 2$ there exists a set S with $|S| = n$ that has $\Theta(n^2)$ collapsible triples.

Our result generalizes to an $\Omega(n^d)$ lower bound for the problem of detecting affine degeneracies in a set of n points in $\mathbf{R}^d$. It also generalizes to other problems, such as an $\Omega(n^2)$ lower bound for testing whether $(A+B)\cap C = \emptyset$ (for integer sets A, B, C of size n), where the only operations allowed are tests of the form $a + b \lesseqgtr c$.

Open Problem Session

reported by MICHA SHARIR

Mark de Berg

Given n pairwise-disjoint triangles in 3-space, we want to compute the visibility graph of their vertices, i.e. report all pairs of mutually-visible vertices. A simple method for doing this is to compute for each vertex v the vertices visible from v. This can be done, using ray-shooting and related techniques, in time $O(n^{4/3+\varepsilon})$ for each vertex v, for a total of $O(n^{7/3+\varepsilon})$ time. The problem is to improve this, hopefully to near-quadratic. There is a method that runs in near-quadratic time and produces a near-quadratic output which represents in a compact manner all pairs of non-visible vertices, but it seems hard to extract the complementary information from this structure.

Leo Guibas

If P is a simple polygon with $n > 3$ vertices, then P has a diagonal; more strongly, for any vertex v of P, either v has a diagonal emanating from it, or its two neighbors can be connected by a diagonal. The problem is to generalize this to simple 'polygons' whose edges are algebraic arcs of constant maximum degree. Here a diagonal would be an algebraic arc connecting two vertices of P within the interior of P and not intersecting any edge of P.

Does there always exists a diagonal whose degree is bounded by the same constant degree of the edges of P? by some other constant degree (independent of the number of edges of P)? Does the stronger property of simple polygons mentioned above extend to the curved case? (If one considers a curved polygon consisting of 4 circular arcs forming a 'bow-tie' pattern, a diagonal must connect two opposite corner and thus must have an inflection point, so it cannot be a second-degree arc.)

Can show: Given a simple polygon with kn (straight) edges so that every k-th vertex is marked, then there is a pair of non-adjacent marked vertices with link distance $\leq k$. Also, from each marked vertex v either there is a k-link path to another non-adjacent marked vertex or the two marked vertices adjacent to v are connected by a k-link path.

Chee Yap

Given a lattice $\Lambda(u, v, w)$ formed by three vectors u, v, w in 3-space, we want to analyze the structure of a Voronoi cell of the lattice (a Voronoi cell in the diagram formed by all lattice points). In the planar case, the cell is a hexagon with vertices on a circle. What is the analogue in three or higher dimensions? In general, give a bound on the complexity of Voronoi cells in d-dimensional lattices.

Bernard Chazelle

Let Q be a unit square and let S be any set of n points in Q so that the distance between any pair of points is $\geq \frac{c}{\sqrt{n}}$. Show that there exists an empty strip intersecting Q and not containing any vertex of Q, whose width is $\omega(1/n)$. Note that if the points of S are lattice points, then the width of such a strip is $1/\sqrt{n}$, and if the points are chosen independently at random from a uniform distribution then there exists an empty strip with width $\Omega(\frac{\log n}{n})$.

Raimund Seidel

Given an $n \times n$ integer grid. We want to find a large set S on the grid having no 3 colinear points and no 4 cocircular points. How large can the size of S be? Clearly $O(n)$ is an upper bound. Can the size be $\Omega(n)$? A class of examples with $|S| = \Theta(n^{2/3-\epsilon})$ is apparently known.

Ken Clarkson

Scattered Data Interpolation: Given a finite set S in 3-space, viewed as points on a surface $z = f(x, y)$. We want to construct a piecewise-linear interpolation of S that will approximate f closely. Specifically, we seek a triangulation T of the xy-projection S^* of S, so that $||f_T - f||_\infty$ is minimized, where f_T is the piecewise-linear function that passes through the points of S and is linear over each triangle of T, and where $||g||_\infty = \max_{x,y} |g(x, y)|$. We assume that f is known. We note that if f is convex (concave), then T is the projection of the lower (upper) hull of S. If $f = x^2 + y^2$ then T is the Delaunay triangulation of S^*. What happens when $f(x, y) = xy$? What is T? How can it be constructed? (Standard heuristics, such as edge flipping or insertion, do not seem to work.) Is there an f for which the problem is NP-hard?

Marc van Kreveld

Optimal construction of the ($\leq k$)-levels in an arrangement of n planes in 3-D: The goal is to construct these levels in time $O(nk^2 + n \log n)$. The best known algorithm, due to Mulmuley, takes time $O(nk^2 \log \frac{n}{k})$. An optimal deterministic solution (with $O(nk + n \log n)$ time) is known in the plane (by Everett, Robert and van Kreveld), and randomized optimal solutions are known for all dimensions ≥ 4.

Pankaj Agarwal

Let Γ be a collection of n x-monotone arcs, each pair of which intersect in at most a constant number s of points. What is the complexity of a single level in the arrangement of Γ. In other words, how many vertices of the arrangement are there, with exactly k arcs passing below them, for some fixed k?

Leo Guibas

Given n triangles in the plane, find the region of the plane covered by at least half of the triangles, or the region covered by most triangles. [These appear to be n^2-hard problems.]

János Pach

We say that k points in the plane are in general position if all $\binom{k}{2}$ distances that they determine are distinct. For a fixed k, almost all k-tuples of any n-element set in the plane are in general position (that is, the fraction of k-tuples in general position tends to 1 as n tends to ∞). This also holds if k is allowed to grow (slowly) with n: It is true if $k = o(n^{1/7})$, and false for $k = n^{1/4}$. How large can $k = k(n)$ grow as a function of n so that the property still holds?

Jack Snoeyink

In 2-D: Given two convex polygons A, B, and a center of rotation, compute the smallest angle by which B has to rotate about the given center to meet A. (Can be done in linear time.)

In 3-D: Given two convex polyhedra A, B, and a line of rotation, compute the smallest angle by which B has to rotate about the line to meet A. Can this be done in subquadratic time? (Parametric searching does not seem to help because B can enter and leave A several times as it rotates.)

Chee Yap

Given two placements of a line segment $s = pq$, we want to move s 'linearly' from one placement to the other, meaning that one endpoint, p, moves at constant speed on the straight segment connecting its initial and final positions, while s rotates simlutaneously about p at constant angular velocity. Given a convex polygon A, we want to compute whether s collides with A during its motion.

Ken Clarkson

How hard is it to compute *optimal* ϵ-nets (ϵ-approximations) for a given set of objects and a collection of ranges? As an abstract problem this is the minimal hypergraph cover problem and so is NP-hard. What happens in geometric settings?

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