Geometric Modelling

Gerald Farin
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Projective Developables

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A developable surface may be defined as the envelope of a one parameter family of planes. This notion suggests a treatment in terms of projective three-space $\mathbf{P}^3$, where they can be written as “control plane” curves.
Cubic developables carry on them a family of conics. We formulate conditions under which two conics may be used as boundaries of a developable.
Applications of this research are in the areas of sheet metal forming and composite materials.

Variational Design of Rational Bezier Curves and Surfaces

Georges-Pierre Bonneau
Hans Hagen
Universität Kaiserslautern

The design of curves and surfaces in CAD systems has many applications in car, plane or ship industry. Because they offer more flexibility, rational functions are often preferred to polynomial functionals to model curves and surfaces.
In this talk, several methods to generate rational Bezier curves and surfaces which minimize some functionals are proposed. The functionals measure a technical smoothness of the curves and surfaces, and are related to the energy of beams and plates in the sense of elasticity theory.

Product Representation for Mechanical Engineering

Susan Bloor
University of Leeds

It is proposed that a central product model would facilitate product information accessibility and consistency. Such a model, developed at the University of Leeds, incorporating a framework and allowing levels of abstraction and instance, and catering for stages of the product life cylce was used as an example.
Representation of geometry is problematic. Ideally there should be a single

highlevel representation from which all others are derived. Current CAD systems collect representations and dimensionalities. The difficulty this causes in connecting the results of analysis from idealised geometries was demonstrated.
Information modelling, based on the STEP methodology, to better understand the relationship of the geometry requirements of computational analyses as a precursor to an integrated, open, CAE complementation was shown to further emphasise current difficulties. Finally, the long term aim that engineers should be able to use high level parameters related to the functional (shape) requirements was voiced.

Curves and Surfaces in an Object-Oriented Framework

G. Greiner R. Klein A. Kolb
R. Pfeifle H.-P. Seidel Ph. Slusallek

In computer graphics and geometric modelling one generally faces the problem to integrate a variety of curve and surface types into a single program. We present an object-oriented framework together with its C++ implementation that starts from an abstract class of differentiable curves and surfaces and in turn refines this design to curves and surfaces that are explicitly given in parametric form. The standard curve and surface types are derived from these abstract classes.
The approach is illustrated by examples from differential geometry, from the construction of blend surfaces, and from scattered data interpolation.

Lean Surface Representation

Wolfgang Schwarz
Institut für CAD-Datenaustauch, Mainz

Observing that many surfaces that occur in CAD applications have an overcrowded or unconvenient representation the knowledge of lean surface representation is introduced. A lean surface representation should allow threatment with minimal effort and storage with minimal resources but meet functional requirements. For exchange in VDAFS format this may mean a ploynomial, non-rational, bicubic tensorproduct surface with a minimal number of patches. Such a representation can be achieved by Hoschek's conversion method. The first step is to find an optimal, geometrically oriented segmentation, each patch having at most one minimum of curvature. This segmentation may be refined during the following approximation steps. The patch boundaries, the interior of the patches and for trimmed surfaces also the representation of the boundary curve in the surface domain are approximated minimizing distances squares at selected points. The result is interactively improved using parameter correction.

GeneSys -A Hybrid Solid Modeling System

Rolf Fischer
Fraunhofer Institut for Computer Graphics

In the Product Development Cycle, tasks like Drafting, Modeling, Virtual Analysis & Simulation, NC Programming and many others generate a series of problems relating the conversion of data, the connection of different hardware and software platforms and user interfaces. Moreover the knowledge of handling and managing each of these tasks are left to experts, arising other semantical communication problems. A Framework that integrates these aspects in a consistent and extensible way in a cooperative environment, is the major target of a Analysis-Aided Geometric Design Systems. Based on the third generation Solid Modeling System-GeneSys, such a Framework was presented with an emphasis of integrating Modeling and Analysis modules through a Semantic History Design Tree, a Hybrid Object Oriented Representation Scheme and a virtual 3D interactive modeling environment.

Interactive Surface Correction Based on a Local Approximation Scheme

Bernd Hamann
Mississippi State University
Brain A. Jean

Typically, CAD/CAM data contain errors. Such errors include overlapping patches, intersecting patches, and gaps between patches. In order to generate a valid surface grid on a given geometry, such errors need to be removed. This talk introduces a method that is based on a local approximation scheme. Given an arbitrary geometry, a user must specify four boundary curves (already existing or newly created) which are used to create a “discrete” bilinear Coons patch. This patch is then “projected” onto the given geometry, and the resulting points on the original geometry are used to create a local C^1 bicubic approximant. Eventually, many such local approximants “around” the given geometry must be connected in an at least C^0 fashion.
Unfortunately, it is necessary to use the same order, same knot vector, and

same number of control points for all approximants. It is planned to achieve the same results by using appropriate automatic paradigms.

Optimal Strategy for Automated Feature Recognition

Mike Pratt
Rensselaer Polytechnic Institute

There are two widely used strategies for the automated recognition of form features from geometric models. The first is a top-down approach in which generalized protrusions and depressions are first identified and the more detailed feature classification is then determined in a hierarchical manner. The initial stage is based on concavity or convexity of edges, and proves unreliable in general situations. The second approach is bottom-up; specific simple types of features are first recognized, and the more difficult cases involving interacting features left until later. This has the advantage that the number of entities unattributed to features may be grately reduced by the time the awkward problems have to be faced. On the other hand, the rule sets used in the early recognition of simple features often contain duplications, as in the case of slots and rectangular pockets, both characterized by pairs of parallel planar faces. The paper proposes an approach based on a generalization of Vanderbrandes generate-and-test strategy, which uses the concept of 'feature hints'. This permits the application of feature recognition rules in a hierarchical manner which avoids duplication and leads to an efficient process.

A knowledge based system for geometric design

Uwe Langbecker
Horst Nowacki
Technische Universität Berlin

A knowledge base for shape definition with problem formulation and problem solution knowledge was developed using the declarative programming language PROLOG embedded in a hybrid environment with MATHEMATICA, X11 and C.

The problem formulation involves statements by the user about geometrical data, mathematical representation, criterion function and discrete as well as integral constraints.

All of these shape generation problems can be viewed in the light of optimization problems with constraints which serves to classify problem types.

Problem solution knowledge in the knowledge base pertains to the choice of

adequate problem solvers for each problem type.
A demonstration system was implemented for the design of curves with multitudinous combinations of criteria functions and constraints.
This prototype system illustrates the advantages of the declarative knowledge based approach which enables the user to make a wide variety of choices about the problem type at session time.
The approach lends itself to achieving great flexibility within a large class of shape generation problems.

Least squares approximation using multiquadric functions

Richard Franke
Naval Postgraduate School
Hans Hagen
University of Kaiserslautern
Gregory Nielson
Arizona State University

This report documents an investigation into some methods for fitting surfaces to scattered data. The form of the fitting function is a multiquadric function with the criteria for the fit being the least mean squared residual for the data points. The principal problem is the selection of knot points (as base points for the multiquadric basis functions), although the selection of the multiquadric parameter also plays a nontrivial role in the process. We first describe a greedy algorithm for knot selection, and this procedure is used as an initial step in what follows. The minimization including knot locations and multiquadric parameter is explored, with some unexpected results in terms of "near repeated" knots. This phenomenon is explored, and leads us to consider variable parameter values for the basis functions. Examples and results are given.

Fairing of B-Spline Curves

Mathias Eck
Jan Hadenfeld
TH Darmstadt

We presented an interactive method for fairing B-Spline Curves. Here, in every iteration step only one control point is changed. This idea is similar to algorithms presented by N. Sapidis and G. Farin.
Now, the changing of the control point is carried out in such a manner that

certain (local) energy integrals are minimized. Therefore, the actual computation is quite simple and has the additional advantage that also an (preselected) error bound can be fulfilled in every step.
Moreover, we are interested in an automatical working algorithm. Therefore, we have to rank the control points in a certain manner. Here, we search that point where the possible improvement of the used energy integral is as large as possible.

Sliding Splines

W. L. F. Degen
Universität Stuttgart

A very simple, effective and robust method to construct a C^1-spline interpolation a given set of data points is presented. The main idea consists of starting from parabola segments interpolating three consecutive data points and then to slide from one segment to the next by a certain affine combination. This method is compared with other well-known solutions of this common task in CAGD.
Furthermore, using affine invariants of Bezier cubics, the shape preserving properties of the “sliding splines” are investigated.

Boolean Operations on General Polyhedra in $\mathbf{R}^d$

Hanspeter Bieri
Universiät Bern

A general polyhedron as defined by W. Nef in 1978 is a point-set in $\mathbf{R}^d$ which can be generated from a finite set of (open) linear halfspaces by forming finitely many complements and intersections. The class of such polyhedra is closed with respect to the operations complement, intersection, union, difference, closure and interior. It includes the “elementary” polyhedra as special cases and allows a notion of “face” which is valid for any dimension d and leads to elegant data structures and algorithms. Forming the complement of a general polyhedron is almost trivial, forming the intersection of two ployhedra can be elegantly done using the idea of the well-known algorithm of Bently and Ottmann which finds all intersections of a finite number of segments in $\mathbf{R}^2$. Finding the closure of a general polyhedron is again very easy, therefore all operations mentioned above can be performed in a transparent and completely general way.

Monotonic Curvature with Involutes of Circles

Bruce Piper
Rensselaer Polytechnic Institute

We review the conditions under which it is possible to interpolate with monotonic curvature given positions, tangents, and curvatures at two endpoints. We then construct an interpolant to such data using portions of involutes of circles joined smoothly together. Advantages and disadvantages of this scheme are disussed.

Localized Radial Basis Methods Using Rational Triangle Patches

Tom Foley
Siram Dayanand
Arizona State University
Dirk Zeckzer
Universität Kaiserslautern

Radial basis methods, such as thin plate splines and multiquadric methods, are generally very accurate and smooth interpolants to scattered data. However, they are global methods and they require solving a linear system of order N, where N is the number of 3D data points. For large N, the computation time can be excessive and the linear systems are often ill-conditioned. A localized approach is presented by decomposing the domain into an arbitrary triangulation that forms overlapping regions. This generalizes the earlier work of Franke who used a tensor product approach. For each region, a radial basis method is applied to a much smaller number of points and the local interpolants are blended using C^1 rational hybrid cubic Bezier triangle functions. Various strategies are discussed for selecting the blending functions and the arbitrary triangulation. Smoothly varying examples are given that are significantly faster than the global methods. The localized approach can also be applied to scattered data interpolants of general topology that are defined on local parametric patch domains.

Automatic Algorithms for constructing Shape-preserving Interpolating Curves

P. D. Kaklis
N. S. Sapidis
M. J. Karavelas
National Technical University of Athens

Motivated by the work of Schweikert (1966) on exponential splines in tension, the family of non-uniform degree polynomial splines (NUDPS) is introduced and an algorithm is presented for constructing planar, C^2-continuous, (locally) convexity-preserving, interpolating curves. Starting from the cubic spline, the algorithm identifies the nodal points and nodal segments, where a set of sufficient conditions, ensuring local convexity for NUDPS, is not fulfilled, and increase the neighbouring segment degrees by one. Grounded on the asymptotic properties of NUDPS, it can easily be proved that the algorithm converges after a finite number of interactions.
The Bezier control polygon of NUDPS possesses the following property: the inner control points are collinear and equidistant. This property permits to readily construct a planar, G^2-continuous, convexity-preserving, interpolatory quintic, using the nodal tangent and curvature values of the corresponding NUDPS.
Finally, the natural 3D extension of 2D NUDPS is introduced and some of its asymptotic properties are presented from the geometrical point of view. Roughly speaking, these properties establish that as the segment degrees increase, 3D NUDPS reflect more and more the intrinsic properties of the corresponding linear interpolant, e. g., the discrete Frenet frame and discrete torsion, in the sense of difference geometry of Sauer (1970). Furthermore, a notion of shape-preservation for 3D curves is introduced, which involves the control of binormal, curvature and torsion distributions as well as handling data containing co-planar and/or collinear subsets.

From the Clay Model to Composite Surfaces

Bernhard Kuhn
Mercdes - Benz

Choosing as an example a dashboard, I showed the problems that designers face when creating a G^2 surface representation of a clay model, in particular in

- digitizing

- curve preparation
- surface construction
- proof technology limits
- handling large numbers of trimmed surfaces

What is Wiggle?

Ron Goldman
Rice University

In computer aided geometric design we are told that a freeform curve or surface must approximate the shape defined by its control points. Just what we are supposed to picture by the phrase "approximate the shape" is generally left somewhat vague, but at the very least we are presumably to understand that the curve or surface does not wiggle too much. But what is wiggle?
In approximation theory the variation diminishing property is invoked to control wiggle. However this property which works very well for explicit functions is not adequate for parametric curves.
Turtle geometry is invoked to provide an intrinsic definition of wiggle for planar C^0 curves. Examples are given to illustrate that the variation diminishing property is an extrinsic property that does not adequately capture either wiggle or turn, but indeed often confounds these two phenomena for parametric curves. Good shape approximation is shown to require a combination of good intrinsic and extrinsic qualities.
Planar Bezier and B-spline curves are not wiggle diminishing, but corner cutting and limiting arguments are used to derive a wiggle-loop diminishing property. Thus Bezier and B-spline planar curves never wiggle or turn more than their control polygons provided that the control polygons do not loop or turn back on themselves.

Interactive Visual surface Design

Roger Anderson
Volvo Data AB

The visual properties of a surface depend on the variation of the normals over the surface. Thus the problem of generating a surface with desired visual properties is a problem of a surface whose normals have a prescribed behaviour.

In the talk we will discuss surface design based on direct modification of renderings of the surfaces or by direct modification of certain curves in offset surfaces.
Selecting a direction field over a parametric surface, modifications of it may be defined by a real-valued function over the parameter domain. In this way, desired visual properties generate first-order partial differential equations for the function. Two different procedures to obtain these equations will be considered.
Having converted the problem of interactive visual design into a PDE-problem, finally, solvability and numerical solutions of the equations will be considered.

Multiresolution Polyhedra

Joe Warren
Rice University

In joint work with Michael Lounsberry and Tony DeRose, we describe an extension of multiresolution analysis to surfaces of arbitrary genus. We develop a new type of wavelet suitable for such surfaces. The definition of this wavelet is unique in that it does not rely on translation and dilation. It may be derived solely from the subdivision procedure. Expansion into this wavelet basis can be done in linear time using standard filter bank methods. We conclude by discussing several applications for such an analysis.

Linear Approximation of Trimmed Surfaces

Pere Brunet
Universidad Politecnica de Cataluna

A new algorithm for the piecewise linear approximation of trimmed surfaces is presented. The algorithm generates a triangulation that approximates the initial surface within a pre-defined tolerance. The approximation is conformal, with no cracks; a closed polyhedron is obtained in the case of a closed initial surface. The algorithm first builds a quadtree-structured bound on the patch curvatures for every surface patch, and then works by first discretizing trimming curves and afterward relaxing the location of a sufficient number of vertices inside the trimmed region in every patch. The resulting triangulation satisfies the max -min angle criterion in the final triangulation. This is a join work with Marc Vigo.

Fair Interpolation using Subdivision Surfaces

Tony DeRose
M. Kass
M. Halstead

We consider the problem of constructing a smooth surface S that is “fair” and interpolates the vertices of a given input polyhedron I. Our definition of fairness is based on the classical thin plate functional. We show that subdivision surfaces (in particular Catmull-Clark surfaces) provide a convenient representation for the surface S.
S can be computed from I by solving a sparse linear system. Performance is good enough to run in (near) real time for polyhedra I of severals hundred vertices.

Infinitesimal Bendings and Stability of Parametric Surfaces

Stefanie Hahmann
Hans Hagen
Universität Kaiserslautern

Infinitesimal bendings have their origin in the mathematics and were mostly developed in the first half of this century.
Infinitesimal bendings are special deformations of surfaces which keep the length of any arbitrary curve on the surface unchanged in the first order. For each infinitesimal bending there exists an unique so-called rotation vectorfield. These vectorfields give a definition of the rigidity/stability of a surface. Using the rotation vectorfields, we propose a measure of stability of parametric surfaces. Finally, we visualize the rotation vectorfield for some Bezier surfaces.

Free-Form Surface Splines

J. Peters
Purdue University

Using a mesh of points in 3D (of arbitrary connectedness), knot spacings (blend ratios) and the connectedness of the points, a space of splines is defined that can model tangent plane continuous surfaces. The surfaces have a parametrization of low degree (2 and 3) and generalize biquadratic and quadratic (box) spline surfaces. A lot of other properties are established: The convex hull property, positivity, partition of unity, intuitive shape parameters in the form of blend ratios etc.
Perturbing the words of the title can lead to some amusing combinations: Spline-free surface forms (advocated in earlier talks), form-free spline surfaces ....

Functional Design of PDE Surfaces in Fluid Dynamics

M. I. G. Bloor
University of Leeds M. J. Wilson

The major difficulty in the analysis inherent in the design of engineering surfaces is being able to represent them by a method which involves few design parameters and which allows easy manipulation in a predictable way. The PDE method is not only capable of producing functionally useful surfaces,

but also of parametrizing them in terms of a relatively small number of 'shape' parameters, owing to its boundary value approach. It is practicable to optimise computationally the surface with respect to its intended functionality. This has been done in a number of model problems in the area of heat transfer, strength and hydrodynamic performance of a yacht hull. In the work presented, more practical and complex geometries are considered with the emphasis on the geometric design parameters and the CAD/analysis interface as a first step towards optimisation.

Minimum Variation Curves, Networks & Surfaces, and Scale Invariance

Henry Moreton
Silicon Graphics
Carlo H. Sequin
U. C. Berkeley

In this work we present a new technique for curve and surface design that combines a geometrically based specification with constrained optimization of a fairness functional. The problem of achieving inter-element continuity is solved simply by incorporating it into the minimization via appropriate penalty functions. Where traditional fairness measures are based on strain energy, we have developed a better measure of fairness; the variation of curvature. In addition to producing objects of clearly superior quality, it is trivial to model regular shapes such as circles and cyclides.
We introduce: curvature variation as a fairness metric, and the minimum variation curve (MVC), network (MVN), and surface (MVS). MVC minimize the arc length integral of the square of the arc length derivative of curvature while interoplating a set of geometric constraints intrinsic to curves. MVN minimize the same functional while interpolating a network of geometric constraints intrinsic to surfaces. Finally, MVS are obtained by spanning the openings of the MVN while minimizing a functional that measures the variation of surface curvature.
Minimum variation shapes are invariant under rigid body transformation and uniform scaling. The values of the curvature functionals, however, change with a change of scale. We have developed curvature variation functionals that are insensitive to scaling. These Scale invariant functionals retain the desirable properties of the earlier functionals while exhibiting greater stability. Scale invariant functionals also make possible the study of canonical curves and surfaces of various topological types, free of interpolatory constraints.
Through difficult interpolation problems and comparisons with traditional

methods we demonstrate the superiority of curvature variation as a fairness metric and the efficacy of optimization as a tool in shape design albeit at significant computational cost.

Scale-Invariant Minimum-Curvature Cost Functionals and Generic Miniumum-Cost Shapes in 2D and 3D

Carlo H. Sequin
U. C Berkeley
Henry Moreton
Silicon Graphics

The use of cost-functionals based on the square of the derivative of curvature rather than on bending energy results in curves and surfaces of improved fairness. However, 1D and 2D minimum variation manifolds still have the same tendency as the classical MEC to run away under certain boundary conditions, forming loops and bubbles of infinite size. This tendency disappears for functionals, such as the MES, that are truly scale-invariant. This scale-invariance can be introduced into the new functionals by multiplication with a suitable power of total arc length or surface area.
The use of scale-invariant cost functionals in curve or surface optimization permits the study of shapes that are determined solely by their topology-free of any external interpolation or arc length constraints. Under minimization of the cost function integral, these shapes will settle in some "natural" unconstrained shape which constitutes either a local or global minimum of the integrated cost in the space of all available degrees of freedom. For smooth curves of any turning number, the global minimum takes on the form of multiply traced circles, or of a "lemnoid" shape for turning number zero. For the SI-MVC functional, curves with turning number ± 1 can fall into a distinct local minimum corresponding to a lemnoid with an inner osculating circle in one of the lobes. The SI-MVC surfaces of low genus (2-5) resemble the shapes obtained with the MES functional, however, they look more pleasing than the "blobby" Lawson surfaces, which are the minimal ME-surfaces.

Cut Locus and Medial Axis

Franz-Erich Wolter
MIT Cambridge, USA

The cut locus C_A of a closed set A in the Euclidean space E is defined as the closure of the set containing all points p which have at least two shortest paths to A. We present a theorem stating that the complement of the cut locus i. e. $E \setminus (C_A \cup A)$ is the maximal open set in $E \setminus A$ where the distance function with respect to the set A is continuously differentiable. This theorem includes also the result that this distance function has a locally Lipschitz continuous gradient on $E \setminus A$. The medial axis of a solid D in E is defined as the union of all centers of all maximal discs which fit in this solid D. We assume in the medial axis case that D is closed and that the boundary ∂D of D is a topological (not necessarily connected) hypersurface of E. Under these assumptions the medial axis of D equals that part of the cut locus of ∂D which is contained in D. The concepts of cut locus and medial axis have recently been found to be important as tools for global shape interrogation and representation in CAD. There exist some computational methods to compute the medial axis and the cut locus in a variety of practically relevant cases. However statements on fundamental topological relation between the shape of a solid and its medial axis mainly exist as conjectures although these relations are crucial for global shape interrogation and representation. Among those are the results that the medial axis has the same homotopy type as its reference solid and that the medial axis can be used to reconstruct its reference solid. We also present a local result stating that the cut locus of a solid's boundary is not dense in any open set in the Euclidean space if the solid's boundary meets certain regularity requirements. We show that the cut locus concept offers a common frame work lucidly unifying different concepts such as Voronoi diagrams, medial axes and equidistantial point sets. In this context we explain that the equidistantial set of two disjoint point sets is a subset of the cut locus of the union of those two sets and that the Voronoi diagram of a discrete point set equals the cut locus of that point set. We present results which imply that a non-degenerate C^1-smooth rational B-spline surface patch which is free of self intersections avoids its cut locus. This implies that for small enough offset distances such a spline patch has regular C^1-smooth offset surfaces which are homeomorphic to the unit sphere. Any of those offset surfaces bounds a solid (which is homeomorphic to the unit disc) and this solid's medial axis is equal to the progenitor spline surface patch. The progenitor spline patch can be "manufactured" with a ball cutter whose center moves along any of those regular offset surfaces and where the radius of the ball cutter equals the offset distance.

Strong Offsets, Weak Offsets and the Quantification of Uncertainties in Geometric Design.

J. Pegna
Rensselaer Polytechnic Institute

A new concept of offset has emerged in the course of experimental works related to the metrology of circles. We shall refer to this mew notion of

offset as “weak offset”, by opposition to the strong offset which refers to the common one.
After a brief description of the experiment and its applications, this paper explores the properties of the two offsets. It is shown that a combination of the two offsets, called “Chebichev zone” unambigously defines the accuracy of measurements on the circle.
We conclude this paper by showing that given a large number of sample points with their corresponding resolution, the precision on the position of the center is greater than that of any of its measurement point.

B-splines and Conversion Methods

Tom Lyche
University of Oslo

We introduce a new elementary matrix approach to B-splines. This formulation gives simple derivations of many B-splines properties. We consider in particular a general approach to conversion problems. This contains knot insertion and degree raising as special cases. We concluded the talk by giving some numerical comparisons of old and new knot insertion methods.

Industrial Requirements for CAGD

D. Ferguson
Boeing, Seattle

We discuss a number of the CAGD issues from the perspective of a large, manufacturing company. Among these are scale, especially in the number of parts and the time that data must be kept for possible reuse. We also describe some mathematical problems related to multi-variate splines that need resolution. Finally, we conclude with a discussion of the pros and cons of NURBS in manufacturing.