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Counting Issues: Theory and Application

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COUNTING ISSUES

in Complexity Theory, Discrete Optimization, and Computational Convexity

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The last few years have seen a rapid growth in research on issues related to counting, both theoretical and applied. Among others, the fields of discrete optimization and more recently, computational convexity (the study of the computational and algorithmic aspects of high-dimensional convex bodies, especially polytopes) have been important sources of new questions involving counting. Furthermore, major new results have been proved about complexity classes based on counting.

This workshop was intended to bring together people from different communities who are interested in counting issues, and to facilitate (and further stimulate) communication amongst the researchers involved.

According to the concept of this conference, the participants belonged to different fields of theoretical computer science and discrete mathematics.

The meeting was attended by 22 participants who each gave a talk. Some of the lectures surveyed new developments in important subfields, others presented recent new results. The lengths of the talks varied between 30 and 60 minutes.

The topics that were discussed at the workshop reflected the wide range of the subject. Some of the lectures dealt with aspects of counting related to structural complexity theory, with complexity classes based on counting, and with counting issues in the theory of communication complexity. Others were devoted to randomized and approximate counting problems, while yet others mentioned algebraic aspects of the field. Another group of contributions focussed on graph theoretic counting problems, and certain complexity issues in enumerative combinatorics. Some other talks gave results on counting issues in integer programming, while counting (and uniqueness) issues in convexity was the subject of another group of lectures. Many of the problems discussed were motivated by practical applications.

In addition various open problems were stated which led to vivid discussions and numerous interactions.

The conference showed that even though the participants belonged to different fields that have quite different tool-boxes, approaches and ideas for solving their problems, there is a deep and close connection which is centered around the concept of counting.

Peter Gritzmann & David Johnson & Victor Klee & Christoph Meinel

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Inductive Counting for Branching Programs

by **Carsten Damm** (joint work with M. Holzer and K.-J. Lange)

Immerman (1987) and Szelepcsényi (1987) proved that for space bounds $s(n) \geq \log n$, $NSpace(s(n))$ is closed under complement. The method they used — it came to be known as “inductive counting” — relies on the fact that the space bound allows to implement a step counter. A consequence of this celebrated result is the collapse of the alternating space hierarchy for space bounds above $\log n$.

In contrast to this it has been proved that for space bounds below $\log n$ (that is $s(n) \in \Omega(\log \log n) \cap o(\log n)$) the alternating space hierarchy is infinite (Liškiewicz/ Reischuk 92; Geffert 92; von Braunmühl 92). The key argument there is uniformity: alternating Turing machines that are sublogarithmically space bounded work the same way on inputs of the form $u1^m v$ and $u1^{m+lm^l} v$.

Can one throw away both obstacles (the need to implement step counters and uniformity) to complement on a nonuniform model of computation? We show that this is possible. We perform inductive counting on nondeterministic branching programs while increasing the width of the programs from w to at most $O(w^3)$. Width restricted branching programs are interesting objects of study in their own since they can be regarded as being half-way between NC^1 (constant width, poly size) and L (poly width, poly size) or NL in the nondeterministic case. Additionally we show that width restricted branching programs are equivalent in computational power to a variant of nonuniform Turing machines, which are generalizations of Barrington’s nonuniform automata and correspond directly to the usual Karp/Lipton model of nonuniformity in case of space bounds $s(n) \geq \log n$. That means nonuniform Turing machines can complement regardless of the space bound.

Counting Lattice Points in Polytopes in Fixed Dimension

by **Martin Dyer** (joint work with Ravi Kannan)

Very recently, Alexander Barvinok gave a polynomial time algorithm for the problem of counting the number of integer points in a polytope in any fixed

dimension d . This improved dramatically a result of Dyer (1991) for the cases $d \leq 4$. Barvinok's method uses complex analysis and a theorem of Brion on exponential sums over polytopes. We show that the machinery of complex variables and Brion's theorem can be dispensed with. However our "improved" algorithm still relies heavily on Barvinok's ideas. We will describe Barvinok's methods, our improvements and the history of this topic.

Approximately Counting Hamilton Cycles in Dense Graphs

by **Alan Frieze** (joint work with Martin Dyer and Mark Jerrum)

Call a graph G on n vertices, dense if it has minimum degree at least $(1/2 + \alpha)n$ for some constant $\alpha > 0$. We consider the problems of computing the number of Hamilton cycles, the number of Hamilton paths, the total number of cycles, the total number of paths. We describe *FPRAS*'s for each problem restricted to dense graphs.

We also show that it is $\#P$ -hard to count Hamilton paths or cycles exactly in dense graphs and NP -hard to determine the existence of a Hamilton path or cycle in graphs with minimum degree $\geq \gamma n$ for constant $\gamma < 1/2$.

Counting and Efficient Data Structures for Boolean Functions

by **Jordan Gergov**

It is proved that any deterministic, nondeterministic, co-nondeterministic and MOD_p (p is a prime) oblivious branching program of linear length for the integer multiplication is of size $2^{\Omega(n)}$.

Counting the number of one's ($\#f^{-1}(1)$) of a Boolean function f given its *FBDD* representation (free binary decision diagram, read-once branching program) can be done easily in polynomial time (e.g. Akers,78). We proved that counting the number of one's given a parity (MOD_2 , *EXOR*) *OBDD* for f is $\#P$ -complete (for motivation e.g. Gergov & Meinel, STACS'93) while the corresponding decision problem is in R .

On the Complexity of Computing (Mixed) Volumes

by Peter Gritzmann

We report on recent results (jointly with M. Dyer and A. Hufnagel) and some of their applications concerning the problem of computing the volume of zonotopes and of computing mixed volumes of polytopes (and more general convex bodies) by means of deterministic and randomized algorithms.

Applications touched upon contain a problem from mixture management and the problem of computing the permanent of an integer matrix.

Hypergraphs and Complexity Classes

by Ulrich Hertrampf

We investigate complexity classes in the area between P and $PSPACE$ which can be defined via leaf languages, like NP (leaf language $0^*1\{0,1\}^*$), $co-NP$ (leaf language 0^*), $\oplus P$ ($\{\omega / \# \text{ 1's in } \omega \text{ is odd}\}$), etc.

Introducing a generalized form of hypergraphs, we give a very general criterion to decide, whether there is an oracle separation between two such classes.

The results include well-known separations like $\exists X: NP^X \not\subseteq co-NP^X$, or $\exists X: MOD_j-P^X \not\subseteq MOD_k-P^X$ (where k is a prime number, not dividing j), but also new ones, like

$$\exists X : C_{>2, \equiv 1(2)}^X \not\subseteq C_{>1, \equiv 0(2)}^X$$

where $C_{>a, \equiv b(2)}$ means the class defined by the following acceptance condition: $\#$ of accepting computations of a nondeterministic polynomial time machine is greater than a , and is congruent to $b \pmod 2$.

Generalization of the Coloring Problem

by **Klaus Jansen**

(joint work with H.L. Bodlaender, P. Scheffler and G.J. Woeginger)

We discuss the Precoloring Extension (PrExt) and the List Coloring (LiCol) problems for trees, partial k -trees, bipartite graphs and cographs in the decision and the construction versions.

PrExt

given: an undirected graph $G = (V, E)$, $m \in \mathbb{N}$ and a m -coloring of a vertex subset $W \subseteq V$.

question: is the coloring extendible to the entire graph G ?

LiCol

given: an undirected graph $G = (V, E)$, subsets $S_v \subset \{1, \dots, m\}$, $v \in V$.

question: is there a m -coloring f of G such that $f(v) \in S_v$, $v \in V$?

Both problems for partial k -trees are solved in linear time, when m is bounded by a constant and by polynomial time algorithms for unbounded m . For trees we improve this to linear time. Moreover, we prove that 1-PrExt is NP -complete for bipartite graphs and $m = 3$. In contrast to that, PrExt and LiCol differ in complexity for cographs while the first has a linear decision algorithm, the second is shown to be NP -complete. We give polynomial time algorithms for the corresponding enumeration problems #PrExt, #LiCol on partial k -trees and trees and for #PrExt on cographs.

Approximately Counting Hamilton Cycles in a Random Regular Graph

by **Mark Jerrum** (joint work with Alan Frieze and Mike Molloy)

The problem of determining whether a cubic (3-regular) graph is a Hamiltonian is known to be NP -complete; it follows that there can be no *fpras* (fully-polynomial randomised approximation scheme) for the number of Hamilton cycles in a cubic graph unless $RP = NP$. This is a "worst case" result. In contrast, we show that there is an *fpras* for Hamilton cycles that succeeds for almost every r -regular graph G , where r is any constant ≥ 3 . (In the event of failure, the algorithm provides a warning message.)

The basic idea of the algorithm is to estimate the number of 2-factors in G ; and then the ratio of Hamilton cycles to 2-factors by Monte Carlo experiment. The verification of the algorithm rests on showing that the ratio of Hamilton cycles to 2-factors in G is likely to be not too small: this involves a difficult second moment calculation combined with an application of the “conditional variance” method.

Counting Issues

by **David S. Johnson**

We provide an introductory survey of the topics of the workshop. We begin with a mock history of counting, leading through the traditional mathematical diversion of counting the number of objects of a given type of “size” n (e.g., the number of trees with n leaves) to the question of counting the number of objects derivable from a given graph, polytope, etc. (e.g., the number of perfect matchings in a given graph). We then introduce the complexity classes P and $\#P$ for classifying such problems, and give a quick survey of $\#P$ -hardness results (prepared by Milena Mihail and Peter Winkler). Next we introduce the concept of a fully polynomial randomized approximation scheme (*FPRAS*) and mention some results about the existence (or non-existence, assuming $NP \neq R$) of such schemes for certain $\#P$ -hard problems. We then briefly discuss the question of listing objects, as opposed to counting them, and the various notions of output-sensitive “polynomial-time” for this task. The next part of the talk is a brief survey of complexity classes related to counting. By this we mean classes defined in terms of the number of accepting computation of a nondeterministic Turing machine (*NDTM*). For polynomial time *NDTM*’s, we get classes such as $\#P$, PP , $\oplus P$, and *Unique-P*. For polynomial time *NDTM*’s obeying additional restrictions on their operation, we obtain classes such as R , BPP , *Few-P*, UP , and (with some additional contortions) IP . The talk concludes with a brief description of other complexity theory issues related to computing, such as the complexity of optimization problems with unique optimal solutions, and the power of boolean circuits augmented by *mod-k* and threshold gates.

Computing Threshold Functions by Depth 3 Threshold Circuits

by **Stasys Jukna**

The threshold gate with threshold value s is any of the two Boolean functions T_s^m and T_{m-s+1}^m where $T_s^m(x_1, \dots, x_m) = 1$ iff $\sum x_i \geq s$. We prove that any depth 3 threshold circuit with threshold values of its gates $\leq s$ which computes T_k^n and has bottom fan-in $\leq t$, must be of size at least $(n/kst)^{k/s}$. We also show that depth 3 threshold circuits with threshold values of its gates $\leq s$ which computes the majority function $T_{n/2}^n$ and has either only *AND*'s or *OR*'s at the bottom must have size $\exp(\Omega(\sqrt{n}/s))$.

Efficient Approximation of Some “Hard” Algebraic Counting Problems

by **Marek Karpinski**

We present some efficient randomized approximation algorithms for a number of (provably) hard algebraic and geometric counting problems, like the problem of estimating the size of an algebraic set or the size of an algebraic curve over $GF[q]$. The proof methods involve the convexity argument on certain cylindrical partitions of the sets of nonzeros of multivariate polynomials over $GF[q]$, and the ratios on the number of nonzeros.

The above problems have been proven, only recently, to be computationally intractable in the exact counting setting.

Some Geometric Uniqueness Problems

by **Victor Klee**

Each of the following problems turns out to be *NP*-hard:

Instance: $n \in \mathbb{N}$, n -parallelotope in \mathbb{R}^n with one vertex at the origin.

Question: Does the Euclidean norm attain its maximum at more than one vertex of the parallelotope?

Instance: $n \in \mathbb{N}$, n -parallelotope in \mathbb{R}^n .

Question: Does the circumsphere contain (in its boundary) more than one diametral pair of vertices?

Instance: $n \in \mathbb{N}$, n -cross-polytope in \mathbb{R}^n .

Question: Does the insphere hit more than one pair of opposite facets?

(The hardness is proved in the following paper: P. Gritzmann & V. Klee, Deciding uniqueness in norm maximization, *Math. Programming* 57 (1992) 203–214.)

Now let $\psi : \mathbb{N} \rightarrow \mathbb{N}$ and $\gamma : \mathbb{N} \rightarrow \mathbb{N}$ be such that $1 \leq \gamma(n) \leq n$ and both ψ and γ are $\Omega(n^{1/k})$ for some $k \in \mathbb{N}$. Then the following problem is NP -hard:

Instance: $n \in \mathbb{N}$, n -polytope P in \mathbb{R}^n given as the convex hull of its $n + \psi(n)$ vertices.

Question: Is the largest* $\gamma(n)$ -simplex in P unique? (* largest with respect to $\gamma(n)$ -measure).

(The hardness is proved in the following paper: P. Gritzmann, V. Klee & D. Larman, Largest j -simplices in n -polytopes, manuscript, 1993.)

On the Power of Single Bits of a $\#P$ Function

by **Johannes Köbler**

(joint work with F. Green, K. Regan, T. Schwentick, S. Toda, J. Torà)

We study the class MP of languages which can be solved in polynomial time with the additional information of one bit of a $\#P$ function. It is shown that the polynomial hierarchy and the classes MOD_kP , $k \geq 2$, are low for MP . They are also low for a class we call $AmpMP$ which is defined by abstracting the “amplification” methods of Toda. As a consequence we get a new upper bound for Barrington’s class ACC which might be useful in separating TC_0 from ACC .

To resolve the question of the computational power of $AmpMP$ we introduce the generalized MOD -class $ModP$. We show that any $\#P$ function can be computed in polynomial time by asking parallel queries to a $ModP$ oracle. Furthermore we prove that $ModP$ is contained in $AmpMP$. This shows that $AmpMP$, $ModP$, and MP are equally powerful.

On the Computational Power of Boolean Circuits vs. Depth 2 Threshold Circuits

by Matthias Krause

The most powerful methods which are known for lower bounding the size of threshold circuits are:

1. the discriminator method giving explicit exponential lower bounds on the number of edges of depth 2 threshold circuits, and
2. the spectral method giving exponential lower bounds on the number of nodes (unbounded number of edges) of depth 2 threshold circuits if the bottom level contains only \oplus -gates (threshold- \oplus circuits).

Using probabilistic arguments we give a more powerful method which works for threshold- MOD^r circuits, r -arbitrary, and which allows to prove new structural results on Boolean- vs. depth-2 threshold circuits. In particular we show that:

- For distinct primes p, q threshold- MOD^q circuits for MOD^p have exponentially many nodes,
 - For all natural r there are $AC_{0,3}$ -functions which need exponential size threshold- MOD^r circuits,
 - All $AC_{0,2}$ -functions can be efficiently realized by threshold- MOD^2 circuits.
- The second result is of special interest because the known lower bound methods (1 and 2) don't provably work for AC^0 -functions. Thus we get a (partial) negative answer to the open question whether there is a more efficient simulation of AC^0 -circuits by small depth threshold circuits better than that given by [YAO 90] yielding $ACC \subseteq TC_{0,3}$.

Counting Rich Cells in Arrangements of Hyperplanes in \mathbb{R}^d

by D. G. Larman

Let H_1, \dots, H_n be an arrangement of n hyperplanes in \mathbb{R}^d . These hyperplanes partition \mathbb{R}^d into regions which we call cells. A cell is rich if every hyperplane H_1, \dots, H_n touches the cell. The maximal number of such rich cells is $\sim n^{d-2}/d - 2$, n large.

How large can n be so that if H_1, \dots, H_n is in general position, a rich cell is guaranteed. In \mathbb{R}^2 , $n = 4$ and conjecture $n = 2d$ in \mathbb{R}^d .

Some Lower Bounds on the Counting Communication Complexity of MOD_m - GAP

by **Christoph Meinel**

We consider the counting versions of the graph accessibility problems MOD_m - GAP , $m > 1$. These problems are complete for the counting classes MOD_m - $LogSpace$. With the aid of rank arguments and certain projection reductions we prove some lower bounds on the counting communication complexity MOD_k -Counting and MAJ -Counting. In detail we show:

$$\begin{aligned} MOD_k\text{-Counting}(GAP_n) &= \Omega(\sqrt{n}) \\ MOD_k\text{-Counting}(MOD_m - GAP_n) &= \Omega(\sqrt{n}) \\ MAJ\text{-Counting}(GAP_n) &= \Omega(\sqrt{n}) \\ MAJ\text{-Counting}(MOD_m - GAP_n) &= \Omega(\sqrt{n}). \end{aligned}$$

This work was done together with Stephan Waack (Göttingen).

Counting Triangle-Free Graphs

by **Hans-Jürgen Prömel**

An important result of Erdős, Kleitman and Rothschild (1976) says that almost every triangle-free graph on n vertices has chromatic number 2. This result allows to derive easily an asymptotic formula for the number of triangle-free graphs. In this talk we study the asymptotic structure of graphs in $Forb_{n,m}(K_3)$, i.e. in the class of triangle-free graphs on n vertices having $m = m(n)$ edges. In particular, we prove that an analogue to the Erdős/Kleitman/Rothschild result is true, whenever $m \geq cn^{7/4} \log n$ for some constant $c > 0$. On the other hand, it is shown that almost every graph in $Forb_{n,m}(K_3)$ has at least chromatic number 3, provided that $c_1 n < m < c_2 n^{3/2}$, where $c_1, c_2 > 0$ are appropriate constants.

This is joint work with A. Steger (Bonn).

On the Number of Graph Automorphisms

by **Jacobo Toran**

We survey some results on the counting properties of the Graph Automorphism and Graph Isomorphism problems, obtained with J. Köbler and U. Schöning. We show that for the case of GA , there is a nondeterministic machine that on input a graph G has exactly $2^{p(|G|)}$ accepting paths if $G \in GA$, and exactly $2^{p(|G|)} + 1$ accepting paths if $G \notin GA$. This implies that GA is in $\oplus P$ and low for the class PP . For the Graph Isomorphism problem we show that there is a machine with a similar accepting mechanism as in the GA case, but now it has 2^p accepting paths if the input graphs are isomorphic, and $2^p + f(|G_1|)$ accepting paths if they are not isomorphic (where f is a poly-time computable function). This implies that GI is low for PP .

We also show some results indicating the difficulty of approximating the function $\#Aut$, counting the number of graph automorphisms, in the sense of enumerability. We show that if $\#Aut$ has a $\log^{1-\varepsilon}(n)$ enumerator then $GA \in P$, and if $\#Aut$ has an $n^{1/2-\varepsilon}$ enumerator then $GA \in R$.

Introducing Interactions into Randomized Optimization Algorithms

by **Umesh Vazirani**

Running a heuristic randomized optimization algorithm k times is conceptually the same as running k simulations simultaneously without interaction. Can one do better by introducing some interaction? We study this question in the context of a concrete model of finding deep vertices on trees. This tree model mimics aspects of the polynomial time behavior of simulated annealing algorithms.

Joint work with David Aldous.

On Different Reducibility Notions for Function Classes

by Heribert Vollmer

We continue research of Toda on problems complete for function classes like $FP^{\#P}$ and $MidP$ under Krentel's metric reductions. We first show that metric reductions wipe out the difference between $MidP$ and other related classes of functions which are probably different from $MidP$. In order to obtain a more detailed classification of naturally arising functional problems we then examine a stricter notion of reducibility and show that a number of problems, among them those proved by Toda to be hard for $MidP$ under metric reductions, are complete for different classes of median functions related to $MidP$ under this stricter reducibility. Finally, we use these results to exhibit new natural complete sets for the well-studied classes of sets PP , PP^{NP} , and P^{PP} .

Counting Issues in a General Theory of Polynomial Time Complexity Classes

by Klaus W. Wagner

Many of the well-studied complexity classes like NP , BPP , PP , $\oplus P$, Σ_k^p , ... can be considered to be the result of the application of an operator to the class P . For example, for any class K we define:

$A \in \exists \cdot K \Leftrightarrow_{def}$ there exist a set $B \in K$ and a polynomial p such that:

$$x \in A \leftrightarrow \exists y (|y| = p(|x|) \wedge (x, y) \in B),$$

and we obtain $NP = \exists \cdot P$.

The main observation of the talk is: all the exciting results on the relationships between the above mention results can be considered as results on the corresponding operators applied to the class P , and they remain valid when the operators are applied to an arbitrary class K fulfilling

- K is closed under union and intersection,
- K is closed under polynomial-time conjunctive and disjunctive tt -reducibility,
- $P \subseteq K$.

For such classes K we can prove that e.g. (assume $K = coK$):

- $BP \cdot K \subseteq \exists \cdot \forall \cdot K \cap \forall \cdot \exists \cdot K$ (Lautemann/Sipser/Gacs)
- $NP^K = \exists \cdot K$
- $\exists \cdot \forall \cdot \exists \dots K \subseteq P^{C \cdot K}$ (Toda)
- ⋮

Randomised Counting Problems

by **Dominic J. A. Welsh**

Each of the following problems is a specialisation of the problem of evaluating the Tutte polynomial at a particular point of the plane:

1. counting subforest
2. counting acyclic orientations
3. counting connected subgraphs
4. determining the chromatic polynomial
 - # of flows over any abelian group
 - Jones polynomial of Knot
 - weight enumerator of linear code.

As we (Jaeger, Vertigan, Welsh 1990) have shown the Tutte plane is $\#P$ -hard at all but one curve $(x-1)(y-1) = 1$ and at 8 special points, it follows that all above are $\#P$ -hard counting problems.

I now consider the possibility of obtaining a *fpras* (fully polynomial randomised approximation scheme). It turns out that 1. and 3. are approximable when the graph is dense (every point has at least $\alpha|V|$ neighbours for some α). The general question of which point has a *fpras* is wide open. The last results are due to Annan (1993), Frieze/Welsh (1993), who show also that reliability and Potts are *fprasable* for dense graphs.

I also give a survey of specific open problems in this area.

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