Domain Theory and its Applications

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This Dagstuhl seminar focused on Domain Theory and Its Applications. Domains were first discovered by Dana Scott in the mid-1960s, and quickly became of central importance in the area of programming language semantics, which remains the primary area of their application. Recently, more diverse applications of domain theory to other areas have been discovered, and this has led to a flourishing of activity in the area. The seminar focused on this diverse range of applications, as well as on the role of domain theory in its more established areas of application.

The Dagstuhl Seminar also was coordinated with the Spring, 1998 meeting of the series, the Mathematical Foundations of Programming Semantics. This series meets annually, mainly in the U.S., and is among the most popular meetings for researchers working in programming semantics and its mathematical underpinnings. Coordinating the Dagstuhl Seminar on Domain Theory with the MFPS meeting allowed us to attract some of the most active researchers in domain theory to both meetings. More importantly, the Dagstuhl meeting was used to assess the state of domain theory, and important insights were gained through the seminar into the status and future possibilities for domain theory and its applications.

Among the various areas within domain theory, we mention a few that were highlighted during the Dagstuhl Seminar:

- *Structure theory of domains:* This "basic" area of domains has had recent impetus through the work of Achim Jung (*Birmingham*) in which the approach of *abstract bases*, originally proposed by M. B. Smyth (*Imperial*), was used to devise a uniform approach to the theory of continuous domains in analogy to the traditional approach to algebraic domains.

- *Domain theory and topology:* Topological aspects have played a fundamental role in domain theory since its founding. In recent years, efforts have been made by M. B. Smyth (*Imperial*), Jan Rutten (*CWI*). Bob Flagg (*Southern Maine*) and others to generalize aspects of domain theory to more general settings. In particular, the basic question of the relationship of domain theory and (quasi-)metric spaces has recently received considerable attention.

- *Domain theory and category theory:* Categorical considerations are of primary importance in domain theory. Domains provided the first (and, as far as we know, the only) categories in which one can solve the equation $X \simeq [X \to X]$ non-trivially. More recently, the work of Kim Wagner (*Cambridge*) has demonstrated that some of these categorical results and related ones in categories of metric spaces have an intimate connection.

- *Synthetic Domain Theory:* The work of Martin Hyland (*Cambridge*) and of Giuseppe Rosolini (*Genoa*) and others on synthetic domain theory – the attempt to establish a theory of sets in which all sets are domains – has had a major influence on the development of the theory and on its relationship to other areas, most notably category theory.

- *Axiomatic Domain Theory:* One of the tantalizing aspects of domain theory is the fact that the only solutions of the equation $X \simeq [X \to X]$ that are known in any cartesian closed category all reside within categories of domains. The work of Peter Freyd (*Pennsylvania*) and of Marcelo Fiore and Gordon Plotkin (*Edinburgh*) has led to a better understanding of this issue.

- *Domain theory and logic:* Domain theory has served as a fertile ground for models for many of the logics that are fundamental to theoretical computer science. The seminal work of Samson Abramsky (*Edinburgh*) has shown how domains are closely linked to program logics. Recent attempts to extend Abramsky's approach has appeared in the work of Marta Kwiatkowska (*Birmingham*) on the modal mu calculus, and Marcello Bonsangue (*Vrei Universiteit Amsterdam*) has provided an extension with the notion of observation frames.

- *Domain theory and concurrency:* Domain theory has been the traditional area in which to devise models for concurrency and process algebra. Recent work by Paul Gastin (*LITP, Paris*) and Volker Diekert (*Stuttgart*) has used domains to provide models for true concurrency, in which parallel composition is taken as primitive, rather than being resolved in terms of sequential composition and nondeterministic choice. These areas were amply represented at the seminar.

- *The Comprox Group:* One of the pioneers of applications of domain theory to other areas in the last few years has been Abbas Edalat (*Imperial College*). Along with such colleagues as Reinhold Heckmann (*Saarbrücken*), he has shown how domains and their topological analogues provide unexpected and remarkable insights into phenomena ranging from fractals to neural networks and learning automata to such established areas as real analysis and the Riemann integral.

An additional important accomplishment of the seminar was the drafting of a *List of Open Problems* on domain theory. This list, which is currently being edited and refined, will be the template for a number of such lists in several areas of theoretical computer science.

This report would not be complete without some comments about the facilities which Schloß Dagstuhl provided, and the tenor of the meeting those facilities afforded. Dagstuhl seminars have become famous for the informal atmosphere and the attentive service that the staff provides to the participants. This environment is ideal for focusing on the main goals of such seminars: to bring together researchers who share similar interests, and to provide them with a stimulating environment in which communication about recent research results can be exchanged and new collaborations can be established. The Organizers wish to thank the staff of the Center, and especially its director, Professor Dr. Reinhard Wilhelm for their support in carrying out the preparations for the seminar, and for the efficient and effective service they provided during the seminar. This allowed us to participate in the seminar activities as if we were participants, and not organizers, and relieved us of the burdens that often make organizing such a meeting an onerous task.

Abstracts

A Set–Theoretic Semantics for the π–Calculus

Michael Baldamus

Recent works on the denotational semantics of the π–calculus have employed category or type theory. Here, a semantics is presented that is based on the theory of non–well–founded sets and on a comparatively simple notion from category theory, namely that of final coalgebras.

We focus on early bisimulation, where the idea is to adopt a two–level approach. On the first level, early ground bisimulation is modeled using Aczel's Special Final Coalgebra Theorem; on the second level, early bisimulation proper can then be modeled by considering functions with name substitutions as arguments and denotations from the first level as values.

A major difficulty consists of the fact that π–calculus processes may be early ground bisimilar although they are not related under CCS–like bisimulation. To solve this problem, a refined SOS is introduced: Early ground bisimulation over the standard SOS is equivalent to classical, CCS–like bisimulation over the refined SOS. The (first–level) final coalgebra semantics ensuing from that, however, does not allow one to interpret the restriction constructor. For this reason, the refined SOS is further refined. The corresponding final coalgebra semantics supports compositional interpretations of all constructors except input prefixing; the second–level semantics built on top of *this* semantics supports compositional interpretations of all constructors.

Recursive Analysis in PER($\mathcal{P}\mathbb{N}$)

Andrej Bauer

The category of partial equivalence relations on the algebraic lattice $(\mathcal{P}\mathbb{N}, \subseteq)$ is locally cartesian closed, has countable coproducts, and the category of countably based T_0–spaces embeds faithfully in it. In PER($\mathcal{P}\mathbb{N}$) there is also a computing operator $\#$.

This category then contains the spaces of classical analysis, such as $\mathbb{R}$ and $\mathbb{R} \longrightarrow \mathbb{R}$, together with their recursive versions $\#\mathbb{R}$ and $\#(\mathbb{R} \longrightarrow \mathbb{R})$. The cartesian closed structure makes it possible to extend the study of computability in analysis to higher types, such as $(\mathbb{R} \longrightarrow \mathbb{R}) \longrightarrow \mathbb{R}$, in a systematic way.

This work is part of a Ph.D. proposal and was done in collaboration with Dana Scott, Steve Awodey and Lars Birkedal.

Relational Semantics Revisited

Eike Best
(joint work with K.M. Richter)

Flat domains containing a "bottom" element, the Plotkin powerdomain and the Egli–Milner ordering can be eschewed (more precisely, replaced by sets, powersets and the subset relation, resp.) in the denotational definition of the relational semantics of programs even if infinite nondeterminism is allowed.

Infinitary Logic of Domains

MARCELLO BONSAGUE

Abramsky's interpretation of domains as finitary propositional theories is very weak in expressive power because it reflects the logic of compact opens. In this talk we present a more expressive interpretation using *infinitary propositional theories*. The key idea is that the Lindenbaum algebra of the new infinitary interpretation forms a completely distributive lattice free over the distributive lattice arising from the Lindenbaum algebra of Abramsky's original interpretation. We show that both interpretations determine the same SFP–domain up to isomorphism, using the framework of *observation frames* and a new characterization of sober spaces in terms of the lattice of saturated sets. As example we treat the Plotkin powerdomain construction and the function space.

Terminal Metric Spaces of Languages

FRANCK VAN BREUGEL

In the area of metric semantics, various metric structures have been proposed to model a wide spectrum of programming notions. In this talk, I concentrate on languages. These sets of (finite and infinite) words are endowed with a combination of the Baire and Hausdorff metric. This only gives us a pseudometric space. By restricting ourselves to compact or closed sets we obtain a complete metric space. These languages are suited for handling finitely branching and image finite labelled transition systems as we will see.

During the last decade the insight gradually grew that systems like labelled transition systems can be described as coalgebras. Among these coalgebras, the terminal ones play an important role. It provides us with definitions and proofs by coinduction. In this talk I show that also the above mentioned languages are terminal coalgebras.

Idealized CSP: Generalizing the Kahn Principle to Fair Networks of Processes

STEPHEN BROOKES

The Kahn principle for deterministic networks of asynchronous processes is well known. It states that each node can be viewed as computing a continuous function from input streams to output streams, and characterizes the behaviour of the network as the least fixed point of a suitable functional. Non–deterministic networks arise naturally, raising issues such as fairness and causing well known difficulties. The continuous stream functions idea cannot be simply generalized to stream relations in a satisfactory manner, as shown by the Brook–Ackermann anomaly. Several approaches have been proposed as generalizations of Kahn's semantics to non–deterministic networks. The prevailing tendency has been to stay close in spirit to the original model, typically retaining continuity in some form. However, these approaches typically fail to deal properly with fairness. We argue that fairness is a crucial feature that should be built into a semantics, in order to permit reasoning about liveness properties of networks, and that in a fair model the computational intuition behind Kahn's model no longer fits. This leads us to formulate a transition–trace semantics for non–deterministic networks: a trace set can be seen as a "stream–relation extended in time", and this forms a natural generalization of Kahn–style stream functions, avoiding the anomalies and building in fairness. An advantage of our approach is the fact that the same trace model can also be used to interpret shared–variable parallel programs, so that we have a unifying framework for paradigms of concurrency which have traditionally been modelled in mutually incompatible ways.

Equational Theories of Recursive Types

Felice Cardone

Consider recursive types for pure λ–terms, defined by the rules:

$$A ::= t \mid A \longrightarrow A \mid \mu t.A$$

($t \in V$, for V denumerable set of type variables). If A is a recursive type, let A^* be the possibly infinite (regular) tree obtained by performing infinitely many steps of unfolding of the recursions. Recursive types A, B are said to be *strongly equivalent* if $A^* = B^*$, written $A \approx B$.

We show that strong equivalence coincides with the equivalence induced by the equality of interpretation (in all environments for type variables) in the following two cases:

(1) recursive types are interpreted as complete and uniform partial equivalence relations (pers) over any domain, of a suitable form, obtained by the inverse limit construction;

(2) recursive types are interpreted as (continuous) closures over the λ–model $P\omega$.

Approximating Traces

Volker Diekert
(joint work with Paul Gastin)

Mazurkiewicz traces are generalised by adding some alphabetical information concerning the possible continuation of a process. This leads to a natural approximation ordering:

$$(r, A) \sqsubseteq (s, B),$$

if the trace r is a prefix of s and if A includes every letter from the suffix $r^{-1}s$ and from B. The bottom element is $(1, \Sigma)$: nothing has happened and everything is possible.

We obtain a coherently complete and prime algebraic domain. The composition is shown to be continuous in the Lawson topology.

Another way to define the model is as follows: For $n \geq 0$ and a trace $r \in M = \Sigma^*/\{ab = ba \mid (a, b) \in I\}$ define the n–th approximation by

$$r[n] = \bigsqcup\{p \in M \mid p \leq r \text{ and } |p| \leq n\}.$$

With a suitable multiplication the set $F[n] = \{(r[n], A) \mid r \in M,\ A \subseteq \Sigma\}$ forms a finite monoid. Our model can be viewed as the projective limit

$$\varprojlim F[n].$$

The results presented in the talk will appear in Acta Informatica.

Asynchronous Cellular Automata and Domains

Manfred Droste
(joint work with Paul Gastin)

Asynchronous cellular automata (ACA) provide a model for concurrent processes. They work over pomsests (labelled partially ordered sets), where the order describes a causality dependence between events and incomparability means independence or concurrency. Given an ACA $\mathcal{A}$, the set of all pomsets for which there exists an accepting run of $\mathcal{A}$ forms (with inclusion) a domain. We investigate the order–theoretic structure of these domains. We show that form particular coherent dI–domains, and any distributive concrete domain arises as the domain associated with a suitable ACA.

Root Finding in Exact Real Arithmetic Using Linear Fractional Transformations

Abbas Edalat Fabian Rico

We present two algorithms for computing the root, or equivalently the fixed point, of a function in exact real arithmetic based on linear fractional transformations and exact floating point. The first algorithm finds the unique attracting fixed point of a function in an interval by iterating the expression tree representing the function. The second algorithm obtains the unique attracting or repelling fixed point of a function by trisection of intervals which can be compared with the well–known bisection method in numerical analysis.

A Conceptual Analysis of Weak Bisimulation

MARCELO FIORE

We provide an analysis of weak bisimulation via open maps in the Kleisli categories of monads induced by suitable adjunctions of the form

$$\text{Existential quantification} \dashv \text{Substitution}$$

in the setting of presheaf (and fibrational) models of concurrency. These Kleisli categories can be seen as categories of transition systems and weak homomorphisms, viz. homomorphisms mapping strong transitions $\rightarrow$ to weak ones $\Rightarrow$, which is useful for treating examples. As a first use of the framework we show that strong bisimulations are always weak ones.

The Appearance of Big Integers in Exact Real Arithmetic based on Linear Fractional Transformations

REINHOLD HECKMANN

One possible approach for exact real arithmetic is to use linear fractional transformations to represent real numbers and computations on real numbers. We show that the bit sizes of the (integer) parameters of nearly all transformations used in computations are proportional to the number of basic computational steps executed so far. Here, a basic step means consuming one digit of the argument(s) or producing one digit of the result.

Domain Theory and Probabilistic Verification

MICHAEL HUTH

We review the semantics of linear temporal logic (LTL) over concurrent Markov chains and discuss its problematic aspects if used as the foundation of a verification tool. If S is the state set, then this semantics is of type

$$\langle D, A\rangle : \mathrm{LTL} \times S \longrightarrow \mathcal{I}$$

where $\mathcal{I}$ is the interval domain over $[0,1]$. We propose a least–fixed point semantics $\langle D', A' \rangle$ where conjunction has a compositional, conservative interpretation. We prove soundness ($\langle D', A' \rangle \leq \langle D, A \rangle$) of that semantics and raise two principal issues:

(a) for which classes of models/formulas does $\langle D', A' \rangle$ render "good" approximations?

(b) one may use $\langle D, A \rangle$ and $\langle D', A' \rangle$ for verifying a hybrid logic which contains probabilities as thresholds or leaves them open as queries.

Do Functional Bisimulations Form a Topos?

Peter Johnstone
(joint work with John Power, Toru Tsujishita,
Hiroshi Watanabe, James Worrell)

A functional bisimulation of labelled transition systems may be regarded as a map of F–coalgebras, where F is the functor $P(A \times -)$ (and a general bisimulation is a relation in the category of F–coalgebras). Motivated by this, we study what can be said about the category (F–coalg), where $F : \mathcal{E} \longrightarrow \mathcal{E}$ is an endofunctor of a 'suitable' category $\mathcal{E}$ having properties like those of $P(A \times -)$ (or, more accurately, of $P_\omega(A \times -)$, where P_ω is the finite–powerset functor). Such functors do not in general preserve pullbacks, but they do have various weak preservation properties, which we investigate. We show that, provided F generates a cofree comonad and preserves weak pullbacks, then (F–coalg) inherits (finite) limits, regularity and a subobject classifier from $\mathcal{E}$; if $\mathcal{E}$ is a Grothendieck topos, then it inherits all the Giraud axioms except for effectiveness of equivalence relations. If F decomposes as $(1 + F_0)$, (F–coalg) cannot be a topos unless F actually preserves pullbacks; but it becomes a topos when we formally 'effectivize' it by splitting its equivalence relations. We give an explicit description of the topos structure when $\mathcal{E} = \mathbf{Set}$ and F is the finite–multiset functor.

The Troublesome Probabilistic Powerdomain

Achim Jung   Regina Tix

Claire Jones showed in her thesis in 1990 that the probabilistic powerdomain of a continuous domain is again continuous. The category of continuous domains, however, is not cartesian closed, and one has to look at subcategories such as "retracts of bifinites" (RB). In 1988, Graham offered a proof that the probabilistic powerdomain construction can be restricted to RB.

We found a counterexample to Graham's argument and in this talk we present our own attempts at proving a closure result for the probabilistic powerdomain construction. We have positive results for finite tress and infinite reversed trees. These illustrate the difficulties rather than being satisfying answers to the question of whether the probabilistic powerdomain and function spaces can be reconciled.

We are more successful with coherent of Lawson–compact domains. These form a category with many pleasing properties but they fall short of supporting function spaces.

Representing Coherent Spaces

Mathias Kegelmann

In Domain Theory in Logical Form prelocales are finitary descriptions of coherent (i.e. stably compact) spaces. The main result of the talk is a theorem that characterizes when a prelocale represents a given coherent space. It outlines the steps necessary for verifying domain constructions in logical form.

Symmetries of the Partial Order of Traces

Dietrich Kuske

The talk deals with the automorphism group of the partial order of finite traces. We show that any group can arise as such an automorphism group if we allow arbitrarily large dependence alphabets. Restricting to finite dependence alphabets, the automorphism groups are profinite and possess only finitely many simple decomposition factors. Finally, we show that the partial order associated with the countable Rado graph as dependence alphabet does not give rise to a homogeneous domain thereby answering an open question from Boldi, Cardone & Sabadini from 1993.

Domain Equations for Probabilistic Processes

Marta Kwiatkowska
(joint work with Christel Baier and Gethin Norman)

In the first part of the talk we consider Milner's CCS enriched with action–guarded probabilistic choice. The calculus is given operational semantics based on probabilistic transition systems and two process equivalences: probabilistic bisimulation of Larsen & Skou, and probabilistic (mutual) simulation as in Segala & Lynch. We extend existing category–theoretic techniques for solving domain equations to the probabilistic case and give two denotational semantics for the calculus. The first, 'smooth', semantics model arises as a solution of a domain equation involving the probabilistic powerdomain and solved in the category $\mathrm{CONT}_\perp$ of continuous domains. The second model also involves appropriately restricted probabilistic powerdomain, but is constructed in the category CUM of complete ultrametric spaces, and hence is necessarily 'discrete'. We show full abstraction for both models, with respect to probabilistic bisimulation and simulation respectively.

In the second part of the talk we introduce a ‘smooth’ metric model for a CSP–like language including external and internal process choice, and action–guarded probabilistic choice. The model is fully abstract for a testing equivalence, coarser than probabilistic bisimulation. The metric, however, is not inductive, and so categorical techniques for solving domain equations do not apply.

Compactness in Domains

Jimmie Lawson

The notion of property M for continuous domains generalises in a rather direct way to quasicontinuous domains. In this more general context a quasicontinuous poset is compact in the Lawson topology if and only if it satisfies property M. Another condition, called DCAP, can also be introduced: Every intersection of finitely generated upper sets admits a representation as a directed intersection of finitely generated upper sets. Condition DCAP is also equivalent to Lawson compactness. On the other hand if a DCPO is a Lawson compact pospace satisfying DCAP, then it is quasicontinuous.

The Fixed Points and Derivatives of Nonmonotonic Mappings in Domain Theory

Keye Martin

A measurement $\mu : D \longrightarrow [0, \infty)^*$ on a continuous dcpo D provides an appropriate setting for computational considerations (complexity and correctness, for example) in domain theory. The primary motivation for the idea is to discuss amounts of information for the expressed purpose of measuring the progress an algorithm makes, and in the process, to prove that it is correct. The same idea, however, leads easily to the notion of a derivative with respect to a measurement, which measures the rate at which a process computes its output: The smaller the derivative is at an output, the faster the algorithm computes it. Another feature of this approach is the availability of fixed point theorems for mappings which are not necessarily monotone, and so in particular, not Scott continuous. Such mappings are fundamental because they arise in practice frequently.

Using Bounded Scott–Closed Sets to Model Topological Spaces

Michael Mislove

The principal results modeling topological spaces in domains are due to Lawson, who characterised the maximal points of ω–continuous domains weak at the top as the Polish spaces, and Edalat & Heckmann who showed any metric span can arise as the maximal points of a continuous poset weak at the top. The natural question is whether coherent domains or Scott domains necessarily model more special spaces. In this talk I show any space which can be modeled as the maximal points of a continuous domain weak at the top in fact can be densely embedded in the maximal elements of a Scott domain. This has the pay–off of allowing continuous maps between such spaces to extend to Scott continuous maps between the respective Scott domains. I also characterize those domains which yield Scott models when the original space is all of the maximal elements as the weak compact domains. Lastly, it is shown when the construction is functorial.

Game PCA's, Almost Everywhere Copy–Cat Strategies and the Longley–Conjecture

Hanno Nickau Luke Ong

We present a systematic investigation of game semantics for pure untyped λ–calculus using the innocent approach of Hyland–Ong and of Nickau. An innovation of this work is a simple and useful encoding of player's views and of innocent functions which we call economical representation.

In the first part (presented by Luke Ong) we study the interpretation of untyped λ–terms in the $\lambda\eta$–algebra $\mathcal{D}$ of innocent strategies over the denumerably–deep and denumerably–branching one–token arena U. Thanks to the economical representation $\mathcal{D}$ can be given a particularly simple description: it is exactly the set of partial functions $f : \mathbb{N}^* \longrightarrow (\mathbb{N} \times \mathbb{N})_{\emptyset}$ that are prefix closed and such that $f(v) = (i, r)$ implies $0 \leq r \leq |v|$. A subalgebra $\mathcal{N}$ of $\mathcal{D}$, consisting of effectively almost–everywhere copy–cat strategies, is a sensible, order–extensional and universal $\lambda\eta$–model. We prove that $\mathcal{N}$ is isomorphic to ${}^{\Lambda^0}\!/_{\mathcal{K}^*}$ the closed λ–term model over the largest sensible consistent λ–theory.

In the second part (presented by Hanno Nickau), by embedding PCF–games and $\mathcal{D}$ in a category of "hybrid arenas" we can prove that there are interpretations of PCF in $\mathbf{Mod}(\mathcal{D}_{\text{rec}})$ and $\mathbf{Mod}(\mathcal{N})$ respectively that are universal for PCF and hence fully abstract. In view of the isomorphism $\mathcal{N} \cong {}^{\Lambda^0}\!/_{\mathcal{K}^*}$ this gives almost a proof of Longley's conjecture "the standard interpretation of PCF in $\mathbf{Mod}(\Lambda^0/T)$, where T is any suitable, sensible λ–theory, is fully abstract". This remains still open, since our interpretation is not "standard" in Longley's sense, i.e. the interpretation of the base type nat is not the lift of the natural numbers object in $\mathbf{Mod}(\mathcal{N}\ )$.

Remarks on Realizability

Giuseppe Rosolini

The categorical approach to realizability has produced a few examples of toposes which are not Grothendieck. Also, the seminal work of Dana S. Scott on the models of λ–calculus suggested that categories of partial equivalence relations could be useful to study the model, indeed generating a host of papers on various examples of that construction.

We show that there is a general, unifying construction which produces all these. It is the free exact category on a category with finite limits.

It is interesting to compare how the work on this relationship between particular instances developed in the applications and the general theory evolves and the football match which took part at the Seminar where two teams meet, one of Teutonic technique, the other based on Latin extravaganza.

Relators and Metric Bisimulations

Jan Rutten

Invoking a result by Carboni, Kelly and Wood on the existence of relators, we show that this explains why the category of a set functor that preserves weak pullbacks is well–behaved. The reason is that such functors extend uniquely to a relator. Next, we generalize these observations to the setting of (generalized) metric spaces, obtaining a notion of a metric bisimulation. Based on this notion, a metric coinduction principle is proved.

Weightable Directed Spaces: Partial Metrics = Generalized (E)valuations

M.P. Schellekens

Partial metrics, or the equivalent weightable quasi–metrics, have been introduced by Matthews as part of the study of the denotational semantics of data flow networks. It is remarked in work by Bukatin and Shorina that the "existence of deep connections between partial metrics and valuations is well known in Domain Theory." Examples of such connections have been discussed by O'Neil, Bukatin, Scott and Shorina, where in each case partial metrics are generated from positive valuations. The interest in (e)valuations in connection to Domain

Theory derives from work by Edalat, Heckmann, Jones and Plotkin. We analyze the precise connections between partial metrics and valuations in Domain Theory.

Since domain theoretic applications typically involve semilattices rather than lattices, the question arises whether the fruitful notion of a valuation can be generalized to this context. We extend our prior work on weightable directed spaces and focus on the class of quasi–uniform semilattices in this context. Quasi–uniform semilattices arise naturally in Domain Theory and include in particular Smyth's totally bounded Scott domains, Matthew's Baire (Kahn) partial metric spaces as well as the complexity spaces.

Partial metric spaces and positive "semivaluations" are shown to be dual notions in this context. This involves the solution of an open problem from Künzi's survey paper "Non-symmetric Topology" (Problem 7) for the class of quasi–uniform semilattices. Each of the above examples is shown to correspond dually to a semivaluation space. We conclude that, in the context of Domain Theory, partial metrics correspond to generalized (e)valuations.

Logics of Types and Computation

Introducation to a Project

Dana Scott

Over the last year, we have been developing a circle of ideas that came up in Scott's 1996 graduate course on domain theory. The aim is to use type theory (and topos theory) *via* realizability to model a constructive logic which accommodates standard types (*e.g.* countably based topological spaces) and domain theory (as certain countably based T_0-spaces), has extensive closure conditions (*e.g.* as a cartesian closed and locally cartesian closed category), and supports polymorphism (in the style of Girard and Reynolds or the calculus of constructions).

Similar approaches having some of these advantages have been suggested many times before, but we now think better results can be achieved using realizability over the graph model for the (untyped) lambda calculus. Moreover, in this proposed interpretation, there is a clear–cut definition of a type operator #A, meaning the formation of the *computable elements* of the type A (which includes computable functions as a type #(A $\longrightarrow$ B)). This operator can also be applied to propositions providing a constructive version of a kind of S4 modal logic. In this way it is possible, for example, to work both with the whole set of real numbers along with the computable numbers and functions.

The major task of the project is to show that this logic is capable of naturally formulating the constructions and proofs required for the questions of semantics of programming languages — especially in justifying the existence and properties of recursively defined types. If this is successful, there should be significant conclusions for constructive logic as well.

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Computational Adequacy in an Elementary Topos

Alex Simpson

Given an elementary topos $\mathcal{E}$ with a natural numbers object N and dominance Σ, the full subcategory of "well–complete" objects provides a candidate category of predomains within the topos. Under the single axiom that N is well–complete, this category has closure conditions sufficient to provide a "set–theoretic" model of any of the usual variants of PCF. For maximum generality, we consider a call–by–value version with sum and product types. As usual, the interpretation is said to be "computationally adequate" if denotational equality is sound with respect to the operational equality of programs. The main result says that computational adequacy holds if and only if the topos is I–consistent (i.e. only true Σ^0_1–sentences are valid according to the internal logic of the topos).

Constructive SFP Domains

Dieter Spreen

In his 1983 paper on the largest cartesian closed category of ω–algebraic cpo's M.B. Smyth conjectured that an analogue of this result holds in an effective environment.

We deal with this question. First we introduce a suitable notion of constructive SFP domain. Then it is shown that the category of these domains is the largest constructively Cartesian closed effectively given full subcategory of the effectively given category of algebraic constructive domains which have a computable completeness test.

Continuous and Algebraic Domain Models

Philipp Sünderhauf

We show how to construct for a given continuous domain D an algebraic domain E with embedding–projection pair $e, p : D \longrightarrow E$ such that the spaces of maximal elements of D

and E in their relative Scott–topologies are homeomorphic via e and p. Our construction maps L–domains to L–domains. Since the interval domain is an L–domain this yields an algebraic model of the reals in a cartesian closed category of algebraic domains.

Clausal Inference in Domain Theory and Logic Programming

Guo–Qiang Zhang William C. Rounds

The purposes of this paper are twofold. One is to introduce an unexpectedly simple, operational representation of the Smyth power–domain. The second is to provide a fixed–point semantics for disjunctive logic programs using a similar construction. These developments hinge upon a combinatorial lemma. Based on this lemma, we prove the necessary completeness results for a clausal inference rule. This rule is more general than the generalized resolution rule of Robinson in that it is not only for deciding inconsistency, but also for deciding logical consequence when explicit negation is not present.