David Mix Barrington, Rüdiger Reischuk, Ingo Wegener (editors):

**Complexity of Boolean Functions**

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Summary of the Dagstuhl Seminar
“Complexity of Boolean Functions”

The complexity of Boolean functions is one of the central and classical topics in the theory of computation. Despite of some breakthrough results (e.g., exponential lower bounds on the monotone circuit complexity, bounded depth unbounded fan-in circuits, and linear depth branching programs, or the classification of bounded-width polynomial-size branching programs by $\text{NC}^1$) there still seems to be a long way to go before successfully establishing large lower bounds in the case of unrestricted circuits over complete bases. Besides the classical lower bound and classification problems people active in this area are working on related topics like communication complexity, neural nets, quantum computing, and learning.

The organizers are happy that 37 researchers followed their invitation to Dagstuhl, they came from Germany (17 including guests from Poland and Lithuania), USA (5), Canada (4), Japan (4), Austria (2), Czech Republic (2), England, Netherlands, and Russia.

The 27 talks captured many of the aspects of Boolean function complexity. There were several talks on branching programs (also for variants with applications in CAD and verification), circuits, communication complexity, and learning. Further talks focussed on algebraic methods. Aspects like randomization and nondeterminism were considered as well as quantum computing and cryptography. Besides some classical automata problems also related topics on algorithms and data structures were discussed.

The schedule contained an open problem session and an evening discussion on new models motivated from biocomputing.

Perhaps more important than the “official” sessions were the many informal discussions inspired by the Dagstuhl atmosphere. The participants also found time for the traditional Wednesday hike, a table tennis tournament, and a wine party.

The organizers

David Mix Barrington, Rüdiger Reischuk, and Ingo Wegener
Seminar Program

Monday, November 1st, 1999

9.10 - 9.45  **Kazuo Iwama, Kyoto**
Oblivious vs. Non-Oblivious Branching Programs

9.50 - 10.20  **Peter Savický, Prague**
A Hierarchy Result for Read-Once Branching Programs
with Restricted Parity Nondeterminism

10.40 - 11.15  **Detlef Sieling, Dortmund**
Lower Bounds for Linear Transformed OBDDs and FBDDs

11.20 - 12.00  **Elizabeta Okol’nishnikova, Novosibirsk**
On Operations over Boolean Functions

15.30 - 16.10  **Thomas Zeugmann, Fukuoka**
On the Complexity of Learning Boolean Functions

16.20 - 17.00  **Peter Damaschke, Hagen**
Learning Boolean Functions with few Relevant Variables by Queries

17.10 - 17.50  **Hans Ulrich Simon, Bochum**
Combinatorial Invariants of Boolean Concept Classes
and Their Relation to Query Complexity

Tuesday, November 2nd, 1999

9.00 - 9.40  **Wolfgang Maass, Graz**
On the Computational Power of Winner-Take-All

9.45 - 10.25  **Anna Gál, Austin**
On the Sensitivity of Multiple Output Boolean Functions

10.45 - 11.25  **Harry Buhrman, Amsterdam**
Quantum Communication Complexity Lower Bounds by Polynomials

11.30 - 12.05  **Thomas Thierauf, Ulm**
The Complexity of Problems in Linear Algebra

15.30 - 16.10  **Pierre McKenzie, Montréal**
Polynomial Replacement Systems

16.15 - 16.40  **Mario Szegedy, Murray Hill**
Dynamic Problems

16.45 - 18.00  **Open Problems Session**

19.30  **Discussion:** From Threshold Circuits to Cortical Circuits
Moderator: Wolfgang Maass
Wednesday, November 3rd, 1999

9.00 – 9.30  Uwe Schöning, Ulm  
Randomized Algorithms for \( k \)-SAT

9.35 – 10.15  Andreas Jakoby, Toronto  
Average Case Complexity of Unbounded Fan-in Circuits

10.40 – 11.20  Matthias Krause, Mannheim  
On the Minimal Hardware Complexity of Pseudorandom Function Generators

11.25 – 12.05  Maciej Liskiewicz, Lübeck  
On Dynamic Process Graphs

Thursday, November 4th, 1999

9.00 – 9.40  Juraj Hromkovič, Aachen  
Communication Complexity and Lower Bounds

9.45 – 10.25  Martin Sauerhoff, Dortmund  
Guess and Verify vs. Unrestricted Nondeterminism

10.50 – 11.20  Stasys Jukna, Dortmund  
Branching Programs with Bounded Uncertainty I

11.25 – 12.00  Stanislav Žák, Prague  
Branching Programs with Bounded Uncertainty II

16.00 – 16.40  Akira Maruoka, Sendai  
Some Properties of Modulo \( m \) Circuits Computing Simple Functions

16.45 – 17.15  Prabhakar Ragde, Waterloo  
The Smallest Common Supertree-Major Problem

17.20 – 17.50  Eric Allender, Piscataway  
Planarity Testing

Friday, November 5th, 1999

9.00 – 9.40  Denis Thérien, Montréal  
Programs over Finite Monoids

9.45 – 10.20  David Mix Barrington, Amherst  
The Complexity of Some Problems on Groups Input as Multiplication Tables

10.45 – 11.15  Georg Schnitger, Frankfurt  
Las Vegas Automata

11.20 – 12.00  Ingo Wegener, Dortmund  
Asymptotically Optimal Lower Bounds for OBDDs
Oblivious vs. Non-Oblivious Branching Programs

Kazuo Iwama (Kyoto University, Japan)

(Joint work with Toshiro Takase and Yasuo Okabe.)

It is shown that oblivious branching programs are exponentially slower than non-oblivious branching programs for some Boolean functions. Namely there exists a Boolean function $f$ such that $f$ needs $\Omega(n \log n)$ depth for oblivious BPs but $O(\log^3 n)$ depth is enough for read-once syntactic BPs.

It is also shown that any BP of depth $d$ can be simulated by an oblivious BP of depth $dn / \log \log n$. Therefore, if there is a Boolean function which can be computed by a BP of depth $O(\log n \log \log n)$ and which needs $\Omega(n \log n)$ depth for oblivious BPs, then it can be regarded as an optimal speedup for BPs. Our current result is weaker than this.

A Hierarchy Result for Read-Once Branching Programs with Restricted Parity Nondeterminism

Petr Savický (Academy of Sciences, Prague, Czech Republic)

(Joint work with Detlef Sieling.)

Restricted branching programs appear to be an important model for representing Boolean functions in applications. Understanding of the properties of different restrictions may help to find stronger models.

We investigate read-once branching programs with restricted parity nondeterminism, which means that the only nondeterministic node is the source and the acceptance mode is the parity one. We prove a proper hierarchy for such b. p. with respect to the out-degree of the source node.

Our result contributes to the understanding of parity nondeterminism and its interaction with adaptive ordering of the variables.
Lower Bounds for Linear Transformed OBDDs and FBDDs

Detlef Sieling (Universität Dortmund, Germany)

Linear Transformed Ordered Binary Decision Diagrams (LTOBDDs) have been suggested as a generalization of OBDDs for the representation and manipulation of Boolean functions. Instead of variables as in the case of OBDDs, linear tests, i.e., tests of parities of variables, may be performed at the nodes of an LTOBDD, where an ordering of the linear tests has to be respected. By this extension, it is possible to represent functions in polynomial size that do not have polynomial size OBDDs, e.g. the characteristic functions of linear codes. We extend the fooling set method in order to prove exponential lower bounds for LTOBDDs, and apply this method to an explicitly defined function.

We also consider two possibilities to introduce linear transformations into read-once branching programs/FBDDs (Free Binary Decision Diagrams) and call the resulting variants of FBDDs LTFBDDs and strong LTFBDDs. We separate these two variants by proving a polynomial upper bound for strong LTFBDDs and an exponential lower bound for LTFBDDs for a modified version of the matrix storage access function. By all the upper and lower bound results we also separate the classes of functions with polynomial size LTOBDDs, LTFBDDs and strong LTFBDDs from the corresponding complexity classes for several other variants of branching programs.

On Operations over Boolean Functions

Elizabeta Okol'nishnikova (Sobolev Institute of Mathematics, Novosibirsk, Russia)

It is shown that the operation of the geometrical projection and the operation of the monotone extension can lead to the increase of the complexities of Boolean functions for some kinds of schemata without restrictions. It is shown that there exists some relation between complexities of Boolean functions for nondeterministic and deterministic read-k-times branching programs on one hand and between complexities of Boolean functions and their geometrical projection for read-k-times deterministic branching programs on the other hand. It is also shown that there exists some relation between the operation of the geometrical projection and the operation of monotone extension.
On the Complexity of Learning Boolean Functions

Thomas Zeugmann (Kyushu University, Fukuoka, Japan)

(Joint work with Rüdiger Reischuk.)

We advocate to analyze the average complexity of learning problems. An appropriate framework for this purpose is introduced. Based on it we consider the problem of learning monomials and the special case of learning monotone monomials *in the limit* and for *on-line predictions* in two variants: from positive data only, and from positive and negative examples. The well-known *Wholist algorithm* is completely analyzed with respect to its average-case behavior with respect to the class of *binomial distributions*. We consider different complexity measures: the *number of mind changes*, *the number of prediction errors*, and *the total learning time*. Tight bounds are obtained implying that worst case bounds are too pessimistic. On the average learning can be achieved *exponentially faster*.

Furthermore, we study a new learning model, *stochastic finite learning*, in which, in contrast to PAC learning, some information about the underlying distribution is given and the goal is to find a *correct* (not only approximatively correct) hypothesis. We develop techniques to obtain good bounds for stochastic finite learning from a precise average case analysis of strategies for learning in the limit and illustrate our approach for the case of learning monomials.
Learning Boolean Functions with Few Relevant Variables by Queries

Peter Damaschke (Fernuniversität Hagen, Germany)

We consider exact learning of monotone Boolean functions by membership queries, in case that only \( r \) of the \( n \) variables are relevant. The learner proceeds in a number of rounds. In each round he submits a set of queries which may be chosen depending on the outputs from previous rounds to the function oracle. In a STOC '98 paper we proved that \( O(2^r + r \log n) \) queries in \( O(r) \) rounds are sufficient. While the query bound is optimal for trivial information-theoretic reasons, it was open whether parallelism can be improved without increasing the amount of queries. Here we prove a negative answer: \( \Theta(r) \) rounds are necessary in the worst case, even for learning a very special type of monotone function. The proof is an adversary argument exploiting a distance inequality in binary codes. More generally, we obtain a lower bound for the queries vs. rounds tradeoff, however it remains open whether this bound is tight. On the other hand, a Las Vegas strategy based on another STOC '98 result can learn monotone functions in \( 2 \log_2 r + O(1) \) rounds, without using significantly more queries. (Actually, more function classes are learnable in the same way.) We also study the constant factors in nearly query-optimal deterministic strategies.

Combinatorial Invariants of Boolean Concept Classes and Their Relation to Query Complexity

Hans Ulrich Simon (Ruhr-Universität Bochum, Germany)

The results, presented during the talk, are part of a joint work with José Balcázar, Jorge Castro, and David Guijarro. An extended abstract of this work will appear in the proceedings of ALT '99 (Springer-Verlag, LNAI Series).

We prove a new combinatorial characterization of polynomial learnability from equivalence queries, and state some of its consequences relating the learnability of a class with the learnability via equivalence and membership queries of its subclasses obtained by restricting the instance space. Then we propose and study two models of query learning in which there is a probability distribution on the instance space, both as an application of the tools developed from the combinatorial characterization and as models of independent interest.
On the Computational Power of Winner-Take-All
Wolfgang Maass (Technische Universität Graz, Austria)

Winner-Take-All and its variants are common computational operations in artificial neural networks, models for biological neural systems, and analog VLSI. We show that Winner-Take-All requires quadratically many gates on any feedforward threshold circuit. On the other hand arbitrary threshold circuits of depth 2 can be simulated by a single Winner-Take-All gate applied to positive weighted sums. Thus Winner-Take-All is shown to be a surprisingly powerful and versatile computational operation.

For details see #119 on http://www.cis.tu-graz.ac.at/igi/maass

On the Sensitivity of Multiple Output Boolean Functions
Anna Gál (University of Texas at Austin, USA)

(Joint work with Adi Rosen.)
The sensitivity of a function on a given input is the number of variables, such that changing the value of just one variable at a time, changes the value of the function. The sensitivity of the function is the maximum of its sensitivity over all inputs. We give an almost tight upper bound on the sensitivity of multiple-output Boolean functions in terms of the sensitivity of each coordinate, and the size of the range of the function.

More formally, we prove the following theorem: Let $F$ be an $m$-output Boolean function such that the sensitivity of each of its coordinate functions is at most $k$. If the range of $F$ contains at most $D$ different values then the sensitivity of $F$ is at most $4k(\log D + 2)$.

Note that the restriction on the size of the range of $F$ can be interpreted as a condition on the "correlation" between the coordinate functions. Our results show that even if the range of $F$ is an arbitrary subset of size $2^q$ of the $m$-dimensional Boolean cube, the sensitivity of $F$ cannot be much larger than the sensitivity of functions whose range is restricted to a $q$-dimensional subcube. Our bound is almost tight, as for $q$ independent coordinates the sensitivity $kq$ is achieved.

Using the above theorem we prove a tight lower bound (up to a small constant factor) on the number of rounds required to privately compute a Boolean function. The lower bound is given in terms of the sensitivity of the function being computed, and the amount of randomness used by the protocol overall.

We believe that the theorem about sensitivity is of independent interest, and may find additional applications.
Quantum Communication Complexity Lower Bounds by Polynomials

Harry Buhrman (CWI Amsterdam, The Netherlands)

(Joint work with Ronald de Wolf.)
The quantum version of communication complexity allows Alice and Bob to communicate qubits and/or to make use of prior entanglement (shared EPR-pairs). Some lower bound techniques are available for qubit communication, but except for the inner product function, no bounds are known for the model with unlimited prior entanglement. We show that the "log rank" lower bound extends to the strongest model (qubit communication + prior entanglement). By relating the rank of the communication matrix to properties of polynomials, we are able to derive some strong bounds for exact protocols. In particular, we prove both the "log-rank conjecture" and the polynomial equivalence of quantum and classical communication complexity for various classes of functions. We also derive some weaker bounds for bounded-error protocols.

The Complexity of Problems in Linear Algebra

Thomas Thierauf (Universität Ulm, Germany)

(Joint work with Thanh Minh Hoang.)
We investigate the computational complexity of some important problems in linear algebra.

- The problem of verifying the characteristic polynomial of a matrix is known to be in the complexity class $C_{=}L$ (exact counting logspace). We show that it is complete for $C_{=}L$.
- The problem of deciding whether two matrices are similar is known to be in the complexity class $AC^0(C_{=}L)$, the $AC^0$-closure of $C_{=}L$. We show that it is complete for this class.

This answers open questions posed by Santha and Tan.
A central open problem in this area remains open however:

- Is $C_{=}L$ closed under complement?

This is known for nondeterministic logspace, $NL$ (Immerman and Szelepcsényi) and for symmetric logspace, $SL$ (Nisan, Ta-Shma). It is trivially true for probabilistic logspace, $PL$, and, at least in the non-uniform setting, for unambiguous logspace, $UL$ (Allender, Reinhardt).

A full paper will be available soon at http://www.informatik.uni-ulm.de/abt/ti/thierauf
Polynomial Replacement Systems

Pierre McKenzie (Université de Montréal, Canada)

(Joint work with Heribert Vollmer and Klaus Wagner.)

We discuss the problems of counting proof trees (as introduced by Venkateswaran and Tompa) and counting proof circuits, a related but seemingly more natural question. We show that counting proof circuits is \#P-complete. The problems of counting proof trees and proof circuits lead to a common generalization of straight-line programs which we call polynomial replacement systems. We suggest a classification of these systems and we consider the complexity of relevant computational problems.

Dynamic Time Complexity

Mario Szegedy (AT&T Labs, Murray Hill, USA)

Dynamic algorithms compute a function for continuously updated inputs. The sequence of updates is potentially infinite. The dynamic time (DYNTIME) for computing a function with respect to a given algorithm is the maximum computation time in between two input-updates maximized over all input sequences. A function is in dynamic time $f(n)$ (DYNTIME($f(n)$)) if there exists an algorithm which computes it within dynamic time $f(n)$. We study the question: $\mathbf{P} \subseteq \text{DYNTIME}(\text{polylog})$? (To avoid trivial counter-examples an arbitrary polynomial time preprocessing is allowed.) We propose several conjectures that would lead to a negative answer to the above question. (Report on an ongoing research with Mikkel Thorup.)
Randomized Algorithms for $k$-SAT

Uwe Schöning (Universität Ulm, Germany)

We consider some very easy randomized algorithms for 3-SAT, more generally for $k$-SAT, and even more generally, for arbitrary (discrete) constraint satisfaction problems. The first algorithm selects a starting assignment at random, and then, by some deterministic backtracking procedure determines if there is a satisfying assignment within Hamming distance $\alpha n$ from the initial assignment. The best choice for $\alpha$ is $1/4$ in the case of 3-SAT and $1/(k + 1)$ in the case of $k$-SAT. It turns out that the complexity is $1.5^n$, and for general $k$-SAT, $(2k/(k + 1))^n$. An improvement is obtained by substituting the backtracking procedure by some random walk. In each step a clause not being satisfied by the actual assignment is selected, then the value of some randomly selected literal in that clause is flipped. It turns out that the complexity is $(4/3)^n$ in the case of 3-SAT, and in case of a constraint satisfaction problem with domain size $d$ and constraint order $\ell$, the complexity is $(d(1 - 1/\ell))^n$.

Average Case Complexity of Unbounded Fanin Circuits

Andreas Jakoby (University of Toronto, Canada)

(Joint work with Rüdiger Reischuk (Med. Universität zu Lübeck).)

Hastad has shown that functions like PARITY cannot be computed by unbounded fanin circuits of small depth and polynomial size. We generalize this result in two directions. First, we obtain the same tight lower bound for the average case. This is done by estimating the average delay – the natural generalization of circuit depth to an average case measure – of unbounded fanin circuits of polynomial size, resp. their error probability given an upper bound on the maximal delay. These bounds are obtained by extending the probabilistic restriction method to an average case setting.

Secondly, we completely classify the set of parallel prefix functions – for which PARITY is just one example – with respect to their average delay in unbounded fanin circuits of a given size. It is shown that only two cases can occur: a parallel prefix functions either has the same complexity as PARITY, that is the average delay has to be of order $\Theta(\log n/ \log \log s)$ for circuit of size $s$, or it can be computed with constant average delay and almost linear size – there is nothing in between. This classification is achieved by analyzing the algebraic structure of the semigroups that correspond to parallel prefix functions.
On the Minimal Hardware Complexity of Pseudorandom Function Generators

Matthias Krause (Universität Mannheim, Germany)

(Joint work with Stefan Lucks.)

A set of \( n \)-ary Boolean functions \( F \) is called a pseudorandom function generator (PRFG) if communicating with a randomly chosen secret function from \( F \) via membership queries cannot be efficiently distinguished from communicating with a truly random function. We ask for the minimal hardware complexity of a PRFG. Apart from the fact that this is an interesting challenge for complexity theory, the study of this question is also motivated by design aspects of secure secret key cryptosystems. On the one hand one chooses encryption algorithms with very fast hardware implementations. But on the other hand one requires that each output bit behaves like a PRFG.

We show that (via widely believed cryptographic hardness and number-theoretic assumptions) \( \text{TC}_0^0 \) contains a PRFG. On the other hand, by giving universal efficient distinguishing algorithms we prove that complexity classes like \( \text{AC}_0^0 \), \( \text{LT}_1 (\text{AC}_0^0) \), \( \text{LT}_1 (\oplus) \), \( \text{AC}_0^0 (p) \), \( p \) prime, and complexity classes induced by depth restricted polynomial size BDDs are too weak for containing PRFGs. Moreover, we relate our concept of distinguishability to learnability of Boolean concept classes and to the concept of natural proofs and strengthen the main observation of Razborov and Rudich on lower bound arguments and cryptographical weakness.
On Dynamic Process Graphs

Maciej Liskiewicz (Med. Universität Lübeck, Germany)

(Joint work with Andreas Jakoby and Rüdiger Reischuk.)

In parallel and distributed computing scheduling low level tasks on the available hardware is a fundamental problem. Traditionally, one has assumed that the set of tasks to be executed is known beforehand. Then the scheduling constraints are given by a precedence graph. Nodes represent the elementary tasks and edges the dependencies among tasks. This static approach is not appropriate in situations where the set of tasks is not known exactly in advance, for example, when different options how to continue a program may be granted.

In this paper a new model for parallel and distributed programs, the dynamic process graph, will be introduced, which represents all possible executions of a program in a compact way. The size of this representation is small – in many cases only logarithmically with respect to the size of any execution. An important feature of our model is that the encoded executions are directed acyclic graphs having a "regular" structure that is typical of parallel programs.

Dynamic Process Graphs embed constructors for parallel programs, synchronization mechanisms as well as conditional branches. With respect to such a compact representation we investigate the complexity of different aspects of the scheduling problem: the question whether a legal schedule exists at all and how to find an optimal schedule. Our analysis takes into account communication delays between processors exchanging data.

Precise characterization of the computational complexity of various variants of this compact scheduling problem will be given in this paper. The results range from easy, that is NL-complete, to very hard, namely NEXPTIME-complete.
Communication Complexity and Lower Bounds
Juraj Hromkovič (RWTH Aachen, Germany)

Communication complexity of two-party (multiparty) protocols has established itself as a successful method for proving lower bounds on the complexity of concrete problems for numerous computing models. While the relations between communication complexity and oblivious, semilective computations are usually transparent and the main difficulty is reduced to proving nontrivial lower bounds on the communication complexity of given computing problems, the situation essentially changes, if one considers non-oblivious or multilective computations. The known lower bound proofs for such computations are far from being transparent and the crucial ideas of these proofs are often hidden behind some nontrivial combinatorial analysis. The aim of this paper is to create a general framework for the use of two-party communication protocols for lower bound proofs on multilective computations. The result of this creation is not only a transparent presentation of some known lower bounds on the complexity of multilective computations on distinct computing models, but also the derivation of new nontrivial lower bounds on multilective VLSI circuits and multilective planar Boolean circuits. In the case of VLSI circuits we obtain a generalization of Thompson’s lower bounds on AT^2 complexity for multilective circuits. The Ω(n^2) lower bound on the number of gates of any k-multilective planar Boolean circuit computing a specific Boolean function of n variables is established for k < 1/2 log_2 n.

Another advantage of this framework is that it provides lower bounds for a lot of concrete functions. This contrasts to the typical papers devoted to lower bound proofs, where one establishes a lower bound for one or a few specific functions.
Guess and Verify vs. Unrestricted Nondeterminism for OBDDs

Martin Sauerhoff (Universität Dortmund, Germany)

It is well-known that an arbitrary nondeterministic Turing machine can be simulated with polynomial overhead by a so-called “guess-and-verify” machine. It is an open question whether an analogous simulation exists in the context of space-bounded computation. In this talk, a negative answer to this question is given for nondeterministic OBDDs. If it is required that all nondeterministic variables are tested at the top of the OBDD, i.e., at the beginning of the computation, this may blow-up the size exponentially.

This is a consequence of the following main result presented here. There is a sequence of Boolean functions $f_n : \{0, 1\}^n \rightarrow \{0, 1\}$ such that $f_n$ has nondeterministic OBDDs of polynomial size with $O(n^{1/3} \log n)$ nondeterministic variables, but $f_n$ requires exponential size if only at most $O(\log n)$ nondeterministic variables may be used.

A preliminary version is available at: http://ls2-www.cs.uni-dortmund.de/~sauerhof

On Branching Programs with Bounded Uncertainty

Stasys Jukna (Universität Dortmund, Germany) and Stanislav Žák (Academy of Sciences, Prague, Czech Republic)

We propose an information-theoretic approach to proving lower bounds on the size of branching programs. The argument is based on Kraft-McMillan type inequalities for the average amount of uncertainty about (or entropy of) a given input during various stages of the computation. We first demonstrate the approach for read-once programs. Then we introduce a strictly larger class of so-called “gentle” b.p. and, using the suggested approach, prove that some explicit Boolean functions, including the Clique function and a particular Pointer function (which belongs to $\text{AC}^0$), cannot be computed by gentle program of polynomial size.

These lower bounds are new since explicit functions, which are known to be hard for all previously considered reading-restricted classes of branching programs (such as $(1, +s)$-programs or syntactic read-$k$-times programs) can be easily computed by gentle programs of polynomial size.

Finally, we argue how the suggested approach could be used to prove a super-polynomial lower bound for unrestricted branching programs, and present one candidate for such hard function based on partial $t$-designs.

A preliminary version is available at: http://ls2-www.cs.uni-dortmund.de/~jukna
Some Properties of Modulo $m$ Circuits Computing Simple Functions

Akira Maruoka (Tohoku University, Sendai, Japan)

(Joint work with Kazuyuki Amano.)

A $\text{MOD}_m$ gate is a Boolean gate with unbounded fan-in which outputs is 0 if and only if the sum of its inputs is divisible by $m$. A $\text{MOD}_m$ circuit is a circuit that consists solely of $\text{MOD}_m$ gates. It is a long standing open problem to derive a superlinear lower bound on the size complexity of depth three $\text{MOD}_m$ circuits that compute some function in $\text{NP}$.

In this paper, we investigate the complexity of constant depth $\text{MOD}_{2^p}$ circuits, where $p$ is an arbitrary prime number. It consists of two parts:

(i) A $(\text{MOD}_p, \text{MOD}_2)$ circuit is a depth two circuit with $\text{MOD}_2$ gates at input level connected to a $\text{MOD}_p$ gate at the output. We give a procedure that converts a $\text{MOD}_{2^p}$ circuit with an arbitrary finite depth to a $(\text{MOD}_p, \text{MOD}_2)$ circuit without changing the function that the $\text{MOD}_{2^p}$ circuit computes.

(ii) We verify that $(\text{MOD}_p, \text{MOD}_2)$ circuits computing non-trivial symmetric functions have rich connection between the input gates and the input variables of the circuits. The statement stated in a technically rigorous way is proved by Fourier analysis.

So if we can show that, for any linear size constant depth $\text{MOD}_{2^p}$ circuit $C$ computing a non-trivial symmetric function, a circuit obtained from $C$ by the procedure described in (i) can not implement the rich connection described in (ii), then we could have a superlinear lower bound on the size of $\text{MOD}_{2^p}$ circuit computing the non-trivial symmetric function.

Although we are not so far successful in obtaining some lower bounds along this strategy, we derive an exponential lower bound on the size of $(\text{MOD}_p, \ldots, \text{MOD}_p, \text{MOD}_{2^p})$ circuits computing non-trivial symmetric functions.
The Smallest Common Supertree-Major Problem

Prabhakar Ragde (University of Waterloo, Canada)

(Joint work with Naomi Nishimura (Waterloo) and Dimitrios Thilikos (UPC Barcelona).)

The diversity of application areas relying on tree-structured data results in a wide interest in algorithms which determine differences or similarities among trees. One way of measuring the similarity between trees is to find the smallest common superstructure or supertree, where common elements are typically defined in terms of a mapping or embedding. In the simplest case, a supertree will contain exact copies of each input tree, so that for each input tree, each vertex of a tree can be mapped to a vertex in the supertree such that each edge maps to the corresponding edge. More general mappings allow for the extraction of more subtle common elements captured by looser definitions of similarity.

We consider supertrees under the general mapping of minor containment, where a graph \( G \) is a minor of a graph \( H \) if it is possible to map each vertex in \( G \) to a connected subgraph of \( H \) such that each edge in \( G \) maps to a path in \( H \). Minor containment generalizes both subgraph isomorphism and topological embedding; as a consequence of this generality, however, it is \( \text{NP} \)-complete to determine whether or not \( G \) is a minor of \( H \), even for general trees. By focusing on trees of bounded degree, we obtain an \( O(n^3) \) algorithm which determines the smallest tree \( T \) such that both of the input trees are minors of \( T \), even when the trees are assumed to be unrooted and unordered.

Planarity Testing

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(Joint work with Meena Mahajan, of the Institute of Mathematical Sciences, Chennai, India.)

We clarify the computational complexity of planarity testing, by showing that planarity testing is hard for \( \text{L} \), and lies in \( \text{SL} \). This nearly settles the question, since it is widely conjectured that \( \text{L} = \text{SL} \). The upper bound of \( \text{SL} \) matches the lower bound of \( \text{L} \) in the context of (nonuniform) circuit complexity, since \( \text{L/poly} \) is equal to \( \text{SL/poly} \).

Similarly, we show that a planar embedding, when one exists, can be found in \( \text{FL}^{\text{SL}} \).
Programs over Finite Monoids

Denis Thérien (McGill University, Montréal, Canada)

An $n$-input program over $M$ is a sequence of instructions $\varphi = (i_1, f_1) \ldots (i_n, f_i)$, where $1 \leq i_j \leq n$, $f_j : A \rightarrow M$. Such program defines a function from $A^n$ to $M$ by $\varphi(x_1 \ldots x_n) = f_1(x_{i_1}) \ldots f_i(x_{i_n})$. Say that a monoid $M$ is universal if $\forall n$ any subset $L \subseteq A^n$ has the form $L = \varphi^{-1}(F)$, $F \subseteq M$, for some $M$-program $\varphi$. Say that a monoid $M$ has the polynomial-length property iff $\exists k \forall n$ any $M$-program is equivalent to a program of the length $n^k$. We conjecture that $M$ has the polynomial-length property iff $M$ is not universal. The forward direction is obvious. We can show that the conjecture is true for groups. For group-free monoids, it is known that monoids in DA have PLP and that monoids divided by the syntactic monoid of $A^*bbA^*$ are universal. We can show that the syntactic monoid of $(ab)^*$ is not universal, but we cannot yet see how to prove it has PLP.

The Complexity of Some Problems on Groups Input as Multiplication Tables

David Mix Barrington (University of Massachusetts, Amherst, USA)

(Joint work with Peter Kadau (Tübingen), Klaus-Jörn Lange (Tübingen), and Pierre McKenzie (Montréal).)

The Cayley Graph Membership problem (CGM) is to input a groupoid (binary algebra) $G$ given as a multiplication table, a subset $X$ of $G$, and an element $t$ of $G$, and to determine whether $t$ can be expressed as a product of elements of $X$. For general groupoids CGM is P-complete, and for associative algebras (semigroups) it is NL-complete. Here we investigate CGM for particular classes of groups. The problem for general groups is in Sym-L, but any kind of hardness result seems difficult because it would require constructing the entire multiplication table of a group.

We introduce the complexity class $\text{FOLL} = \text{FO}(\log \log n)$ of problems solvable by uniform circuit families of polynomial size, unbounded fan-in, and depth $O(\log \log n)$. Since parity is not in $\text{FOLL}$, no problem in $\text{FOLL}$ can be complete for any class containing parity, such as $\text{NC}^1$, $\text{L}$, or $\text{Sym-L}$. But $\text{FOLL}$ is not known to be contained even in $\text{Sym-L}$.

We show that CGM for cyclic groups is in $\text{FOLL} \cap \text{L}$, and that CGM for abelian groups is in $\text{FOLL}$. We conjecture that CGM for solvable groups is in $\text{FOLL}$ as well. We also consider the problem of testing for various properties of a group input as a table: we prove that cyclicity and nilpotency can each be tested in $\text{FOLL} \cap \text{L}$. 

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Las Vegas Automata

Georg Schnitger (Johann Wolfgang Goethe-Universität Frankfurt, Germany)

(Joint work with Juraj Hromkovič.)

We consider two-way finite automata and investigate the number of required states to accept languages with a deterministic, nondeterministic or Las Vegas computation. Let $s(L)$ (resp. $ns(L)$ or $lvs(L)$) denote the minimal number of states required by a two-way deterministic (resp. nondeterministic or Las Vegas) automaton to recognize the language $L$. We obtain the following results:

1. $lvs(L) = \Theta(ns(L) + ns(L))$.
2. There is a family $L_n$ such that $ns(L_n) \leq n$, but $lvs(L_n) = \Omega(n^2)$. Thus there is an at least quadratic gap between nondeterministic and Las Vegas two-way automata.
3. There is a family $L_n$ such that $lvs(L_n) \leq n$, but $s(L_n) = \Omega(\frac{n^2}{\log n})$. Thus there is an almost quadratic gap between deterministic and Las Vegas two-way automata.

Asymptotically Optimal Lower Bounds for OBDDs

Ingo Wegener (Universität Dortmund, Germany)

(Joint work with Beate Bollig.)

It is well-known that one-way communication characterizes the $\pi$-OBDD size for Boolean functions. Nevertheless, we have only unsatisfactory lower bounds on the OBDD size of certain important functions, e.g., multiplication. We solve some open problems concerning the OBDD size. They are motivated by the application of OBDDs as data structure and by automata theory.

- There are functions $f_n$ and $g_n$ essentially depending on all their variables such that
  the OBDD size of $h_n = f_n \oplus g_n$ and an optimal variable ordering is of the order of the product of the $\pi$-OBDD size of $f_n$ and $g_n$.

- The QOBDD (quasi-reduced OBDD) size and the ZBDD (zero-suppressed OBDD) size of the multiplexer is only $\Theta(n^2/(\log n))$ and it is not optimal to test all control variables before all data variables.

- There are explicitly defined functions $f_n$ essentially depending on $n$ variables such that
  the OBDD size of $f_n$ is linear while its QOBDD size is quadratic.

- There are explicitly defined functions $f_n$ such that $|f_n^{-1}(1)| = n$ and the OBDD size of $f_n$ is $\Theta(n^2)$.

The proofs contain some new methods how to obtain asymptotically optimal lower bounds for OBDDs.
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