Computational Geometry

18.03.2001 to 23.03.2001

organized by

Rolf Klein (Bonn) and Günter Rote (FU Berlin)

51 participants attended this meeting. As in previous Dagstuhl seminars about Computational Geometry, the exchange of ideas and information about the latest developments in the fields of computational and combinatorial geometry was the main purpose of this meeting. In addition, there was a focus on a few topics on individual days of the meeting, featuring longer keynote talks as well as more technical contributions:

- Geometric shape recognition, shape reconstruction and shape matching (talks 9, 10, 14, 26, 31, 33)
- Surface and volume modeling; optimal triangulations and meshing (2, 6, 7, 25, 29, 34, 35); visualization and rendering (1, 2, 11, 32)
- Robust and exact geometric computations (16, 17, 18)

We had originally planned to have also a focus on geometric methods in bioinformatics, exploring how and to what extent geometry-based methods can play a role in certain computational problems of biology such as molecular modeling, protein folding, drug design etc. However, we could not attract the bioinformatics experts to come to a week-long seminar on a field which is not central to their study, because it seems that people who are active in bioinformatics are too busy in their own field.
In the core area of computational geometry, the contributions can be classified according to their subjects and techniques as follows.

- proximity and covering (1, 5, 8, 12, 19, 20)
- combinatorial geometry (3, 4, 13, 14, 15, 24, 27)
- dynamic geometry (3, 21, 22, 23, 25)
- data structures (28, 30)

The program left ample opportunity for discussion in the lunch breaks and in the evenings, as well as the traditional excursion on Wednesday afternoon.

Dagstuhl-Seminar 01121, ‘‘Computational Geometry’’

Monday, 19. 3. 2001

8:55 - 9:00	**Opening**
9:00 - 9:25	**Kokichi Sugihara**, University of Tokyo Digital-picture approach to generalized Voronoi diagrams and its applications
9:30 - 9:55	**Lutz Kettner**, University of North Carolina 4D Data Visualization Using Iso-Surfaces and a Control Plane
	10:00 Coffee break
10:30 - 10:55	**Erik Demaine**, University of Waterloo How to fold a map
11:00 - 11:20	**Ferran Hurtado**, Universitat Politècnica de Catalunya On the reflexivity of point sets
11:25 - 11:50	**Christian Icking**, FernUniversität Hagen Smallest color-spanning objects
	12:15 Lunch
15:00 - 15:30	**Jonathan Shewchuk**, UC Berkeley Three-dimensional mesh generation for domains with small angles
15:35 - 16:00	**Matthias Müller-Hannemann**, Universität Bonn Simulation of the human mandible: A case study in hexahedral mesh generation
	16:05 Coffee break
16:30 - 16:55	**David Kirkpatrick**, University of British Columbia Bounded curvature traversals of narrow roadways
17:00 - 17:25	**Christian Knauer**, Freie Universität Berlin Minimizing the Fréchet distance between two polygonal curves
17:30 - 17:55	**Carola Wenk**, Freie Universität Berlin Approximate matching of polygonal curves with respect to the Fréchet distance
	18:00 Dinner

Tuesday, 20. 3. 2001

9:00 - 9:25	**Tetsuo Asano**, JAIST - Ishikawa, Japan Digital Halftoning As a Combinatorial Optimization Problem with Polynomial-time Algorithms
9:30 - 9:55	**Michiel Smid**, Universität Magdeburg Translating a planar object to maximize point containment
	10:00 Coffee break
10:30 - 10:55	**Oswin Aichholzer**, TU Graz Enumerating order types for small point sets
11:00 - 11:25	**Peter Brass**, FU Berlin Combinatorial geometry in point pattern recognition
11:30 - 11:55	**Bernd Gärtner**, ETH Zürich One line and *n* points
	12:15 Lunch; 15:30 Coffee and Cake
16:05 - 16:50	**Chee K. Yap**, Courant Institute, New York Exact geometric computation and randomized theorem proving
16:55 - 17:20	**Kurt Mehlhorn**, Max-Planck-Institut für Informatik, Saarbrücken A Separation Bound for Real Algebraic Expressions
17:25 - 17:50	**Ulrich Kortenkamp**, Freie Universität Berlin Geometric straight-line programs
	18:00 Dinner
20:00	**Dan Halperin**, Tel Aviv University (Chairman) OPEN PROBLEM SESSION

Wednesday, 21. 3. 2001

9:00 - 9:25	**Laura Heinrich-Litan**, Freie Universität Berlin Nearest neighbor search in high dimensions
9:30 - 9:55	**Piotr Indyk**, Massachusetts Institute of Technology Better algorithms for high-dimensional proximity problems via asymmetric embeddings
	10:00 Coffee break
10:30 - 10:55	**Leonidas Guibas**, Stanford University Discrete mobile centers
11:00 - 11:25	**Pankaj Kumar Agarwal**, Duke University Data structures for moving objects
11:30 - 11:55	**Bettina Speckmann**, University of British Columbia Kinetic collision detection for simple polygons
	12:15 Lunch

Thursday, 22. 3. 2001

9:00 - 9:25 **Emo Welzl,** ETH Zürich
Incarnations of the generalized lower bound theorem

9:30 - 9:55 **Jack Snoeyink**, University of North Carolina
Bounded-degree pseudotriangulations of points

10:00 Coffee break

10:30 - 10:55 **Bernard Chazelle**, Princeton University
Three-dimensional shape matching

11:00 - 11:55 **Micha Sharir**, Tel Aviv University
New bounds on incidences and related problems

12:15 Lunch; 15:30 Cake and Coffee

16:10 - 16:35 **Joseph S. Mitchell**, SUNY at Stony Brook
Binary space partitions for orthogonal segments and hyperrectangles

16:40 - 17:05 **Jeff Erickson**, University of Illinois - Urbana
Nice point sets can have nasty Delaunay triangulations

17:15 - 17:40 **Olivier Devillers**, INRIA - Sophia Antipolis
The shuffling buffer: How to use randomized incremental constructions with a nonrandom order

18:00 Dinner

Friday, 23. 3. 2001

9:00 - 9:25 **William Evans**, University of British Columbia
Reconstruction of line arrangements from linear probes

9:30 - 9:55 **Marc van Kreveld**, Utrecht University
Schematization in road networks

10:00 Coffee break

10:30 - 10:55 **Edgar Ramos**, Max-Planck-Institut für Informatik, Saarbrücken
Smooth surface reconstruction in almost linear time

11:00 - 11:25 **Monique Teillaud**, INRIA - Sophia Antipolis
Walking in a triangulation

11:30 - 11:55 **Günter Rote,** Freie Universität Berlin
The polytope of minimum pseudotriangulations and the quest for the Delaunay pseudotriangulation

12:15 Lunch

Contents

1 Digital-Picture Approach to Generalized Voronoi Diagrams and Its Applications

Kokichi Sugihara, University of Tokyo

Digital-picture approach to geometric problems is first to convert the problems into pixel representations, next to solve it, and finally to convert the solutions reversely to the original world. This approach has merits in that it is simple, quick and robust, while the solution is an approximation depending on the resolution of the pixel representation. This talk considers two problems for constructing generalized Voronoi diagrams for which the digital-picture approach is effective.
The first problem is the construction of a crystal Voronoi diagram, which is the partition of the plane generated by crystals that grow from individual seed points in different speed. When two crystals meet, they stop growing in that direction; thus the crystals grow avoiding the other crystal regions. Consequently, the "distance" defining this diagram is measured along the shortest path avoiding the growing crystals, which makes computation of the diagram difficult. Hence, we make use of the finite-difference method for the partial differential equation, more specifically, the first marching method for the Eikonal equation, to simulate the growth of crystals. We also apply it to path planning for a robot that moves among enemy robots.
The second problem is to generate the "sports players' Voronoi diagram", which is defined as the partition of the field court into players' regions according to which player can reach the point fastest. For this purpose, a player's motion model is constructed and the parameters in the model were adjusted using experimental data. Each player's possible motion is represented by a distorted cone in the location-time space with the apex at the player's current position and with the distortion made by the current velocity of the player. From the scene of the distorted cones, the players' Voronoi diagram is generated by graphics hardware for hidden-surface removal. Finally, several quantitative indices are extracted to evaluate each player's contribution to the teamwork. The validity of the method was shown by experiment using the video data of a field hockey game.
This is joint work with Kei Kobayashi and Akira Fujimura.

2 4D Data Visualization Using Iso-Surfaces and a Control Plane

Lutz Kettner, University of North Carolina

We outline data structures and algorithms for a client/server system for the visualization of time-varying volume data of one or several variables as they occur in scientific and engineering applications. We partition the data dimensions on the client side into a 3D viewer for iso-surfaces and a 2D control plane where each point selects a particular iso-surface in 3D. We describe geometric data structures based on an unstructured mesh of 4-simplices on the server side to support remote storage and real-time interaction with the data. We report on an initial prototype for the user interaction at the client. The user interaction includes annotating the control plane with additional information, here the number of connected components of the iso-surface corresponding to each point in the control plane, and a pre-computed and fast overview window tracking the mouse cursor in the control plane.
This is joint work with Jarek Rossignac and Jack Snoeyink.

3 How to Fold a Map

Erik D. Demaine, University of Waterloo

Our work is motivated by the common practical problem of refolding a road map while obeying the existing directions of the creases. More precisely, given a rectangle with horizontal and vertical creases, each marked “mountain” or “valley”, when can it be folded by a sequence of folds along lines? We show that this problem can be solved in roughly linear time for various models of folding, depending on how many layers of paper can be folded at once. However, if the rectangle also has diagonal creases at 45 degrees, the decision problem becomes (weakly) NP-complete. Deciding foldability is also NP-complete for an orthogonal polygon with orthogonal creases. Both of these problems remain NP-complete even when every crease is free to fold as either mountain or valley, and it is only required that every crease is eventually folded.
This is joint work with Esther M. Arkin, Michael A. Bender, Martin L. Demaine, Joseph S. B. Mitchell, Saurabh Sethia, and Steven S. Skiena.

4 On the Reflexivity of Point Sets

Ferran Hurtado, Universitat Politècnica de Catalunya

We introduce a new measure for planar point sets S. Intuitively, it describes the combinatorial distance from a convex set: The *reflexivity* $\rho(S)$ of S is given by the smallest number of reflex vertices in a simple polygonization of S. We prove various combinatorial bounds and provide efficient algorithms to compute reflexivity, both exactly (in special cases) and approximately (in general). Our study naturally takes us into the examination of some closely related quantities, such as the *convex cover* number $\kappa_1(S)$ of a planar point set, which is the smallest number of convex chains that cover S, and the *convex partition* number $\kappa_2(S)$, which is given by the smallest number of disjoint convex chains that cover S. We prove that it is NP-complete to determine the convex cover or the convex partition number, and we give logarithmic-approximation algorithms for determining each. This is joint work with Esther M. Arkin, Sándor P. Fekete, Joseph S. B. Mitchell, Marc Noy, Vera Sacristán and Saurabh Sethia.

5 Smallest Color-Spanning Objects

Christian Icking, FernUniversität Hagen

We are given a set of n point sites in the plane and $k \leq n$ colors, each site is associated one color. An object in the plane is called *color-spanning* if it contains at least one site of each color. For several kinds of objects we want to find the smallest color-spanning one of that kind. The following instances can be solved.

- The smallest color-spanning *circle* can be found by using the upper envelope of the one-colored Voronoi diagrams; there are also generalizations to other metrics.
- The narrowest color-spanning *strip*, i. e., two parallel lines with minimum distance that contain all colors in between them, can be found in time $O(n^2\alpha(k)\log k)$.
- The smallest color-spanning axis-parallel *rectangle*, by perimeter or area, is the most interesting. Our algorithm uses the dynamical maintenance of maximal elements as a subprocedure; its running time is $O(n(n-k)\log^2 k)$.

Many other instances are still open, e. g. the smallest color-spanning convex polygon, triangle, or rectangle with arbitrary orientation, to name just a few.

This is joint work with Manuel Abellanas, Ferran Hurtado, Rolf Klein, Elmar Langetepe, Lihong Ma, Belén Palop, and Vera Sacristán.

6 Three-dimensional Mesh Generation for Domains with Small Angles

Jonathan Shewchuk, UC Berkeley

Consider the problem of generating a tetrahedral mesh that conforms to a domain (object or region) that has interior boundaries which the tetrahedra must respect. For instance, an object may be composed of several materials, and any given tetrahedron cannot pierce the boundaries separating those materials. If there are boundaries separated by small dihedral angles, and boundary edges separated by small angles as well, it is difficult to mesh the domain with Delaunay tetrahedra. The difficult is compounded if the tetrahedra must be "well-shaped", as numerical solvers require.

I discuss four ingredients that together yield a practical and theoretically sound approach to the problem of small input angles in three-dimensional mesh generation. First, I have generalized constrained Delaunay triangulations to three or more dimensions. They offer most of the advantages of ordinary Delaunay triangulations, but also make it possible to enforce the presence of boundary facets in the tetrahedralization. Second, I offer efficient algorithms for constructing constrained Delaunay tetrahedralizations. Third, a segment-splitting algorithm ensures that a constrained Delaunay tetrahedralization of a given domain exists. (Not all polyhedra have a tetrahedralization at all, unless additional vertices are permitted.) Fourth, a "lookahead" Delaunay refinement algorithm uses these tools to construct a high-quality mesh. It uses a lookahead step to anticipate and avoid circumstances in which Delaunay refinement may fail to terminate because of small angles in the input domain.

The four ingredients have been the subject of my research over the last four years, all of which was driven by the wish to solve the problem of small input angles in three-dimensional mesh generation.

7 Simulation of the Human Mandible: A Case Study in Hexahedral Mesh Generation

Matthias Müller-Hannemann, University of Bonn

We provide a case study for the generation of pure hexahedral meshes for the numerical simulation of physiological stress scenarios of the human mandible (jawbone).
This test case is used as a running example to demonstrate the applicability of a combinatorial approach for the generation of hexahedral meshes by means of successive dual cycle eliminations. We report on the progress and recent advances with this method. The given input data, a surface triangulation obtained from computed tomography data, requires a substantial mesh reduction and a suitable conversion into a quadrilateral surface mesh as a first step, for which we use mesh clustering and b-matching techniques.
Several strategies for improved cycle elimination orders are proposed. They lead to a significant reduction in the mesh size and a better structural quality. Based on the resulting combinatorial meshes, gradient-based optimized smoothing with the condition number of the Jacobian matrix as objective together with mesh untangling techniques yielded embeddings of a satisfactory quality.
To test our hexahedral meshes for the mandible model within an FEM simulation we used the scenario of a bite on a "hard nut." Our simulation results are in good agreement with observations from biomechanical experiments.
This is joint work with Cornelia Kober, FU Berlin.

8 Bounded Curvature Traversals of Narrow Roadways

David Kirkpatrick, University of British Columbia

We consider the problems of existence and construction of bounded (without loss of generality, unit) curvature paths in roadways (domains bounded by two curves of fixed separation w – the width of the roadway). Roadways can be viewed as the space swept by a disk of diameter w. If the boundaries do not self-intersect the roadway is said to be planar; non-planar roadways correspond to fixed width strips embedded in 3-space. For both planar and non-planar roadways we specify thresholds w^* with the properties:

i) for every roadway of width $w \geq w^*$ there exists a unit curvature traversal (i.e. a path separating the roadway boundaries) whose number of bends is proportional to the number of bends in the shortest polygonal (i.e. unbounded curvature) separating chain; and

ii) for any $\epsilon > 0$, there exist roadways of width $w^* - \epsilon$ which do not admit unit curvature traversals.

The exact values of w^* are 1.4641...($2\sqrt{(3)} - 2$) and 1.5051...(root of $r^3 - 5r^2 - 16r + 32 = 0$) for the planar and non-planar cases respectively.
This is joint work with Sergei Bespamyatnikh.

9 Matching Polygonal Curves with Respect to the Fréchet Distance

Christian Knauer, Free University of Berlin

We provide the first algorithm for matching two polygonal curves f and g under translations with respect to the Fréchet distance. If f and g consist of m and n segments, respectively, the algorithm has runtime $O((mn)^3(m+n)^2 \log(m+n))$.

10 Approximate Matching of Polygonal Curves with Respect to the Fréchet Distance

Carola Wenk, Free University of Berlin

The task of comparing two two-dimensional shapes arises naturally in many applications. Often two-dimensional shapes are given by the planar curves forming their boundaries. Two possible distance measures to assess the resemblance of two curves are the Hausdorff distance and the Fréchet distance.
We show that there exist reference points that allow a fast matching of two curves with respect to the Fréchet distance under translations. Furthermore we consider planar curves where the arc length between any two points on the curve is at most a constant κ times their Euclidean distance, which we call κ-straight curves. We show that the Fréchet distance of such curves is at most $(1+\kappa)$ times their Hausdorff distance.

11 Digital Halftoning As a Combinatorial Optimization Problem with Polynomial-time Algorithms

Tetsuo Asano, Japan Advanced Institute of Science and Technology

Digital halftoning is an important technique for printers to convert a continuous-tone image to a binary image. In this paper we formulate it as a combinatorial optimization problem. The optimization criterion is to minimize the difference between the perceived displayed continuous-tone image and the perceived displayed halftone image. Although a number of algorithms have been proposed so far toward this goal, there is no study on its inherent computational difficulty to the best knowledge of the authors, except for the one given by three of the authors which established NP-hardness of some simplified version of the problem. In this paper we consider how to relax the optimization criterion to have a polynomial-time algorithm. A key idea is to describe the computational complexity using geometric characteristics of a family of regions which defines the criterion. We also show experimental results of our polynomial-time algorithm based on a minimum-cost maximum flow algorithm for comparison with the results of the conventional methods.
This is joint work with Naoki Katoh (Kyoto Univ.), Tomomi Matsui (Univ. of Tokyo), Hiroshi Nagamochi (Toyohashi Univ. of Technology, Koji Obokata (JAIST) and Takeshi Tokuyama (Tohoku Univ.)

12 Translating a Planar Object to Maximize Point Containment: Exact and Approximation Algorithms

Michiel Smid, University of Magdeburg

Let C be a closed and bounded set in the plane and let S be a set of n points in the plane. We consider the problem of computing a translate of C that contains the maximum number of points of S. We start by giving two basic algorithms for this problem whose running times are roughly quadratic in n. Then we show how random sampling can be used to transform any deterministic algorithm for this

optimal-placement problem into a Monte Carlo approximation algorithm. For sets C that satisfy a certain fatness condition, we use a simple bucketing technique to transform any algorithm for the optimal-placement problem into an algorithm that solves the same problem. The bucketing transformation can be implemented in the algebraic computation-tree model, or in a more powerful model using hashing. Applying the random-sampling and bucketing transformations to the two basic algorithms, we obtain a variety of results. For example, if C is a convex polygon with m vertices, we can solve the optimal-placement problem in the algebraic computation-tree model in $O(n \log n + mn \log(mk^*) + nk^*)$ time, where k^* is the number of points in an optimal placement of C. Also, for any constants $\epsilon, c > 0$, we can compute a placement of C that contains at least $(1-\epsilon)k^*$ points in $O(m + n \log(mn))$ time with error probability at most $e^{-c\sqrt{k^*}}$. Employing hashing techniques, we can implement this algorithm so that its running time is bounded by $O(m + n \log m)$. This is joint work with Torben Hagerup and Rahul Ray.

13 Enumerating Order Types for Small Point Sets

Oswin Aichholzer, TU Graz

Order types are a means to characterize the combinatorial properties of a finite point configuration. In particular, the crossing properties of all straight-line segments spanned by an planar n-point set are reflected by its order type. We establish a complete and reliable data base for all possible order types of size $n = 10$ or less. The data base includes a realizing point set for each order type in small integer grid representation. To our knowledge, no such project has been carried out before.

We substantiate the usefulness of our data base by applying it to several problems in computational and combinatorial geometry. Problems concerning triangulations, simple polygonalizations, complete geometric graphs, and k-sets are addressed. This list of possible applications is not meant to be exhaustive. We believe our data base to be of value to many researchers who wish to examine their conjectures on small point configurations.

14 Combinatorial Geometry Problems in Pattern Recognition

Peter Brass, Free University of Berlin

The general situation of point pattern matching is that two point-sets A (the pattern) and B (the background) in d-dimensional space are given, as well as an equivalence relation on subsets defined by a group G (translations, congruences, homotheties, similarities, affinities) Then we want to find all subsets of B that are equivalent to the given pattern A. In this talk I surveyed the current state for all of these situations, in each case giving bounds for the maximum number of possible matches and for the time needed to find them. Always these two questions, the combinatorial geometry problem of 'how many equivalent subsets are there' and the algorithmic problem 'how do we find them' turn out to be very strongly related. Then I presented some results on a preprocessing-variant of this problem, which leads to a different kind of geometric question, and proposed a new model for approximate point pattern matching ('window matching': the matching subset should actually be a section of B, cut out by a convex 'window'), for which I have some first results.

15 One Line and N Points

Bernd Gärtner, ETH Zürich

We describe and analyze a simple randomized process involving one line and n points in the plane. The process models the behavior of the RANDOM-EDGE simplex algorithm on $(n-2)$-polytopes with n facets. We obtain a tight $\Theta(\log^2 n)$ bound for the expected number of pivot steps. This is the first nontrivial analysis of RANDOM-EDGE for a whole class of polytopes. Previous results were restricted to special polytopes.
This is joint work with Joszef Solymosi, Falk Tschirschnitz, Pavel Valtr and Emo Welzl.

16 Exact Geometric Computation and Randomized Theorem Proving

Chee K. Yap, Courant Institute, New York

The phenomenon of numerical nonrobustness is an important issue for many scientific and engineering applications. Among the many approaches proposed, the idea of Exact Geometric Computation (EGC) is one of the most promising with a sound basis. We review the key challenges facing EGC and some recent results on root bounds and filters. We then describe our ongoing work on the "Core Library" which aims to bring EGC techniques within easy reach of any programmer.
The rest of the talk describes an unexpected application in randomized theorem proving. We generalize the well-known Schwartz's Lemma for randomized zero testing of multivariate polynomials. The generalization allows the input expressions to have division and radical operations. This then allows us to randomly test the truth of geometric theorems based on ruler-and-compass constructions. Our theorems have the form $(D, H) \Rightarrow (P = 0)$ where H is a set of polynomial equations, D a set of degeneracy conditions (either polynomial inequalities or inequations) and P a polynomial. Because of the possibility of inequalities, such theorems are strictly speaking theorems about "real geometry" (even when the corresponding theorem about complex geometry may be true).
We have implemented such a prover using the Core Library. Our experimental results show that when such a theorem is also true in complex geometry, then Wu's method may be much faster for "real geometry". On the other hand, our method is the fastest method available for such real theorems. Another useful feature of our system is its ability to reject false theorems very quickly.
This is joint work with Daniela Tulone and Chen Li.

17 A Separation Bound for Real Algebraic Expressions

Kurt Mehlhorn, MPI für Informatik, Saarbrücken

(Joint work with Christoph Burnikel, Stefan Funke, Stefan Schirra and Susanne Schmitt)
Real algebraic expressions are expressions whose leaves are integers and whose internal nodes are additions, subtractions, multiplications, divisions, k-th root

operations for integral k, and taking roots of polynomials whose coefficients are given by the values of subexpressions. We consider the sign computation of real algebraic expressions, a task vital for the implementation of geometric algorithms. We prove a new separation bound for real algebraic expressions and compare it analytically and experimentally with previous bounds. The bound is used in the sign test of the number type `leda_real`.

18 Geometric Straight-line Programs

Ulrich Kortenkamp, Free University of Berlin

Straight-line programs are a common way to represent polynomials in a compact fashion. If we want to use them as a description tool for geometry, we are in need of square (and cubic) roots as soon as we introduce intersections of circles (or general conics). Unfortunately, roots are not determined but ambiguous, which leads to several different possible instances for a straight-line program with given input values. A major problem is the following decision problem: Does there exist a continuous path from one instance of a straight-line program to another one? We show that this problem is at least as hard as deciding whether a polynomial given by a straight-line program is the zero polynomial.

19 Nearest Neighbor Search in High Dimensions

Laura Heinrich-Litan, Free University of Berlin

The talk presents algorithms for solving the nearest neighbor problem with respect to the L_∞-distance. The first algorithm requires no preprocessing and storage only for the point set P. Its average runtime assuming that the set P of n points is drawn at random from $[0,1]^d$ under uniform distribution is essentially $\Theta(nd/\ln\ n)$ thereby improving the brute-force method by a factor of $\Theta(1/\ln\ n)$. Several generalizations of the method are also presented, in particular to other "well-behaved" probability distributions and to the important problem of finding the k nearest neighbors to a query point. It is shown how to use a partition of the point set P into monotone subsequences, which is computed in the preprocessing phase, in order to speed up the query algorithm. The talk presents also a method which provides time-space tradeoffs for L_∞-nearest-neighbor-search.

20 Better algorithms for high-dimensional proximity problems via asymmetric embeddings

Piotr Indyk, Massachusetts Institute of Technology

In this talk we give approximation algorithms for several proximity problems in high dimensional space. In particular, we give a Las Vegas data structure for $(1+\epsilon)-$nearest neighbor with polynomial space and query time polynomial in dimension d and $\log n$, where n is the database size. We also give a deterministic 3-approximation algorithm with similar bounds. Finally, we show a general reduction from the furthest point problem to the nearest neighbor problem. Our results are unified by the fact that their key component is a dimensionality reduction technique for Hamming spaces.

21 Discrete mobile centers

Leonidas Guibas, Stanford University

We propose a new randomized algorithm for maintaing a set of clusters among moving points in the plane. Given a specified cluster radius, our algorithm selects and maintains a variable subset of the nodes as cluster centers. This subset has the property that:

- balls of the given radius centered at the chosen nodes cover all the others, and
- the number of centers selected is a constant-factor approximation of the minimum possible

As the nodes move, an event-based kinetic data structure updates the triangles as necessary. This kinetic data structure is shown to be responsive, efficient, local, and compact. The produced cover is also smooth, in the sense that wholescale cluster re-arrangements are avoided. The algorithm can be implemented without exact knowledge of the node positions, if each node is able to sense its distance to other nodes up to the cluster radius. Such a kinetic clustering can be used in numerous applications where mobile devices must be interconnected into an ad-hoc network to collaboratively perform same task.

22 Data Structures for Moving Objects

Pankaj K. Agarwal, Duke University

With the rapid advances in positioning systems, e.g., GPS, ad-hoc networks, and wireless communication, it is becoming increasingly feasible to track and record the changing position of continuously moving objects. These developments have raised a wide range of challenging geometric problems involving moving objects, including efficient data structures for answering proximity queries, for clustering, and for maintaining connectivity information. This talk describes a few data structures for answering various queries on moving objects and states a number of open problems in this area.

23 Kinetic Collision Detection for Simple Polygons

Bettina Speckmann, University of British Carolina

We design a simple and elegant kinetic data structure for detecting collisions between polygonal (but not necessarily convex) objects in motion in the plane. Our structure is compact, maintaining an active set of certificates whose number is proportional to a minimum-size set of separating polygons for the objects. It is also responsive; on the failure of a certificate invariants can be restored in time logarithmic in the total number of object vertices. Furthermore we describe an extension of this structure that incorporates a hierarchical representation of convex chains. This permits us to define and maintain an adaptive hierarchical outer approximation for simple polygons. The latter constitutes a succinct approximation of the relative convex hull of the objects whose size is linearly related to the size of the minimum link subdivision separating the objects. This representation can be exploited to give separation sensitive complexity bounds for kinetic collision detection.

24 Incarnations of the Generalized Lower Bound Theorem

Emo Welzl, ETH Zürich

We briefly discuss the Generalized Lower Bound Theorem for polytopes, and relate it to recently established extremal results in combinatorial geometry.

Generalized Lower Bound Theorem. Consider a simplicial d-polytope with f_i the number of its i-faces, $0 \leq i \leq d-1$. We know that the f-vector $(f_0, f_1, \ldots, f_{d-1})$ has to satisfy certain conditions. For example, when $d = 3$, we have $f_0 - f_1 + f_2 = 2$, $f_0 \geq 4$, and $f_1 = 3f_0 - 6$. These facts generalize to other dimensions, e.g. *Euler's Relation*, $\sum_{i=0}^{d-1}(-1)^i f_i = 1 + (-1)^{d-1}$, for all $d \geq 0$, which is just one of the so-called *Dehn-Sommerville Relations.* Moreover, $f_0 \geq d + 1$, for $d \geq 0$, and $f_1 \geq d\, f_0 - \binom{d+1}{2}$ for $d \geq 3$. The later is called *Lower Bound Theorem*, due to D. Barnette, 1970. The two inequalities are the first of a whole sequence – known as the Generalized Lower Bound Theorem, first proved by R. Stanley in 1980 (as part of the necessity part of the g-Theorem that was conjectured by P. McMullen). It claims that all expressions $g_1 := f_0 - (d+1)$, $g_2 := f_1 - d\, f_0 + \binom{d+1}{2}$, $g_3 := f_2 - (d-1)f_1 + \binom{d}{2}f_0 - \binom{d+1}{3}$, $\ldots g_i := \cdots$, are nonnegative, for $i \leq d/2$. (We agree on $f_{-1} := 1$. Then $g_i := \sum_{j=1}^{i+1}(-1)^{j-1}\binom{d+j-i}{j-1}f_{i-j}$, which is $h_i - h_{i-1}$ given the h-vector defined by the identities $f_{d-i-1} = \sum_{j=0}^{d}\binom{j}{i}h_j$, $0 \leq i \leq d$.)

Balanced Lines. Let R and B be two disjoint sets of n points each in the plane, so that $R \cup B$ is in general position (i.e. no three on a common line). A line is called *balanced*, if it contains a point in R and a point in B, and, on both sides of the line, the number of points from R and B are equal. In 2000, J. Pach and R. Pinchasi proved that *there are always at least n balanced lines*, answering a conjecture of G. Baloglou's. The bound is tight. Their proof required quite some effort and is rather technical, probably more than such a simple statement might deserve – one may be tempted to believe. However, we show that this fact is equivalent to

$$g_{d/2} \geq 0, \text{ for } d\text{-polytopes with at most } d+4 \text{ vertices and } d \text{ even.} \qquad (*)$$

On one hand, that allows now a simpler proof of the balanced lines result. On the other hand, the GLBT has so far resisted an 'elementary' proof, even for the special case which is of interest here.

Halving Triangles. Given a set Q of $2n+1$ points in general position in 3-space (i.e., no four coplanar), a *halving triangle* is a triangle t spanned by three of the points in Q, with the same number of points from Q on both sides of the plane carrying t. It has been shown that *there are always at least n^2 halving triangles;* this is tight, e.g., when the points are in convex position. It follows from the (at most j)-facets result in 3-space by Welzl, presented at the '99 Computational

Geometry Dagstuhl Meeting. Again, it is equivalent to the fact about balanced lines, and to (*).
Joint work with Micha Sharir.

25 Bounded-degree Pseudo-triangulations of Points

Jack Snoeyink, University of North Carolina

A pseudo-triangle is a planar polygon that has three convex vertices, and possibly other vertices with internal angles greater than π. A pseudo-triangulation (PT) for n points in the plane is a partition of their convex hull into pseudo-triangles using these points as vertices. PTs have been studied because of their applications to visibility, collision detection data structures, and motion of a linkage.
We show several basic properties: 1. The minimum PTs on n points in general position have $n-2$ pseudo-triangles, $2n-3$ edges, and one reflex angle at each point. 2. A canonical minimum PT can be computed by a simple sweep algorithm. 3. The family of all minimum PTs is connected under a natural edge flip operation. 4. Every point set has a minimum PT with maximum vertex degree 6; in a PT wheel with 12 points, some vertex has degree at least 5.
We would like to determine whether every point set has a PT with maximum degree 5; we have implemented a test in software that enumerates minimum pseudotriangles. We would also like to determine, for points moving with linear trajectories, how to use edge flips to preserve some degree bound.
This was joint work with Lutz Kettner, Andrea Mantler, Bettina Speckmann, and Fumihiko Takeuchi.

26 Three-dimensional Shape Matching

Bernard Chazelle, Princeton University

I will discuss a new method for computing shape signatures for arbitrary (possibly degenerate) 3D polygonal models. The approach is based on reducing the shape matching problem to the comparison of probability distributions. Experimental evidence suggests that the method compares favorably to more traditional

shape matching methods that require pose registration, feature correspondence, or model fitting.
This is joint work with David Dobkin, Tom Funkhouser, and Robert Osada.

27 New Bounds for Incidences and Related Problems

Micha Sharir, Tel Aviv University and New York University

Given a set P of m distinct points in the plane and C a set of n curves, we denote by $I(P, C)$ the number of point-curve incidences between P and C, and by $I(m, n)$ the maximum possible value of $I(P, C)$, for $|P| = m$, $|C| = n$. We review known bounds for $I(m, n)$ for the case of lines, unit circles, and general circles, present several techniques that lead to simple proofs of these bounds, and derive new bounds for the case of general circles and for several other related cases. The talk reviews the recent techniques due to Székely and to Tamaki and Tokuyama, and presents several improvements of their bound. For example, we show that n circles or parabolas can be cut into $O(n^{3/2+\epsilon})$ arcs, each pair of which intersect at most once. For pairwise-intersecting pseudo-circles the bound improves to $O(n^{4/3})$. We also discuss duality in pseudoline arrangements, and use this to compute efficiently incidences between points and pseudolines or circles or parabolas.
This is joint work with B. Aronov, J. Pach, R. Pinchasi, E. Nevo, P. Agarwal and S. Smorodinsky. Results: Complexity of many faces for arrangements of curves as above, Erdős' repeated distances and distinct distances problems, the number of congruent simplices in a point set in higher dimensions, incidences involving parabolas or graphs of polynomials of constant maximum degree, some algorithmic connections, and more.

28 Binary Space Partitions for Axis-Parallel Segments, Rectangles, and Hyperrectangles

Joseph S. Mitchell, SUNY at Stony Brook

We provide a variety of new results, including upper and lower bounds, as well as

simpler proof techniques for the efficient construction of binary space partitions (BSP's) of axis-parallel segments, rectangles, and hyperrectangles. (a) A consequence of the analysis in [1] is that any set of n axis-parallel and pairwise-disjoint line segments in the plane admits a binary space partition of size at most $2n-1$. We establish a worst-case lower bound of $2n-o(n)$ for the size of such a BSP, thus showing that this bound is almost tight in the worst case. (b) We give an improved worst-case lower bound of $\frac{9}{4}n-o(n)$ on the size of a BSP for isothetic pairwise disjoint rectangles. (c) We present simple methods, with equally simple analysis, for constructing BSP's for axis-parallel segments in higher dimensions, simplifying the technique of [2] and improving the constants. (d) We obtain an alternative construction (to that in [2]) of BSP's for collections of axis-parallel rectangles in 3-space. (e) We present a construction of BSP's of size $O(n^{5/3})$ for n axis-parallel pairwise disjoint 2-rectangles in $\mathbb{R}^4$, and give a matching worst-case lower bound of $\Omega(n^{5/3})$ for the size of such a BSP. (f) We extend the results of [2] to axis-parallel k-dimensional rectangles in $\mathbb{R}^d$, for $k < d/2$, and obtain a worst-case tight bound of $\Theta(n^{d/(d-k)})$ for the size of a BSP of n rectangles. Both upper and lower bounds also hold for $d/2 \leq k \leq d-1$ if we allow the rectangles to intersect.
This is joint work with Adrian Dumitrescu and Micha Sharir; to appear in the Proceedings of the ACM Symposium on Computational Geometry, June 2001.

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29 Nice Point Sets Can Have Nasty Delaunay Triangulations

Jeff Erickson, University of Illinois - Urbana

We consider the complexity of Delaunay triangulations of sets of points in $\mathbb{R}^3$ under certain practical geometric constraints. The *spread* of a set of points is the ratio between the longest and shortest pairwise distances. We show that in the worst case, the Delaunay triangulation of n points in $\mathbb{R}^3$ with spread Δ has complexity $\Omega(\min\{\Delta^3, n\Delta, n^2\})$ and $O(\min\{\Delta^4, n^2\})$. For the case $\Delta = \Theta(\sqrt{n})$, our lower bound construction consists of a uniform sample of a smooth convex surface with bounded curvature. We also construct a family of smooth connected surfaces such that the Delaunay triangulation of any good point sample has near-quadratic complexity.

30 The Shuffling Buffer

Olivier Devillers, INRIA - Sophia Antipolis

The complexity of randomized incremental algorithms is analyzed with the assumption of a random order of the input. To guarantee this hypothesis, the n data have to be known in advance in order to be mixed what contradicts with the on-line nature of the algorithm.
We present the *shuffling buffer* technique to introduce sufficient randomness to guarantee an improvement on the worst case complexity by knowing only k data in advance.
Typically, an algorithm with $O(n^2)$ worst-case complexity and $O(n)$ or $O(n \log n)$ randomized complexity has an $O(\frac{n^2 \log k}{k})$ complexity for the shuffling buffer. We illustrate this with binary search trees, the number of Delaunay triangles or the number of trapezoids in a trapezoidal map created during an incremental construction.
This is joint work with Philippe Guigue (INRIA).

31 Reconstruction of Line Arrangements from Linear Probes

William S. Evans, University of British Columbia

We are given an arrangement of n invisible input lines in the plane. None of the input lines are vertical. Our task is to determine the arrangement by probing it with vertical probe lines. If we draw a probe line through the input arrangement, we obtain the coordinates of the intersections of the probe line with the invisible

lines.
If we could place the probes one after the other then we could determine the hidden arrangement using only three probes. The real problem is to find a set of k vertical probes that will work to determine any arrangement of n input lines. We know that any $n+1$ probes suffice and that any $n-1$ probes do not suffice - meaning that there are two different arrangements of n lines that "look the same" according to the $n-1$ probes.
If we strengthen the probes so that they report the number of input lines that pass through an intersection then the example showing that $n-1$ probes are insufficient no longer works. For these stronger probes, we show that the number of probe lines must be at least $\log n$. In particular, k probes can be confused by $6^{k/3}/2$ input lines (for k divisible by 3).
This is joint work with R. Beals, M. de Berg, J. Bose, D. Bremner, J. Friedman, and L. Narayanan.

32 Schematization of Network Maps

Marc van Kreveld, Utrecht University

We study the problem of computing schematized versions of network maps, like railroad maps. Every path of the schematized map has two or three links with restricted orientations, and topologically, the schematized map must be equivalent to the input map. Our approach applies to several types of schematizations, and certain additional constraints can be added. In the general case our algorithm takes $O(n \log^3 n)$ time, and when all paths in the input are monotone in some (not necessarily the same) direction, it runs in $O(n \log n)$ time.
This is joint work with: Mark de Berg, Sergio Cabello, Steven van Dijk, and Tycho Strijk.

33 Smooth-Surface Reconstruction in Almost Linear Time

Edgar Ramos, Max-Planck-Institut für Informatik, Saarbrücken

A surface reconstruction algorithm takes as input a set of sample points from a closed and smooth surface in 3-d space and produces a piece-wise linear approximation of the surface which contains the sample points. This problem has received considerable attention in computer graphics and more recently in computational geometry. In the latter area, recently four different algorithms (by Amenta and Bern '98; Amenta, Choi, Dey and Leekha '00; Amenta, Choi and Kolluri '00; Boissonnat and Cazals '00) have been proposed. These algorithms have a correctness guarantee: if the sample is sufficiently dense then the output is a good approximation to the original surface. Unfortunately, these algorithms are based on the construction of 3-d Voronoi diagrams or Delaunay tetrahedrizations, and hence have a worst-case running time that is quadratic in the size of the input. We describe a new algorithm that also has a correctness guarantee but its worst-case running time is almost linear (in fact, $O(n \log n)$ where n is the input size). The piece-wise linear approximation produced by the algorithm is a subcomplex of the 3-d Delaunay tetrahedrization (as it was in two of the previous algorithms), but it is obtained while avoiding to compute the complete tetrahedrization. The reconstruction problem for a set of samples that is distributed uniformly on the surface is easy; in contrast, local oversampling complicates the connections that the algorithm needs to make between samples to produce the reconstruction. Making use of this observation, the new algorithm proceeds in four major steps: First, it estimates the normal and a *decimation* radius for each sample point (estimation); then redundant samples are eliminated (decimation) to obtain a subset of sample points that is "locally uniform"; from this subsample a reconstruction can be obtained using a simple algorithm (basic reconstruction); finally, the eliminated sample points are incorporated in the reconstruction (consolidation).

34 Walking in a Triangulation

Monique Teillaud, INRIA - Sophia Antipolis

Given a triangulation $\mathcal{T}$ of n vertices in the plane and a point p, finding the triangle of $\mathcal{T}$ containing p is a fundamental problem in computational geometry. There exist different strategies, starting from the triangle containing a source point q. The simplest strategy consists in visiting all triangles along the line segment qp. A second strategy, *the orthogonal walk*, visits the triangles along an isothetic path moving from q to p by changing one coordinate at a time.
Finally, we call *visibility walk* the following strategy: from a triangle t not con-

taining p we move to the neighbor of t through an edge e if the line supporting e separates t from p; if there are two such edges, we may move to any of these two neighbors. In the case of a non Delaunay triangulation, the walk may loop. We consider the *stochastic walk* in which we decide that if we can choose between two neighbors of t, then the choice is done at random.

All these walking strategies generalize to higher dimensions. Our purpose is to study the performances of the different strategies from both theoretical and practical points of view, in $\mathbb{R}^2$ and $\mathbb{R}^3$.

This is joint work with Olivier Devillers and Sylvain Pion.

35 The Polytope of Minimum Pseudotriangulations and the Quest for the Delaunay Pseudotriangulation

Günter Rote, Freie Universität Berlin

A minimum pseudotriangulation of a planar point set is a maximal set of non-crossing edges between these points such that every point is incident to an angle $> \pi$. It decomposes the convex hull into pseudotriangles, i.e., polygons with three convex vertices and an arbitrary number of reflex vertices. Besides having useful applications, mentioned in a few other talks, pseudotriangulations have very nice combinatorial properties. In many respects, they appear even more attractive than (ordinary) triangulations. For instance, we define a polyhedron whose vertices are in one-to-one correspondence with the pseudotriangulations of a given point set. The proof is not difficult, but it still contains some mysteries which are not fully understood. For point sets in convex position, pseudotriangulations coincide with ordinary triangulations, and so we get another geometric representation of the associahedron (the polytope whose vertices correspond to the different possibilities of inserting brackets into the formula $a * b * c * \cdots * z$). By optimizing a suitably chosen linear function over the polytope, we obtain a canonical pseudotriangulation for every point set, which might play the role which the Delaunay triangulation plays among the triangulations. However, for convex point sets, this triangulation does not coincide with the Delaunay triangulation, but it has some other interesting properties, like being invariant under affine transformations.

This is joint work with Ileana Streinu and Francisco Santos.