Dagstuhl Seminar 01251

Graph Decompositions and Algorithmic Applications

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organized by
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Contents

1 Preface

There are many notions of graph decomposition which arise in the literature. Some decompositions involve decomposing a graph using separators of special types (balanced or polynomially bounded, star cutsets, clique cutsets), others involve identification of special sets (substitution or splits), while others involve tree decomposition (treewidth, cliquewidth, branchwidth) or tree composition (Cartesian product, lexicographic product).

These decompositions are of fundamental importance for solving optimization and recognition problems on classes of graphs. For example, substitution decomposition is closely related to such problems as solving problems expressible in monadic second order logic quantifying over vertices and/or edges and comparability graph recognition and optimization. Treewidth and its generalizations are of special importance due to the Robertson-Seymour results on tree decomposition and existential proof of existence of algorithms. Clique cutsets and star cutsets are fundamental tools used in the study of chordal and perfect graphs. Particular tools for working with these decompositions, such as partition refinement and lexicographic breadth first search, have recently been improved and generalized in this context.

The second Dagstuhl seminar on Graph Decompositions and Algorithmic Applications was designed to bring together researchers working on a variety of aspects of graph decomposition. Talks were given reporting on recent results concerning the cliquewidth of graphs and its algorithmic use, the connection between cliquewidth and treewidth, studying special classes of graphs, new decomposition techniques and optimization algorithms, and data structures which allow faster decomposition algorithms.

A. Brandstädt
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2 Talks

Decomposition by Clique Minimal Separators

Anne Berry, LIMOS, Université Clermant-Ferrand II, France
Jean-Paul Bordat, LIRMM, Montpellier, France

Clique minimal separator decomposition consists in copying a separator which is a clique into the different connected components it defines. This decomposition is interesting because it is hole-preserving, but Tarjan in 1985 left open the question of defining a unique decomposition.

We explain why using only clique separators which are also minimal separators yields a unique decomposition, by nearly partitioning the minimal separators of the graph into the subgraphs obtained. As a result, problems related to minimal separation (such as triangulation, minimal separator enumeration, class recognition ...) can be run separately on the atoms of this decomposition.

A generic implementation consists in first computing a minimal triangulation of the graph, a process which preserves clique minimal separators. This is one of the startling and important applications of minimal triangulation.

In the absence of clique minimal separators, the decomposition can be forced by using a non-clique minimal separator and adding the necessary edges to make it into a clique.

Because of our invariant, this process will in the end yield the same fill as repeatedly choosing a non-clique minimal separator and making it a clique. Both processes thus force the graph into respecting Dirac's characterization for chordal graphs: 'A graph is chordal iff every minimal separator is a clique.' Both processes yield a minimal triangulation of the input graph, which make them remarkable coercion mechanisms, further illustrating the power of minimal separation as a graph property exploration tool.

On the clique width and structure of graph classes defined by some forbidden P_4 extensions

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Dieter Kratsch, Univesité de Metz, Metz, France
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It is well-known that every algorithmic graph problem expressible in Monadic Second Order Logic quantifying only over sets of vertices can be efficiently solved on graph classes having bounded clique width. The problems Maximum Stable Set, Maximum Clique and Minimum Dominating Set, for instance, are of this type.

We give a survey on recent results classifying graph classes defined by some forbidden one-vertex extensions of the P_4 such as the P_5, chair, P, C_5, bull, gem and their complements. The results improve and unify recently published work, in particular on the Maximum Stable Set problem by showing that several of these classes have bounded clique width due to the simple structure of their prime subgraphs.

Some Optimization Problems on Intersection Graphs

Kathie Cameron, Wilfrid Laurier University, Waterloo, Ontario, Canada
Chính T. Hoàng, Wilfrid Laurier University, Waterloo, Ontario, Canada
R. Sritharan, University of Dayton, Dayton, U.S.A.
Yingwen Tang, University of Dayton, Dayton, U.S.A.

Recently Gavril introduced a new class of intersection graphs called interval-filament graphs. A graph is an interval-filament graph if it is the intersection graph of a set of curves C in the xy-plane with left endpoint, $l(C)$, and right endpoint, $r(C)$, on the x-axis such that C lies in the plane above the x-axis between $l(C)$ and $r(C)$. These include cocomparability graphs and polygon-circle graphs (the intersection graphs of polygons inscribed on a circle), which include circular-arc graphs (the intersection graphs of arcs of a circle), circle graphs (the intersection graphs of chords of a circle), chordal graphs, and outerplanar graphs.

Two related classes of graphs, not known to be intersection graphs of any nice family are asteroidal-triple-free graphs (which include cocomparability graphs) and weakly chordal graphs (which include chordal graphs).

I will prove the strong perfect graph conjecture holds for polygon-circle graphs.

I will prove a bound on the clique-covering number for interval-filament graphs in terms of the size of a largest independent set of vertices in the graph.

An induced matching in a graph G is a matching M such that no two edges of M have any third edge of the graph between them; that is, an induced matching is a matching which forms an induced subgraph of G. I will show that the maximum induced matching problem is polytime-solvable for interval-filament graphs, asteroidal-triple-free graphs, and weakly chordal graphs. The maximum induced matching problem is known to be NP-complete for bipartite graphs and for planar graphs.

The work on the strong perfect graph conjecture and on the clique-covering bound is joint with Chinh Hoang. The result on induced matchings in weakly chordal graphs is joint with R. Sritharan and Yingwen Tang. The result on induced matchings in asteroidal-triple-free graphs was obtained independently by Jou-Ming Chang.

On the relationship between clique-width and treewidth

Derek Corneil, University of Toronto, Toronto, Canada
Udi Rotics, Netanya Academic College, Netanya, Israel

Treewidth is generally regarded as one of the most useful parametrizations of a graph's construction. Clique-width is a similar parametrization that shares one of the powerful properties of treewidth, namely: If a graph is of bounded treewidth (or clique-width), then there is a polynomial time algorithm for any graph problem expressible in Monadic Second Order Logic, using quantification on vertex sets (in the case of clique-width you must assume a clique-width parse expression is given). In studying the relationship between treewidth and clique-width, Courcelle and Olariu showed that any graph of bounded treewidth is also of bounded clique-width; in particular for any graph G with treewidth k, the clique-width of $G \leq 4 * 2^{k-1} + 1$. (Johansson's result on NLC width shows that the "+" is not needed.)

In this paper we improve this result to the clique-width of $G \leq 3 * 2^{k-1}$ and more importantly show that there is an exponential lower bound on that relationship. In particular, for any k, there is a graph G with treewidth $= k$ where the clique-width of $G \geq 2^{\lfloor k/2 \rfloor - 1}$

Bull-reducible Berge graphs are perfect

Celina de Figueiredo, Universidade Federal do Rio de Janeiro, Rio de Janeiro, Brazil
Hazel Everett, LORIA – Université Nancy 2, Nancy, France
Sulamita Klein, Universidade Federal do Rio de Janeiro, Rio de Janeiro, Brazil
Bruce Reed, Université Pierre et Marie Curie, Paris, France

A graph is perfect if for every induced subgraph H of G, the chromatic number, $\chi(H)$, of H is equal to its clique number $\omega(H)$ (i.e., the size of the largest clique it contains). Berge's well known SPGC (Strong Perfect Graph Conjecture) states that the class of perfect graphs coincides with the class of graphs containing no induced odd cycle of length at least 5 or the complement of such a cycle. A graph in this second class is called Berge. It is easy to see that every perfect graph is Berge so the SPGC actually states that every Berge graph is perfect. A bull is a graph with five vertices a, b, c, d, e and five edges ab, bc, cd, be, ce. Chvátal and Sbihi proved Every bull-free Berge graph is perfect. We extend this result and prove that if no vertex in a Berge graph G is in two bulls then G is perfect. We also provide a polynomial-time recognition algorithm for such graphs, which we call bull-reducible Berge graphs (following the definition of P_4-reducible introduced by Jamison and Olariu).

Modular decomposition of directed graphs and 2-structures: a linear-time algorithm

Fabien de Montgolfier, Université de Montpellier, France

The modular decomposition is a powerful tool for graph algorithms or drawing. There exist $O(n+m)$ (linear) algorithms for the decomposition of undirected graphs [Cournier and Habib 1994, McConnell and Spinrad 1994], but as far as we know the best algorithms for the directed graphs run in $O(n^2)$ [Ehrenfeucht, Gabow, McConnell, Sullivan 1994] or $O(m \log n)$ [Dahlhaus, Gustedt, McConnell 1997]. In this talk we present a $O(n + m)$ algorithm.

After a theorem bounding the sum of the size of the strong modules by the size of the graph, we give a simple way to get a factorizing permutation of a tournament, allowing their linear-time decomposition. Then we present our general algorithm. It is a reduction to the undirected case, and we work on their modular decomposition tree.

That algorithm extends, with few modifications, to the Prime Tree Decomposition of the 2-structures.

Planar Domination Graphs

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R. Sritharan, University of Dayton, Dayton, U.S.A.

A graph G is a domination graph if each induced subgraph of G has a pair of vertices such that the open neighborhood of one is contained in the closed neighborhood of the other in the subgraph. No polynomial time algorithm or hardness result is known for the problem of deciding whether a graph is a domination graph. We show that the class of planar domination graphs is equivalent to the class of planar weakly chordal graphs, and thus can be recognized in polynomial time. Furthermore, we prove that planar domination graphs are a subclass for which each graph in the class either is a clique or has two nonadjacent dominated vertices.

On the Hierarchy of Interval, Probe and Tolerance Graphs

Martin Charles Golumbic, University of Haifa, Haifa, Israel
Marina Lipshteyn, Bar-Ilan University, Ramat-Gan, Israel

Tolerance graphs and interval probe graphs are two generalizations of the well known class of interval graphs. It is known, and easy to show, that every interval graph is a probe graph, and that every probe graph is a tolerance graph. For all three classes, we can place additional restrictions of requiring that all intervals be of unit length or that no interval properly contains another. Clearly, a unit representation is also a proper representation, but not conversely. However, it is well known that unit tolerance graphs do not equal proper tolerance graphs, but that unit interval graphs do equal proper interval graphs. One of our results shows that unit probe graphs equal proper probe graphs.

In this Dagstuhl lecture, we present the complete hierarchy of all possible subclasses taken from <unit, proper, general> × <interval, probe, tolerance> and the class of bounded tolerance graphs, together with examples separating different classes. We survey those results which are known and prove those which are new.

Bichromatic P_4-Composition Schemes for Perfect Orderability

Ryan Hayward, University of Alberta, Edmonton, Alberta, Canada
Bill Lenhart, Thompson Computing Labs (TCL), Williams College, Williamstown, U.S.A.

A P_4 is an induced path with four vertices. A *bichromatic P_4 composition scheme* is as follows: (1) start with two graphs with vertex sets of different colour, say black ● and white ○ (2) select a set of allowable four-vertex bichromatic sequences, for example {●●●●, ○○○○, ●○○●, ○●●○} (3) add edges between the graphs so that in the composed graph each P_4 is coloured with an allowable sequence. Answering a question of Chvátal, we determine all such schemes which preserve perfect orderability. In the process, we characterize some graph classes which can be defined in terms of bichromatic P_4 structure.

Our main result is that the set of allowable bichromatic P_4's of any such scheme is a subset of one of the following sets (or their colour-exchange equivalents), where the symbol * indicates a sequence together with the reverse sequence.

1. {●●●●, ●●○●*, ●○●○*, ●○○●, ●○○○*, ○○○○}
2. {●●●●, ●○○●, ○●●○, ○○○○}
3. {●●●●, ●●○●*, ○●○○*, ○○○○}
4. {●●●●, ●●●○*, ●○○○*, ○○○○}
5. {●●●●, ●●●○*, ●○●○*, ●○○○*}.

On the divisibility of graphs

Kathie Cameron, Wilfrid Laurier University, Waterloo, Ontario, Canada
Chính T. Hoàng, Wilfrid Laurier University, Waterloo, Ontario, Canada

A k-division of a graph G is a partition of its vertex-set into sets $V_1, V_2, \ldots, V_k$ such that no V_i contains a largest clique of G. A graph is k-divisible if each of its induced subgraphs with at least one edge has a k-division. We prove the complements of interval-filament graphs are α-divisible. This is joint work with Kathie Cameron.

$\triangle$-free graphs, medians, retracts and fixed boxes

Wilfried Imrich, Montanuniversität Leoben, Leoben, Austria

This talk is concerned with tree-like graphs. It establishes connections to median graphs, retracts of products of graphs and fixed boxes, as well as to clique-graphs and Helly graphs.

Several classes of graphs are finally exhibited that have the same (or almost the same) recognition complexity as the class of triangle-free graphs. For example, cube-free median graphs have exactly the same recognition-complexity, whereas graphs of finite index have almost the same.

Finally it is asserted that C_4-free graphs can be recognized in $O(n^3)$ time; without using matrix multiplication algorithms.

$\log n$-Approximative NLC$_k$-Decomposition in $O(n^{2k+1})$ time

Öjvind Johansson, Royal Institute of Technology, Stockholm, Sweden

NLC$_k$ for $k = 1, \ldots$ is a family of algebras on vertex-labeled graphs introduced by Wanke. The operations are union and relabeling, and the former allows edges to be drawn between the two involved graphs. Once two vertices have been given the same label, they can no longer be differentiated by these operations. An NLC-decomposition of a graph is a derivation of this graph from single vertices using the above operations, and the width of the decomposition is the number of labels involved. Many difficult graph problems can be solved efficiently with dynamic programming if an NLC-decomposition of low-width is given for the input graph. This is also the case for the similar clique-decomposition introduced by Courcelle and Olariu.

In the presentation, I have described how an NLC-decomposition of width at most $\log n$ times the optimal width k can be found in $O(n^{2k+1})$ time. It follows that clique-decomposition of width at most $2 \log n$ times the optimal can be found in this time as well.

Cheap Fixed-Parameterized-Complexity Thrills

Hans Bodlaender, Utrecht University, Utrecht, The Netherlands
Maw-Shang Chang, National Chung Cheng University, Chia-Yi, Taiwan
Gregory Gutin, University of London, London, U.K.
C. M. Lee, National Chung Cheng University, Chia-Yi, Taiwan
Jim Liu, University of Lethbridge, Alberta, Canada
Ton Kloks, Vrije Universiteit Amsterdam, Amsterdam, The Netherlands

In my talk I give some new kernelization results for four problems. I report on joint work with Hans Bodlaender, Maw-Shang Chang, Gregory Gutin, C. M. Lee, and Jim

Liu. I start with the k-LEAF SPANNING TREE problem. I show that there exists an algorithm that runs in time $O(c^k + m + n)$ time to solve this problem for some constant c. A similar result was recently obtained by Fellows et al.

The other three results I show deal with planar graphs. The main tool used for obtaining these results is my recent proof that for planar graphs: $tw(G) = O(\sqrt{\gamma(G)})$. During my talk I show a kernelization for k-INDEPENDENT DOMINATION which allows an algorithm that runs in $O(c^{\sqrt{k}} + n)$ time.

Next I show that a planar graph with a feedback vertex set of at most k vertices has treewidth at most $\sqrt{k}$. Hence k-FEEDBACK VERTEX SET can be solved in time $O(c^{\sqrt{k}} + n)$. Here we use a dedicated treewidth algorithm that solves this problem for planar graphs.

Finally I mention a recent result concerning the k-VERTEX DISJOINT CYCLES problem. We showed that this problem can also be solved in $O(c^{\sqrt{k}} + n)$ time using similar techniques.

We have convinced ourselves of the correctness of the following conjecture: If a planar graph allows a maximum of k vertex disjoint cycles, then it has a feedback vertex set with at most Ck vertices for some constant C.

On Subfamilies of AT-free graphs

Derek Corneil, University of Toronto, Toronto, Canada
Ekkehard Köhler, Technische Universität Berlin, Berlin, Germany
Stephan Olariu, Old Dominion University, Norfolk VA, U.S.A.
Lorna Stewart, University of Alberta, Edmonton, Alberta, Canada

We introduce two subfamilies of AT-free graphs, namely, path orderable graphs and strong asteroid free graphs. Path orderable graphs are defined by a linear ordering of the vertices that is a natural generalization of the ordering that characterizes cocomparability graphs. On the other hand, motivation for the definition of strong asteroid free graphs comes from the fundamental work of Gallai on comparability graphs.

We show that cocomparability graphs $\subset$ path orderable graphs $\subset$ strong asteroid free graphs $\subset$ AT-free graphs. In addition, we settle the recognition question for the two new classes by proving that recognizing path orderable graphs is NP-complete whereas the recognition problem for strong asteroid free graphs can be solved in polynomial time.

On (P_5, gem)-free graphs

Andreas Brandstädt, Universität Rostock, Germany
Dieter Kratsch, LITA, Université de Metz, Metz, France

A P_5 is an induced path on five vertices and a gem is a graph consisting of a P_4 and a vertex adjacent to all vertices of the P_4.

Using modular decomposition we obtain the following structure theorem:
A connected and co-connected graph is (P_5, gem)-free if and only if

1. the homogeneous sets of G are P_4-free, and
2. the characteristic graph G^* of G is either
   (a) a specific graph (on at most nine vertices), or
   (b) a matched co-bipartite graph on at least six vertices, or
   (c) a co-chordal gem-free graph, or
   (d) a graph with C_5 fulfilling certain suppositions ("recursive class").

This theorem allows modular decomposition based efficient algorithms for recognition of (P_5, gem)-free graphs, maximum weight independent set on (P_5, gem)-free graphs, and some other optimization problems on (P_5, gem)-free graphs.

On the Structure of Bull-free Perfect Graphs

Celina de Figueiredo, Universidade Federal do Rio de Janeiro, Rio de Janeiro, Brazil
Frédéric Maffray, CNRS, Laboratoire Leibniz – IMAG, Grenoble, France
Oscar Porto, Pontifícia Universidade Católica do Rio de Janeiro, Rio de Janeiro, Brazil

In 1987, Chvátal and Sbihi studied bull-free Berge graphs (i.e. graphs that contain no bull[1], no odd hole and no odd antihole) and proved that they are perfect.

Later, Reed and Sbihi showed how to decide in polynomial time if a bull-free graph is Berge.

We prove that every prime bull-free Berge graph that contains a hole has a special structure called the Box Partition, which is like a "bipartite" layout of the graph. The Box Partition can then be exploited to find some objects that are useful for coloring optimally the graph:

- every box of size at least two contains an even pair of G or of $\overline{G}$;
- if G has no antihole, then every box of size at least two, different from a clique, contains an even pair of G;
- in fact if G has no antihole then G is transitively orientable.

Ryan Hayward eventually proved that every bull-free graph with no hole and no antihole is perfectly orderable. It follows that every bull-free Berge graph with no antihole is perfectly orderable; this was a conjecture of Chvátal (1989).

When G has a hole and an antihole, the Box Partition can still be exploited to find a homogeneous pair, which can be used in a decomposition scheme in order to find an optimal coloring for the graph G.

[1]bull: graph with vertices a,b,c, d,x and edges ab, bc, cd, xb, xc.

Approximation algorithms for graphs with small octupus

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Dieter Kratsch, LITA, Université de Metz, Metz, France
Haiko Müller, University of Leeds, Leeds, Great Britain

For a graph G a *d-octopus* $T = (W, F)$ is a subgraph of G such that

- T is the union of d shortest paths having one common endpoint
- W is a dominating set of G.

We consider some NP-complete graph problems that are hard to approximate in general when restricted to graphs with d-octopus and extract a structural property enabling the design of efficient constant-factor approximation algorithms assuming that the input graph is given together with d-octopus. The following table lists some problems and the worst case performance ratios of our linear-time approximation algorithms.

problem	ratio	
k-clustering	$3d$	for $k = 2$
	$2d$	for $k \geq 3$
bandwidth	$8d$	
topological bandwidth	$8d$	
λ-coloring	$4dp$	for $\lambda_{p,1}$
interval max degree	$8d$	
chordal max degree	$8d$	
domino pathwidth	$8d$	
domino treewidth	$8d$	

Moreover we obtain exact algorithms for the restriction of some domination problems to graphs with d-octopus. The running time of these algorithms is bounded by a polynomial of degree linear in d. Finally we study the complexity of finding and approximating a d-octopus of a graph.

On the use of graphs recognition techniques for distance labelling in graphs

Christophe Paul, Université Bordeaux I, LABRI, Talence, France

We consider the problem of labelling the nodes of a graph with short labels in such a way that the distance between any two nodes u, v can be approximated efficiently by nearly inspecting the labels of u and v (without using any other information).

We generalize the optimal distance labelling scheme proposed by Gavaille, Peleg et al. by using decomposition techniques (split decomposition, tree-decomposition).

Recognition of AT-free graphs

Jerry Spinrad, Vanderbilt University, Nashville TN, U.S.A.

This paper develops the first $o(n^3)$ algorithm for recognizing asteroidal triple-free graphs. The same techniques apply to a number of other problems, such as finding a clique separator in a graph.

Finding houses and holes in graphs

Chính T. Hoàng, Wilfrid Laurier University, Waterloo, Ontario, Canada
R. Sritharan, University of Dayton, Dayton, U.S.A.

A *house* is the complement of an induced path on five vertices. A *hole* is an induced cycle on five or more vertices. A *domino* is the cycle on six vertices with a long chord. A graph is HH-free if it does not contain a house or a hole. A graph is HHD-free if it does not contain a house, or a hole, or a domino.

We present $O(n^3)$ algorithms to recognize HH-free graphs and HHD-free graphs. The previous best algorithms for the problems run in $O(n^4)$ time.

On Clique Traversals and Clique Independent Sets

Jayme L. Szwarcfiter, Universidade Federal do Rio de Janeiro, Rio de Janeiro, Brazil
Santosh Vempala, MIT, Cambridge MA, U.S.A.

A clique traversal of a graph G is a subset of vertices intersecting all cliques of G. A clique independent set is a subset of disjoint cliques. Denote by $\tau_c(G)$ and $\alpha_c(G)$ the cardinalities of a minimum clique traversal and maximum clique independent set. A graph is clique-perfect when $\tau_c(H) = \alpha_c(H)$, for every induced subgraph H of G. Similary, G is c-clique-perfect when $\tau_c(H) = \alpha_c(H)$ for every clique-induced subgraph H of G. We characterize c-clique-perfect in terms of its clique graph. A relationship between c-clique-perfect graphs and perfect graphs is proved. Further, we prove that c-clique-perfect properly contain clique-perfect graphs, for chordal graphs. In general, the class of perfect graphs, clique-perfect graphs and c-clique-perfect graphs pairwise overlap.

Treewidth of planar graphs: connections with duality

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Frédéric Mazoit, LIP ENS-Lyon, Lyon Cedex 07, France
Ioan Todinca, LIFO, Universite d'Orleans, Orleans Cedex 2, France

We give a simple proof of the following result obtained by D. Lapoire: the treewidth of a planar graph and the treewidth of its dual differ by at most one unit.

We show that the minimal separators of a planar graph can be seen as Jordan curves in the plane. A minimal triangulation (chordal embedding) of a planar graph corresponds to a family of pairwise parallel Jordan curves. We introduce a notion of block region formed by these curves and show how to associate maximal cliques of the triangulation to minimal block regions.

Eventually we construct a larger family of curves which yields a triangulation of the dual graph, and whose clique size is at most the clique size of the initial triangulation plus one.

On a Cograph's recognition algorithm, possible generalizations

Vassilios Giakoumakis, Université de Picardie Jules Verne, France
Vadim V. Lozin, RUTCOR, Rutgers Center for Operations Research, Piscataway, New Jersey, U.S.A.
Jean-Marie Vanherpe, Université de Picardie Jules Verne, France

In 1985, Corneil, Perl and Stewart proposed an incremental approach for Cograph's recognition in linear time.

In the main step of this algorithm, necessary conditions are checked and a new decomposition tree is provided.

In this talk, we show that the necessary conditions can be weaker and thus this method can also be applied for the recognition of other graph classes in linear time.

We also give two examples of such recognition concerning classes of graphs which can be decomposed using a method devoted to bipartite graphs, namely the canonical decomposition.

Deciding clique-width for graphs of bounded tree-width

Egon Wanke, Universität Düsseldorf, Düsseldorf, Germany

We show that there exists a linear time algorithm for deciding whether a graph of bounded tree-width has clique-width k for some fixed integer k. Since every graph of tree-width r has clique-width at most $3 \cdot 2^{r-1}$, there is also a linear time algorithm for computing the clique-width of a graph of bounded tree-width by testing "clique-width at most k" for $k = 1, \ldots, 3 \cdot 2^{r-1}$. It remains open whether the clique-width k property is expressible in MSO_2-logic and whether "clique-width at most k" is decidable in polynomial time for arbitrary graphs.

3 Open Problems

3.1 Characterization of a subclass of interval filament graphs (Elias Dahlhaus)

I mentioned the following subclass of interval filament graphs. Lines are allowed to cross once or to touch. Two lines touch if they do not cross and if their intersection is a subpath of both lines. The question is to characterize these graphs.

3.2 Local conditions for edge-colouring chordal graphs and cographs (Celina de Figueiredo)

We say that a graph G is *overfull* when the number n of vertices is odd and $\Delta\frac{n-1}{2} < m$, where m is the number of edges and Δ is the maximum vertex degree. A graph G is *subgraph-overfull* when it has an overfull subgraph H with $\Delta(H) = \Delta(G)$. If the overfull subgraph H can be chosen to be a *neighbourhood*, that is, induced by a Δ-vertex and all its neighbours, then we say that G is *neighbourhood-overfull*. Notice that overfull, subgraph-overfull and neighbourhood-overfull graphs are in C2, i.e., cannot be edge-coloured with Δ colors. Our goal is to find classes of graphs for which the C2 graphs are precisely the overfull graphs [1, 2]. We have the following conjecture about edge-colouring chordal graphs:

Conjecture. Every C2 chordal graph is neighborhood-overfull.

The validity of this conjecture implies that the edge-colouring of chordal graphs can be solved in polynomial time. We have established necessary conditions for the validity of this conjecture: Firstly, we need all chordal graphs that are subgraph-overfull to be also neighbourhood-overfull. We have established this fact for two subclasses of chordal graphs split graphs and proper interval graphs.

Secondly, we need all odd maximum degree chordal graphs to be C1. We have established this fact for proper interval graphs, for interval graphs and for dually chordal graphs.

The edge-colouring problem is NP-hard for comparability graphs. For complete multipartite graphs, it is known that that every C2 graph is overfull. We propose to study the overfull condition for the edge-colouring of a superclass of multipartite graphs, the class of cographs, graphs with no induced path on 4 vertices.

References

[1] C. M. H. de Figueiredo, J. Meidanis, and C. P. de Mello. Local conditions for edge-coloring. *J Combinatorial Mathematics and Combinatorial Computing* **32** (2000), 79–91.

[2] M. M. Barbosa, C. P. de Mello, and J. Meidanis. Local conditions for edge-colouring of cographs. *Congressus Numerantium* **133** (1998), 45–55.

3.3 Gallai numbers and Tree-width (Feodor Dragan)

A *tree-decomposition* of a graph $G = (V, E)$ is a pair (T, W), where T is a tree and $W = (W_t : t \in V(T))$ is a family of subsets of $V(G)$, satisfying

(P1) $\bigcup_{t \in V(T)} W_t = V(G)$, and every edge of G has both ends in some W_t, and

(P2) if $t, t', t'' \in V(T)$ and t' lies on the path from t to t'' then $W_t \bigcap W_{t''} \subseteq W_{t'}$.

The *width* of a tree-decomposition is $\max(|W_t| - 1 : t \in V(T))$, and *tree-width* $tw(G)$ of G is the minimum width of a tree-decomposition of G.

Many optimization problems on graphs with constant bounded tree-width can be solved in linear time.

There are several equivalent characterizations of the notion tree-width. The (probably) most well known equivalent characterization is by the notion "partial k-tree". Also, Seymour and Thomas have proved in [1] that if a graph G has tree-width $k - 1$, then it can be monotonely searched by $< k + 1$ cops, and cannot be searched at all by $< k$ cops.

Here we are interested in the following characterization of tree-width obtained by Seymour and Thomas in the same paper [1]. The tree-width of a graph G equals the maximum k such that there is a collection of connected subsets of $V(G)$, pairwise intersecting or adjacent, such that no set of $\leq k$ vertices meets all of them.

One can define the *Gallai number* $gn(G)$ of a graph G to be the minimum number k such that, for any collection of connected subsets of $V(G)$, (necessarily) pairwise intersecting, there can be found a set of $\leq k$ vertices which meets all subsets of the collection. (A similar notion is known in geometry/topology.)

Clearly, $gn(G) \leq tw(G) + 1$ for any graph G. Therefore, if tree-width of a graph G is bounded by a constant, then the Gallai number of G is bounded, too.

The following questions naturally arise from this definition and the relation to the notion of tree-width.

- Do there exist equivalent characterizations of the notion Gallai number similar to those of the notion tree-width?
- Which graph problems can be solved efficiently on graphs with constant bounded Gallai number? How?
- How large is the class of graphs with bounded Gallai number and unbounded tree-width?

References

[1] P.D. Seymour and R. Thomas, Graph Searching and a Min-Max Theorem for Tree-width, *J. Combin. Theory Ser. B* **58** (1993), 22–33.

3.4 Stable Cutsets (Van Bang Le)

In a graph $G = (V, E)$, a stable cutset is a a stable set $S \subset V$ such that $G - S$ is disconnected. Chvátal [J. Graph Theory 8 (1984) 51-53] has proved that deciding whether a given graph has a stable cutset is NP-complete.

Chen and Yu (1998) have proved that every graph with n vertices and $m \leq 2n - 4$ edges has a stable cutset. Le and Randerath (2001) have proved that deciding whether a graph with n vertices and $m \leq \frac{5}{2}n$ edges has a stable cutset is NP-complete.

Problem. Find the maximum number $f(n)$ such that deciding whether a graph with n vertices and $m \leq f(n)$ edges has a stable cutset can be done in polynomial time.

3.5 Strong Perfect Graph Conjecture on Dominating Pair Graphs (Dieter Kratsch)

Corneil, Olariu and Stewart call a pair of vertices (x, y) a dominating pair if every x, y-path is dominating.

A graph $G = (V, E)$ is said to be a dominating pair graph if every connected induced subgraph of G has a dominating pair.

Question. Is the Strong Perfect Graph Conjecture true for dominating pair graphs?

3.6 Recognition and robust optimization algorithms for interval filament graphs (Jerry Spinrad)

Interval filament graphs are a new class of intersection graphs introduced by Gavril. Vertics correspond to "filaments" on the real line, where a filament is a curve which starts at a left endpoint, end at the right endpoint, and is a continuous curve in the region above the line and between these two endpoints. He shows that some optimization problems can be solved on interval filament graphs, if the filament model is given as part of the input.

Question 1. Can interval filament graphs be recognized in polynomial time?

Question 2. Can you design robust optimization algorithms for interval filament graphs, where a robust algorithm takes an arbitrary graph as input, and either answers the problem correctly on the graph or answers that the graph is not in the class?

Relevant references are F. Gavril, Maximum weight independent sets and cliques in intersection graphs of filaments, IPL 73 (2000) 181-188 for interval filament graphs and V. Raghavan and J. Spinrad, Robust Algorithms for Restricted Domains, SODA 2001 for robust algorithms.

3.7 Convergence of the iterated clique graph; Helly circle graphs (Jayme L. Szwarcfiter)

1. The clique graph $K(G)$ of a graph G is the intersection graph of the maximal cliques of G. The iterated clique graph of G is the one denoted by $K^i(G)$ and defined as

$$K^0(G) = G \qquad \text{and} \qquad K^i(G) = K(K^{i-1}(G)).$$

   Say that G is *divergent* when the limit of the order of $K^i(G)$ is infinite when i tends to infinite.

   Say that G is *convergent* when $K^i(G)$ is the one vertex graph, for some finite i.

   Question. Given G, is G divergent? Is G convergent? Is any of these problems decidable?

2. A circle graph G is the intersection graph of the chords of a circle. Say that G is *Helly* when there exists a circle model for G, whose chords satisfy the Helly property.

   Question. Is it true, that G is Helly if and only if G is a circle graph having no diamonds as induced subgraphs?

3.8 Conjecture on Bicliques (Ian Todinca)

Consider a graph G and two disjoin sets of vertices A and B. We say that (X, Y) is a *biclique relative to* (A, B) if $X \subseteq A$, $Y \subseteq B$ and each vertex of X is adjacent to each vertex of Y. Let us denote by $bc(A, B)$ the minimum number of edge-disjoint bicliques relatives to (A, B), whose union covers all the edges between A and B.

Conjecture. Let (A_1, B_1, A_2, B_2) be a partition of the vertices of G. Then

$$bc(A_1, B_1) + bc(A_2, B_2) \leq bc(A_1 \cup A_2, B_1 \cup B_2) + bc(A_1 \cup B_1, A_2 \cup B_2)$$

If the conjecture is true, we can prove that a certain greedy algorithm would give an approximation for the clique-width problem.