Report of the Dagstuhl seminar on

Geometric Modelling

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organized by

Guido Brunnet, TU Chemnitz, Germany
Gerald Farin, Arizona State University, USA
Ron Goldman, Rice University Houston, USA
Stefanie Hahmann, LMC-IMAG Grenoble, France

Geometric Modelling is the branch of Computer Science concerned with the efficient representation, manipulation, and analysis of geometry on a computer. The origin of this discipline is curve and surface design for CAD/CAM systems. Today, Geometric Modelling is a well established field with a wide range of applications, including computer graphics, scientific visualization, virtual reality, simulation, and medical imaging, and it attracts researchers with backgrounds in computer science as well as mathematics and engineering.

The 5th Dagstuhl seminar on geometric modelling was attended by 51 participants. The participants came from 3 continents and 13 countries, and included 6 industrial scientists as well as the leading academic experts in the field. Several young invited researchers were funded by the HLSC program of the European community. A very special event during the conference was the award ceremony for the John Gregory Memorial award. This time Prof. Hans Hagen, Prof. Gerald Farin, Prof. Joseph Hoschek, and Prof. Tom Lyche have been awarded with this price for their fundamental contributions to the field of geometric modelling. After the conference, as with all previous Dagstuhl Seminars on Geometric Modelling, a conference proceedings will be published.

The were a total 42 technical presentations at the conference related to the following diverse topics:

- curve and surface modelling
- non-manifold modelling in CAD
- multiresolution analysis of complex geometric models
- surface reconstruction
- variational Design
- computational geometry of curves and surfaces
- 3D meshing
- geometric modelling for scientific visualization
- geometric models for Biomedical application

Despite the large number of presentations during the conference and the high attendance at these talks, there was ample time for scientific discussions and research.

The organizers would like to thank all the attendees for their participation. They would also especially like to thank the team of Schloss Dagstuhl for helping to make this workshop a success. As always, we enjoyed the warm atmosphere of the Schloss, which supports formal discussions along with informal exchanges of ideas.

Contents

Surface approximation using regular triangulation

D. Ayala and N. Pla and M. Vigo
Universitat Politècnica de Catalunya, Barcelona

A method that approximates a surface by a triangular mesh has been presented. It uses a given tolerance and is adaptive to the surface curvature. The input are faces of a solid which are trimmed parametric patches. The corresponding algorithm first computes bounds for each parametric point and then performs an incremental Delaunay triangulation. Bounds on the curvature (a function, R) are used to determine if an edge is admissible and, if not, a point is inserted and the re-triangulation is performed. In the present work, we improve this algorithm by using these bounds also in the refining edges process and in the re-triangulation. Now, we use regular triangulation, a generalization of Delaunay triangulation that maintain both their

relationship with convex hulls and with Voronoi diagrams and that allows to associate a weight to each point. In our case a function of $\mathbb{R}$ is used as the weight for each point. The experimental results we have obtained give triangular meshes with less points when regular triangulation is used and a theoretical bound for this number of points is outlined.

Mesh Processing via Diffusion of Normals

Alexander Belyaev

Max-Planck-Institut für Informatik, Saarbrücken

An iterative nonlinear mesh filtering method is introduced. Each iteration of the method consists of the following two steps:

1. weighted nonlinear averaging (diffusion) is applied to the mesh normals;
2. the current positions of the mesh vertices are updated in order to fit the mesh to the field of modified normals.

The method is controlled by a user-specified parameter and, if the value of the parameter is properly chosen, suppresses small-scale oscillations while preserving and enhancing salient mesh creases. The method can be used for robust detection of salient shape creases and natural shape segmentation.

Spline Curve Approximation and Design by Optimal Control over Knots

Michel Bercovier and Rony Goldenthal
School of Computer Science and Engineering, Hebrew University of Jerusalem

Given a parametrization vector S and a knot vector T, the resulting least square approximation (resp. interpolation) of the corresponding B-Spline over the knots T can be defined as a "state" of a control problem, where S or T can be considered as the control variables (Cf. Alhanaty-Bercovier, CAD, 33 : 167-182 , 2001). Cost functions can be minimal length , minimal bending energy etc.... Thus the design process automatically computes the resulting control vector . In this talk we introduce the analysis of T as a control vector, and extend it to the combine choice of S and T in an alternating process. The knots can be moved during the minimization in such a way that there may be no

parameter belonging to S between a pair of knots, or knots could coalesce also. The first case results in a singular least square approximation problem that is solved by using SVD decomposition at each stage and including the null space basis functions in the control stage. The resulting algorithm is then stable. Numerous numerical examples are given to illustrate the method. The high non linearity due to the choice of the knot vector as the control variable may lead to some to undesirable local minima, unless some bound on how the knots can change is introduced.

Multiresolution Analysis for Subdivision Surfaces and Volumes

Martin Bertram
Universität Kaisreslautern

Multiresolution modeling has emerged from the need of representing and exploring large-scale scientific data sets, such as numerical simulations of turbulent hydrodynamics, terrain models, computer tomography, and highly detailed CAD models. View-dependent visualization and progressive transmission methods for scientific data sets rely on multiresolution data representations providing efficient access to local geometry at demand-driven levels of detail. The ability to access data at multiple levels of resolution makes it possible to explore large data sets using small hardware.

Biorthogonal wavelet transforms provide an ideal multiresolution framework for compression, progressive transmission and visualization of scientific data. We construct wavelets with local support defined on subdivision surfaces and volumes, generalizing hierarchical B-spline representations to surfaces and volumes of arbitrary topology. Combined with adaptive meshing algorithms for the parametrization of manifold geometries, our wavelet scheme provides a highly efficient tool for the computation, compression, progressive transmission and rendering of highly detailed geometries and of functions defined thereon.

Non-Manifold Geometric Modeling

Robert M. Blomgren
SMLIB, Bellevue

A non-manifold boundary representation was developed by Kevin Weiler in 1986. Our work has used this radial edge structure to generalize the Boolean operator into a Merge operator. One of the primary benefits of the Merge operator is the ability to combine surfaces into a well defined brep object. This enables all the intermediate steps in various high level constructions to be well defined as non-manifold objects.

Five important modeling applications that benefit from or require the Merge operator are discussed; they are: 1. An extended Boolean operator. 2. A non-manifold sweep operation. 3. A reverse engineering utility. 4. Intermediate fillets and blends. 5. The shell offset process.

Geometric Constraints for Conceptual Design

Beat Bruderlin
Technical University of Ilmenau

The design decisions in the early phases of product design have the highest impact on the product performance and production cost. Despite the importance of the early design phase, there are still very few software tools available for conceptual design.

We give examples of mechanism design, where the functionality is often defined at a high level of abstraction, using "solution principles." These can be represented by simple geometry and geometric constraints. With the appropriate user interface and a geometric constraint solver, users can put together new mechanisms interactively, and analyze the performance, and make informed design decisions, early on.

In the next design phase (embodiment design), three dimensional shape is associated with the solution principles. The constraints defined in the previous phase are used to define the shape parameters of predefined parts. New shapes can be defined by hand sketches and dimensioned, by using geometric constraints.

New methods for generating and retouching free form curves surfaces are shown. An efficient approach for solving constraint problems for B-splines was developed for this purpose.

Discrete Multiresolution Models, Tiles and Points

C. Andujar, P. Brunet, J. Esteve, E. Monclus, I. Navazo, A. Vinacua
Polytechnical University of Catalonia, Barcelona

We present a new discrete representation scheme for complex geometric models based on tiles and oriented points. The goal is to obtain a minimum number of large, planar polygonal faces (tiles) with bounded approximation error. To avoid small faces in curved regions, oriented points are used to represent the object surface in these curves zones. The approach is based on splitting the voxels of a discrete band into five tetrahedra with integer weights at their vertices. The discrete band is a face-connected set of voxels that contains the surface of the initial object. The surface extraction algorithm is robust and efficient due to the extensive use of integer arithmetics. The key idea is the use of a large set of integer represented candidate planes. Tiles are generated by a region growing algorithm that works by iterating the insertion of new neighbour relevant tetrahedra while updating the convex set of feasible discrete planes. The algorithm recovers information at sub-voxel precission, and exact edges are detected and reconstructed.

Reconstruction from Contours - Boissonnat's method revisited -

Guido Brunnett
Technical University Chemnitz

In this talk we report on a reconstruction project that is conducted in collaboration with the Institute of Anatomy in Gttingen. In the data acquisition phase the 'Huge Image system' (developed by the University of Gttingen and Fa. Zeiss) is used to integrate series of high resolution photos of each slice of a certain species into one image. This image is then loaded to a CAD system where contours are created and medical phenomena are marked. The graphics group in Chemnitz is concerned with the automated contour creation and the 3D reconstruction of the object from the slice data. In this context our goal is to evaluate the most promising reconstruction methods, to create reference implementations and to provide sophisticated tools for user interactions in situations where automatic methods fail.

Especially, we report on our experiences with Boissonnat's method for reconstruction from slice data. Our contribution to this approach is threefold. First, we show that the list of criteria for tetrahedra removal has to be extended in order to avoid situations like boundary fold-overs that have not been mentioned in the literature. Second, we point out the necessity of creating internal contour edges in a novel manner. As a consequence of this approach we have to modify the usual triangle split in order to ensure that the interior and exterior Voronoi skeletons do not cross contour edges. Third, we present a criterion that has been successfully used to decide whether an internal contour has to be included or not. At the end of the talk a live presentation of our reconstruction system is given.

Fairness Criteria for Algebraic Curves

Pavel Chalmoviansky
University of Linz

Constructions of algebraic curves, such as algorithms for scattered data fitting, often produce algebraic curves of higher degree. Using suitable techniques for bivariate Hermite interpolation, these curves can be approximated by algebraic spline curves of lower degree. This process, however, often produces wiggles and oscillations, and fairing techniques are therefore needed. Two approaches to fairing are considered. First, the defining function of the algebraic curve is faired via minimization of second order discontinuities, subject to certain boundary conditions. This approach does not produce an optimal curvature distribution. The second method is based on truly geometric fairness criteria, such as the elastic bending energy, and takes feasibility criteria for the algebraic curve segment into account. Fairness Criteria for Algebraic Curves

Improving the computation of the medial axis for smooth curves

W. L. F. Degen
University of Stuttgart

Let $\Omega \subset \mathbb{R}^2$ be a compact (multi-)connected domain with $\partial\Omega$ being of class C^3. By the decomposition of Choi et al. the computation of the medial axis transform ($MAT(\Omega)$) can be reduced to the parts, having singular disks at the ends and only regular disks between them.

The known bisector algorithms for the pair of boundary curves $\mathcal{C}_1, \mathcal{C}_2 \subset \partial\Omega$ of that part do not make other use of these curves than considering their points and tangents. In contrast, we take the curvature circles C_1 and C_2 at the two contact points $p \in \mathcal{C}_1$ and $q \in \mathcal{C}_2$ of a certain regular disk D_s into account. We think of a discretisation with step control, where the "next" disk D_{s+h} has to be computed.

This configuration leads to an interesting connection with Dupin cyclides: It is the intersection with one of its symmetry planes (together with the family of enveloping spheres). So the analogous problem has to be solved for “Dupin circle families”; but this can easily derived from the theory of Dupin cyclides. Projecting the solution back to the curves $\mathcal{C}_1$ and $\mathcal{C}_2$ yields an excellent estimate (a “predictor”) for the latter case. A refinement by one of the corrector algorithms (“dracon”- or “pnecil”-algorithm) that were also presented, yields a fast, stable and high-accuracy predictor/corrector algorithm (similar to those used in the numerical annalysis of ODE's).

The examples confirm the high accuracy of the predictor part, which reduces the number of corrector iterations to very few steps (3 .. 5 mostly).

The Kernel of Freeform Planar Parametric Curves

Gershon Elber
CS Technion, Haifa

Freeform curves are employed in virtually all contemporary geometric modeling environment. Yet, little has been published on a whole variety of geometric properties of planar freeform shapes such as the kernel of the curve, its diameter, etc.

This paper extends the knowledge of computing these properties for piecewise linear, polygonal, planar shapes, and offers algorithms to compute the kernel of a planar curve.

The propose approach suggests the reduction of the problem in hand into a different polynomial problem whose zero set or the projection of the zero set aid in finding the solution of the problem.

Barycentric coordinates: Wachspress vs Sibson

Gerald Farin
Arizona State University, Tempe

We discuss two kinds of barycentric coordinates. One is based on a method by E. Wachspress and finds the coordinates of a point inside a convex polygon by multiplying together appropriate triagle areas of triangles generated by the point which is inserted into the polygon. The other method is generated by a method due R. Sibson, using the Voronoi diagram of the polygon and the inserted point. We compare both methods and conclude that Sibson's is more stable when the points are not evenly distributed on a circle.

Numerical differentiation and finite difference schemes on non-uniform grids

Michael S. Floater
SINTEF, Oslo

New, optimal error bounds for numerical differentiation based on Lagrange interpolation over arbitrarily spaced points are derived. These results are used to generalize the finite difference method for two-point boundary-value problems to non-uniform grids without loss of accuracy.

Musings on multisided Bezier patches

Ron Goldman
Rice University, Houston

Standard Bezier patches come in two distinct forms: rectangular and triangular. These polynomial surfaces are particularly important in geometric design because they have convenient geometric properties and they possess powerful analysis algorithms. Bezier patches are affine invariant, nondegenerate, lie in the convex hull of their control points, and their boundaries are the Bezier curves determined by their boundary control points. Standard algorithms include evaluation, differentiation, subdivision, and blossoming procedures based on a bivariate version of the de Casteljau algorithm.

Multisided Bezier patches – Bezier patches with more than four sides – are less well known, but they too have many potential applications in geometric design including filling n-sided holes. Currently there are several different schemas for generating multisided Bezier patches, including S-patches, toric patches, T-patches, and Hosaka-Kimura patches. In contrast to the standard triangular and tensor product polynomial Bezier surfaces, multisided Bezier patches are typically rational patches. Nevertheless these multisided patches generally share the same properties and analysis algorithms as standard Bezier patches.

The key to constructing multisided Bezier schemes is to generalize the bivariate de Casteljau algorithm for three sided and four sided patches to convex polygons and multisided arrays of control points. To do so requires both barycentric coordinate functions for convex polygons as well as some notion of a multisided array of control points. Barycentric coordinates on convex polygons are well known in computational geometry. In this talk we concentrate on how to construct multisided arrays of control points that can be used to generate multisided Bezier patches. Gnomons, tessellations, spider webs, fractal gaskets, and Minkowski sums are investigated. Some of these constructs, such as Minkowski sums, encompass known schemes such as S-patches, but from a new perspective; others such as fractal gaskets lead to entirely new schemes. We speculate here on the potential utility of these new constructions.

Efficient Coding of 3D Mesh Data

Craig Gotsman
Technion, Haifa

3D graphic data is an important component in synthetic image and animation generation, in CAD/CAM and in computer games, on and off the Web. Hence efficient methods to code, progressively transmit and render this type of data have increasing importance. This talk will survey methods developed by the speaker and others over the past few years to code and render 3D graphic data, specifically the connectivity and geometry components of 3D meshes.

A parametric quartic spline interpolant to position, tangent and curvature

Thomas A. Grandine and Thomas A. Hogan
Boeing, Seattle

In their paper "High accuracy geometric Hermite interpolation," de Boor, Hollig and Sabin describe a parametric cubic spline interpolation scheme that matches prescribed position, tangent and curvature data at each knot. Among the more important merits of their resulting interpolant are that it is twice continuously differentiable with respect to arclength, can be constructed locally, preserves convexity and is 6 th -order accurate. But there are two attributes of the scheme that we found problematic: the first is that the interpolant does not depend continuously on the prescribed data, and the second is the abstruse nature of the admissibility conditions on the data.

This talk describes a scheme for interpolating given tangent vectors and curvature values at distinct positions by a twice continuously differentiable parametric quartic spline. The restrictions placed on the data by the [BHS] scheme are simplified by using quartic segments instead of a cubic. This provides much greater freedom than is necessary and the additional degrees of freedom are exploited to make the scheme especially robust for circular arcs. What's more, the interpolant depends continuously on the data and, under appropriate assumptions, preserves convexity and is 6 th -order accurate.

Area Preserving Deformation of Multiresolution Curves

Stefanie Hahmann, Georges-Pierre Bonneau*, Basile Sauvage
LMC-IMAG, Institut National Polytechnique de Grenoble
* iMAGIS-GRAVIR, Université Joseph Fourier, Grenoble

We describe a method for deformation of periodic planar multiresolution curves that keeps the enclosed area constant. We use a wavelet based multiresolution representation of the curves which are represented by a finite number of control points at each level of resolution. A deformation can then be applied to the curve by modifying one or more control points at any level of resolution. This process is generally known as multiresolution editing to which we add the constraint of constant area. A multiresolution representation for the area moment is also developed. We make sure that all computations are fast and that the deformations can be performed interactively. Diverse types of deformation are discussed.

The space of n-gons in the analysis of subdivision

Ioannis Ivrissimtzis and Hans-Peter Seidel
Max-Planck-Institut für Informatik, Saarbrücken

We use the theory of n-gons in the analysis of the simplest scheme and the sqrt(3) subdivision scheme. The faces of the simplest scheme, or the star of the vertices in the case of the sqrt(3) scheme, are projected on the Euclidean plane and represented as vectors with n complex components. At each subdivision step these vectors are multiplied by a circulant matrix. We study these transformations by writing the vectors in the base formed by the universal eigenvectors of the circulant matrices. The result is that in the most cases the limit shape of these polygons is the affine image of a regular polygon.

Also, we show that there is one more potential source of degeneracy for the sqrt(3) scheme, namely, the updated position of a vertex may lie outside its star. These degeneracies are resolved after few subdivision steps.

Bitangency

John K. Johnstone
University of Alabama, Birmingham

Bitangency is central to many geometric problems that involve the interrelationship between many curves or many surfaces, such as visibility analysis, lighting, convex hulls and shortest path motion. We discuss dual representations for the tangent space of a curve and surface, called the tangential curve and the tangential surface, that lend themselves particularly well to the computation of bitangency.

Once the bitangent structure of a scene has been built (bitangent lines for a 2D scene, bitangent developables for a 3D scene), we can in turn use it to build the secondary structures that are needed for visibility, lighting and motion. We illustrate this refinement process through the construction of the convex hull of a smooth curve, using a giftwrapping technique that can be interpreted as a smooth version of the Jarvis march algorithm for the convex hull of a finite set of points.

Bounds on the Distance between Planar Parametric Bézier Curves and their Control Polygon

M.I. Karavelas, P.D. Kaklis and K.V. Kostas
National TU Athens

Employing the approach introduced in Nairn at al. (1999), we first derive sharp bounds on the distance between a planar parametric Bézier curve $\mathbf{b}(t) = \sum_{i=0}^{d} \mathbf{b}_i B_i^d(t)$, $t \in [0,1]$, of degree d, and its control polygon.

The distance is measured by the maximum value of the $p-$norm ($p = 1, \infty$) of the vector $\mathbf{b}(t) - \mathbf{l}(t)$, where $\mathbf{l}(t)$, is a linear-spline parameterization of the control polygon of $\mathbf{b}(t)$, with knot vector that induced by the Greville abscissas. Polygonal envelopes of $\mathbf{b}(t)$, of complexity $O(d)$, can then be constructed by calculating the Minkowski sum of the control polygon and the unit $p-$sphere, scaled appropriately by the aforementioned bounds.

Since, however, these unit p-spheres are axis aligned, we present and numerically illustrate algorithms for finding the orientation angles for which these bounds become minimal. These algorithms run currently at a cost $d^2\alpha(d)$, where $\alpha(.)$ denotes the functional inverse of Ackerman's function.

REFERENCES: D. Nairn, J. Peters and D. Lutterkort, Sharp Quantitative Bounds on the Distance between a Polynomial Piece and its Bézier Control Polygon, CAGD, 16, no. 7 (1999) pp. 613-631.

Convex Hull Computation for Freeform Space Curves and Surfaces

Myung-Soo Kim, Joon-Kyung Seong, and Gershon Elber*
School of Comp Science and Eng., Seoul National University
* Dept. of Computer Science, Technion, Haifa

We present an algorithm that computes the convex hulls of freeform space curves and surfaces. The problem is reformulated as one of finding the zero-sets of polynomial equations in multi-variables; using these zero-sets we characterize developable surface patches that belong to the boundaries of the convex hulls.

Feature sensitive mesh processing

Leif Kobbelt
RWTH Aachen

Many mesh processing algorithms assume the actual geometry of a triangle mesh to be characterized by the vertex positions only. ¿From the manifold point of view however, triangle meshes have to be considered as continuous piecewise linear surfaces.

In sufficiently smooth and flat regions of the surface this observation doesn't really matter since any triangulation will yield a decent approximation to the underlying geometry. In the presence of sharply curved features however, this is not true. Here, severe alias artifacts can affect the perceived surface quality and can lead to quite bad approximation behavior.

In my talk I will discuss several consequences of this observation and present recently developed algorithms for feature sensitive mesh generation and re-meshing techniques. I will report recent results in feature sensitive surface extraction from volume data, surface anti-aliasing by remeshing of blend regions in technical data sets, and diffusion based remeshing of triangle meshes.

Medial Axis Homotopy

Andre Lieutier
Dassault Systemes Aix-en-Provence and LMC-IMAG Grenoble

Medial Axis Transform is sometimes used as an intermediate representation in algorithms for meshing or recognition of shapes from digitized data. This raises the question whether the Medial Axis captures fundamental topological invariants of the object. The (positive) answer has been known already in the case of smooth objects. The result presented here is the homotopy equivalence of any bounded open subset of $\mathbb{R}^n$ with its Medial Axis. We explain why the deformation retract technique can not be applied in the general sitiuation (for npon C^2 boundaries in the plane or even non convex polyhedra in 3D. Some key ideas of the proof are introduced including the non continuous vector field ∇ extending the gradient of the function distance to the boundary.

Wavelet-based Multiresolution with "nth-root-of-2" Subdivision

Lars Linsen, Valerio Pascucci*, Mark A. Duchaineau*, Bernd Hamann, and Kenneth I. Joy
Center for Image Processing and Integrated Computing (CIPIC),
Department of Computer Science, University of California, Davis

* Center for Applied Science (CASC), Data Science Group,
Lawrence Livermore National Laboratory

Multiresolution methods for representing data at multiple levels of detail are widely used for bivariate functions defined over large-scale two-dimensional meshes. We present a hierarchical data representation for tri- and quadrivariate functions defined over three- and four-dimensional grids. The hierarchies are used for dealing with large-scale volume data and time-varying volume data. The four-dimensional approach, where time is treated as the fourth dimension, supports a hierarchy with spatial and temporal scalability.

The multiresolution data organization is based on "nth-root-of-2" subdivision. Applied to n-dimensional data, the "nth-root-of-2"-subdivision scheme only doubles the overall number of grid points in each subdivision step. This fact leads to fine granularity and high adaptivity. For high-quality data approximation on each level of detail, we use linear B-spline wavelets. We present a linear B-spline wavelet lifting scheme based on "nth-root-of-2" subdivision to obtain narrow masks for the update rules. Narrow masks provide a basis for out-of-core data exploration techniques and view-dependent visualization.

A Simple Derivation of the General Recurrence Relation for Calculating with B-splines

Tom Lyche and Kyrre Strøm*
University of Oslo
* SimSurgery, Oslo

For calculating with B-splines there are algorithms for evaluation, knot insertion, knot removal, degree raising, and degree reduction. These algorithms can be derived from a general recurrence relation for representing the product of two splines as a linear combination of B-splines of higher degree on a suitable knot vector. We use the differentiation formula for multivariate cone splines to give a simple derivation of this recurrence relation. As an additional bonus we obtain a formulation which includes the differentiation formula and which is valid for a class of splines containing polynomial, trigonometric, and hyperbolic B-splines. Hence with one algorithm one can perform all the above mentioned tasks. The main advantage of these recurrence relations is their numerical stability when applied to evaluation, knot insertion, and degree raising.

Quasi-interpolants with tension properties from and in CAGD

Carla Manni
Universita di Torino

Recently, some quasi-interpolation schemes based on C^2 functions which generalize classical cubic B-splines have been proposed. These basis functions are constructed using the so-called "parametric approach" consisting in looking at the given function as a particular parametric (cubic) curve. Imposing geometric continuity of order two for such a curve allows to introduce parameters that act as "tension parameters". The tension parameters make it possible to construct quasi-interpolant having a distance from the given data less than any prescribed tolerance. Moreover, they allow to control the oscillations of the constructed approximant.

In this talk, first we discuss a geometric construction of the above mentioned basis functions, then we present and we analyze some new quasi-interpolants based on them.

The application of the obtained quasi-interpolants for approximation of parametric curves will be also described.

Control polygons as geometric filters

Géraldine Morin
Freie Universität Berlin

In this talk, we present how control polygons of curves can be used as geometric filters in exact computational geometry. A first level of filtering is provided by the convex hull property. Then, the curve can be approximated by its control polygon. To achieve filtering, a bound on the distance between the curve and its control polygon is necessary. We recall the upper bounds existing for polynomial curves and splines, and approximation bounds to the space $(1, x, cosx, sinx)$, and to the corresponding spline space. We also derive the improvement of the garantied approximation under subdivision, for these four different spaces.

Geometric Modeling on Discrete Displacement Fields

Heinrich Müller, Jörg Ayasse
University of Dortmund

A displacement field can be defined by a vector field and a height field on an orientable surface S, describing the direction and the amount of displacement to be applied to S in order to get a surface represented by the displacement field. S together with the vector field can also be considered as a representation of a volume by a "surface crust". The crust allows the application of sculpturing operations for designing detailed structure. For conventional height fields over the plane, sculpturing can be reduced to the task of finding the minimum or maximum of the height values of a surface and the height values stored. Over the plane, depth-buffering with parallel projection has been used for calculation. For displacement fields over curved surfaces the projection is more complicated. In this contribution, an efficient solution is presented for displacement fields over triangular meshes. The central task is to insert a triangle in space into the curved depth-buffer represented by the displacement field.

T-spline

Ahmad Nasri
Dept. of math. & Computer Science American University of Beirut

(in collaboration with Thomas W. Sederberg, Jinamin Zheng and Almaz Bakenov
at Brigham Young University)

Non Uniform B-spline surfaces can be generalized to a new type of surfaces called "T-spline". The control mesh of a T-spline can have T-junctions making the surface locally refinable: points can be inserted in the mesh without propagating an entire row and / or column of control points. This allows the merging of several non-uniform B-spline patches with different knot vectors. Cubic and quadratic T-splines are discussed with various applications such as rendering, producing sharp features (darts and creases), and devising subdivision schemes that can integrate non-uniform Catmull-Clark subdivision with cubic NURBS.

Marching through the Cubes

Gregory M. Nielson
Arizona State University, Mesa

The Marching Cubes (MC) algorithm is an effective and popular method for computing a triangular mesh surface approximation to the isosurface of 3D discrete data given on a rectilinear grid. The original method as presented by Lorensen & Kline (SIGGRAPH 1987) is flawed and can lead to surfaces with gaps and holes. Over the years there have been several "fixes" and enhancements proposed. Unfortunately, some of these proposed algorithms are also flawed. We give an historical account of the literature on this subject leading to a new three-level algorithm that is guaranteed to produce a triangular mesh surface with the same connectivity/separation of vertices as the piecewise trilinear interpolant. The first level is based upon linear interpolation on edges, the second level examines all cases arising from bilinear interpolation on faces and the third level utilizes trilinear interpolation on the interior of each voxel. Under the operation of rotation, the totality of all possible cases is organized into a total of 57 equivalence classes of unique configurations which allows for a simple, effective implementation of the three level algorithm.

Spline Concenpts in the Finite Element Method

Ulrich Reif
TU Darmstadt

The finite element method is a well established tool for approximating elliptic boundary value problems. Since the triangulation of complicated domains is rather involved, meshless methods are gaining more and more interest. Here, the major problem is the handling of essential boundary conditions.

We present a new approach to the problem which is based on **w**eighted **e**xtended **b**-splines (web-splines) as approximation subspace. The resulting method does not require any mesh generation process, admits to satisfy boundary conditions of Dirichlet or Neumann type, and provides high approximation order with relatively few coefficients. Further, the web-basis is stable in the sense that the condition number of the Galerkin matrix is of order $O(h^{-2})$ as the grid width h tends to zero. Due to this moderate growth, standard iterative solvers can be applied to solve the Galerkin system efficiently. But moreover, it can be shown that w-cycle multigrid leads to uniform convergence, i.e., a fixed number of iterations for achieving a given tolerance independent of the grid width h. The features of the web-methods are illustrated by a couple of examples.

Conformal Maps on the Horosphere

Alyn Rockwood
Colorado School of Mines

The horosphere is isotropic to Euclidean n-space, which is defined by embedding the Euclidean space into a space two dimensions higher. The embedding and subsequent calculations are based on a Clifford Algebra that contains signatures of 1 and 0, i.e. an algebra that admits null vectors. As homogeneous coordinates linearize affine maps by embedding them in a space of one dimension higher, so the horosphere linearizes the conformal group in a space of two dimensions higher. This leads to important implications in defining and computing with conformal maps.

Conformal maps are important in geometric design because many applications are simplified with them such as FE meshing, forward and inverse function integration steps, minimally distorted texture mapping and steady state, boundary value problems in 2D.

Geometric Modeling of Design Constraints in Complex Arrangements

Nickolas S. Sapidis, Gabriel Theodosiou*
University of the Aegean, Syros
* National Technical University of Athens

Complex products and systems, like an aircraft, ship, or machinery plant, involve a large number of components which are arranged under spatial constraints/relationships in design space. Current practices approximate these relationships with simplified "dimensional constraints" aiming at formulating the design problem as a system of (in)equalities to be solved automatically, e.g., by a geometric-constraint solver. This research proposes informationally-complete models for design-constraints based on an analysis of geometric and non-geometric properties of the related space volumes. Also, an Extended Product Model is proposed describing the system's structure and components as well as related procedures and constraints to be used as a system life-cycle model.

Trivariate Spline Interpolation

Larry L. Schumaker
Vanderbilt University, Nashville

In this talk we discuss a convenient space of trivariate splines which can be used for approximating trivariate functions. Such spaces have applications in interpolation, data fitting, volume matching, volume morphing, and elsewhere. The aim is to construct a space with global C^1 continuity, but using relatively low degree polynomials (in this case quintics). To achieve this, we consider a special class of partitions which we call type-4 paritions and which is obtained by starting with a collection of boxes and dividing each box into five subtetrahedra. The main results involve dimension formulae, local bases, the construction of a quasi-interpolation operator providing full approximation power, and techniques for performing both Hermite and Lagrange interpolation.

Approximating curves with C^2 splines

Timothy L. Strotman
EDS PLM Solutions, Milford

C^2 spline curve to a given tolerance. This method is local, producing the largest segment that can interpolate the position, tangent, and curvature vectors of the procedural curve at the segment ends and remain within tolerance. We achieve a C^2 join of the segments by constraining the tangential component of the second derivative at the joins. The fit is shown to be near optimal. Two approaches are presented. The first produces splines with better parametrizations, while the second achieves better approximation order.

Shape Parameterisation for Design Optimisation of Thin-Walled Structures

Hassan Ugail
Department of Applied Mathematics, University of Leeds

Parametric representation of shapes of thin-walled structures, especially those used in foood Packaging, is considered. Such objects are created in great numbers especially in the food packaging industry. In fact these objects are produced in such vast numbers each year that one important task in the design of these objects is the minimisation of the amount of plastic used, subject to some functional constraints.

An efficient technique for parameterising geometry, based on PDE surfaces, is utilised here. This enables us to create a wide variety of possible shapes of thin-walled strucutures in an intuitive and interactive environment. Once the geometry is parameterised efficiently an appropriate surface mesh can be created on which the necessary functional analysis can be performed which enable us to quantify the stregth of the structure. The shape parameters along with the results of the functional analysis are used within an automatic optimisation routine enabling to find a design which satisfies user requirements.

Thus, an efficient technique for design optimisation of thin-walled structures is described which would reduce the need to perform inefficient and expensive "trial and error" experimentation using physical prototypes.

An Automatic Segmentation of a Digitized 3D Object

Marek Vanco
TU Chemnitz

We present an approach for automatic object segmentation and surface recognition and fitting. Our method is based on k-nearest neighbors, which are the fundamental data structures for estimating important surface properties as normal vectors and principal curvatures. We divide the segmentation in two passes: the first pass (normal vectors based segmentation) helps to find sharp edges or highly curved areas and provides the first estimation of segmentation parameters for the next pass. The second pass (curvature based segmentation) is able to detect parts of the basic surface types (as planes, cones, spheres, ...), which are extended in the following postprocessing step, in order to facilitate stable and proper fitting of the object's surfaces.

Segmentation of smooth, multiple point regions

Tamas Varady
Computer and Automation Research Institute, Hungarian Academy of Sciences

One of the most crucial phase in the reverse engineering of geometric shapes is segmentation. Segmentation aims at separating a large point cloud into smaller regions in such a way, that the regions will become pre-images of the individual faces in the final boundary representation model. Each region is to be approximated by a single implicit or parametric surface.

While locating sharp edges within the point cloud is relatively easy, segmentation between smoothly connected regions is fairly difficult. Take the class of conventional engineering objects, which are bounded by planes, cylinders, cones, spheres, tori, translational and rotational surfaces. An automatic segmentation method called direct segmentation is introduced, which is based on a hierarchy of various filters. The point cloud is subdivided into so-called "stable" regions while "unstable" point ribbons are excluded. The filters are based on various indicator values characterizing planarity, dimensionality, local translational direction, best apex, local axis, etc.

Once the segmentation is complete special surface fitting methods, including constrained fitting of multiple surfaces need to be applied to assure smooth connection between adjacent surface elements. Segmentation of smooth, multiple point regions

Interference Analysis of Quadric Surfaces

Wenping Wang
University of Hong Kong

Given two quadric surfaces, their characteristic equation is studied to determine the topological type of the arrangement formed by the quadrics. The obtained results are applied to detecting collision between two moving ellipsoids. Also, a new method is presented for computing a separating plane of two disjoint ellipsoids.

Dual Contouring for Hermite Data

Joe Warren, Tao Ju, Frank Lossaso, Scott Schaefer
Rice University, Houston

This paper describes a new method for contouring a signed grid whose edges are tagged by Hermite data (i.e; exact intersection points and normals). This method avoids the need to explicitly identify and process "features" as required in previous Hermite contouring methods. Using a new, numerically stable representation for quadratic error functions, we develop an octree-based method for simplifying contours produced by this method. We next extend our contouring method to these simplified octrees. This new method imposes no constraints on the octree (such as being a restricted octree) and requires no “crack patching”. We conclude with a simple test for preserving the topology of the contour during simplification.

Characterizing and Moulding Shape

F.- E. Wolter
Lehrstuhl Graphische Datenverarbeitung, Universität Hannover

The author explains how stable umbilics, the spectra of the Laplace - and of the Laplace - Beltrami - operator and also how the Medial Axis can be used to classify the shape of surfaces and solids. The approach suggesting to distinguish shapes by using the Spectra of the Laplace - and of the Laplace Beltrami operator appears to be completely new in the areas of geometric modeling, CAD and Computer Graphics in general. In the second part of the talk it is shown that the medial axis transform provides geometric concepts that might be employed to develop intuitive user interfaces useful to mould shape via haptic man/machine interaction.

Applications of geometric computing to drug discovery and design

Hans Wolters
Signature Bioscience Inc., Hayward

In this talk I present an overview of the computer aided drug design process. In particular, ligand and structure based design approaches are discussed and applications of computational geometry and geometric modeling concepts are highlighted. Starting from purely 2D methods that view molecules as graphs I progress to 3D shape descriptors. Furthermore, different representations for molecular shape are introduced. These representations are then used to perform geometry processing such as rapid overlay calculations as well as molecular docking.