Approximately Counting Graph Homomorphisms and Retractions

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Abstract

A homomorphism from a graph G to a graph H is a function from the vertices of G to the vertices of H that preserves the edges of G in the sense that every edge of G is mapped to an edge of H. By changing the target graph H, we can capture interesting structures in G. For example, homomorphisms from G to a k-clique H correspond to the proper k-colourings of G. There has been a lot of algorithmic work on the problem of (approximately) counting homomorphisms. The goal is to figure out for which graphs H the problem of approximately counting homomorphisms to H is algorithmically feasible. This talk will survey what is known. Despite much work, there are still plenty of open problems. We will discuss the problem of approximately counting *list homomorphisms* (where the input specifies, for each vertex of G, the list of vertices of H to which it can be mapped). Because the lists add extra expressibility, it is easier to prove that counting homomorphisms to a particular graph H is intractable. In fact, we have a full trichotomy (joint work with Galanis and Jerrum, 2017). Here, the complexity of homomorphism-counting is related to certain hereditary graph classes. The trichotomy will be explained in the talk – no prior knowledge of the area will be assumed. In more recent work, with Focke and Živný, we have investigated the complexity of counting *retractions* to H – this problem falls between homomorphism-counting and list-homomorphism counting. Here we have only a partial classification, which applies to all square-free graphs H. So again, there are plenty of open problems.

2012 ACM Subject Classification Mathematics of computing → Graph theory

Keywords and phrases Graph homomorphisms, counting

Digital Object Identifier 10.4230/LIPIcs.FSTTCS.2021.3

Category Invited Talk

41st IARCS Annual Conference on Foundations of Software Technology and Theoretical Computer Science (FSTTCS 2021).
Editors: Mikołaj Bojańczyk and Chandra Chekuri; Article No. 3; pp. 3:1–3:1

Leibniz International Proceedings in Informatics
Schloss Dagstuhl – Leibniz-Zentrum für Informatik, Dagstuhl Publishing, Germany