

Sparsity in Algorithms, Combinatorics and Logic

Edited by

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Abstract

This report documents the program and the outcomes of Dagstuhl Seminar 21391 “Sparsity in Algorithms, Combinatorics and Logic”. The seminar took place in a hybrid format from September 26 – October 1, 2021 and brought together 61 researchers. This report includes a discussion of the motivation of the seminar, presentation of the overall organization, abstracts of talks, and a report from each of the working groups.

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1 Executive Summary

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Motivation

It was realized already in the early days of computer science that structures (networks, databases, etc.) that are sparse appear ubiquitously in applications. The sparsity of input can be used in a variety of ways, e.g. to design efficient algorithms. This motivates a theoretical study of the abilities and limitations of sparsity-based methods. However, a priori it is not clear how to even define sparsity formally. Multiple sparsity-oriented paradigms have been studied in the literature, e.g. bounded maximum or average degree models, topologically constrained classes of graphs or graphs with bounded width parameters. However, many of those paradigms suffer from being either too restrictive to model real-life applications, or too general to yield strong tractability results.



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In the late 2000's, Nešetřil and Ossona de Mendez proposed a new framework of uniform structural sparsity for classes of graphs that generalized existing definitions and initiated the development of a toolbox of sparsity-based methods for analyzing graphs. The central notions of their framework are bounded expansion and nowhere dense classes. It quickly turned out that the proposed notions can be used to build a mathematical theory of sparse graphs that offers a wealth of tools, leading to new techniques and powerful results. This theory has been extensively developed in the recent years.

It is particularly remarkable that the concepts of classes of bounded expansion and nowhere dense classes can be connected to fundamental ideas from multiple other fields of computer science, often in a surprising way, providing several complementary viewpoints on the subject. On one hand, foundations of the area are grounded in structural graph theory, which aims at describing structure in graphs through various decompositions and auxiliary parameters. On the other hand, nowhere denseness seems to delimit the border of algorithmic tractability of first-order logic, providing a link to finite model theory and its computational aspects. Finally, there is a fruitful transfer of ideas to and from the field of algorithm design: sparsity-based methods can be used to design new, efficient algorithms, especially in the paradigms of parameterized complexity, approximation algorithms, and distributed computing. Further, classic techniques for designing algorithms on sparse inputs inspire new combinatorial results on sparse graphs.

The aim of this Dagstuhl Seminar is to bring together researchers working on various aspects of sparsity in their own fields, in order to facilitate the exchange of ideas, methods and questions between different communities. So far, the synergy effect between graph theory, logic, and algorithm design has led to fundamental developments in the theory of sparse graph classes. Our goal is to inspire a new wave of developments by “stirring in the pot” of researchers working on different facets of sparsity. An important part of the seminar will be the discussion of the (still fledgling) area of real-life applications of sparsity-based methods, where theory and practice could meet.

Seminar organization

Due to the on-going COVID pandemic, the seminar was held in hybrid format. In total the seminar was attended by 61 participants around the world (from North America to Europe to Asia). 32 of the participants were on-site and 29 remote. To make the hybrid format a success and in particular allow all members to participate in talks and working groups, the following measures were taken.

1. To accommodate both on-site and remote participants, a mix of on-site and remote talks were scheduled. The talks were scheduled in the early to late afternoon (MEZ local time), allowing remote audience members from all parts of the world to attend.
2. Both on-site and remote talks were streamed via zoom. The zoom session was projected onto a whiteboard in the seminar room. The remote participants could see and hear the on-site whiteboard and slide presentations. They could interrupt and ask questions or ask questions in the chat, which were then read by the organizers. This turned out to be a quite successful setup in which all participants could discuss in real-time.
3. On the first day of the seminar we had a short introduction of all participants, one invited tutorial lecture, one contributed talk and the open problem session. In total, we had 5 tutorial lectures and 12 contributed talks spread over the week. The topics and speakers were chosen to create a joint understanding of the state of the art in the fields that were brought together in the seminar.

4. The remaining program put a strong emphasis on open time for ad-hoc discussions and working in groups. After the open problem session on Monday, several groups of on-site and remote participants were formed, who approached the posed problems.
5. A discord server was set up to coordinate further communication and to keep track of the progress of the working groups.
6. A social event was organized online on Tuesday evening.

Work on open problems

Following the open problem session on the first day of the workshop, spontaneous groups working on selected open problems emerged. These typically included a mix of on-site and online participants, working in either synchronous or asynchronous manner using the Discord platform as a mean of communication. Below we list a selection of directions that were pursued during the seminar.

Model-checking on interpretations of locally well-behaved structures. It is known that model-checking First Order logic (FO) can be done in fixed-parameter time on classes of graphs that are locally well-behaved, for instance have locally bounded treewidth. However, the question is whether this is still true if the input graph is “logically disguised”, or more precisely, has been additionally mapped through some FO transduction. The aim of this research group was to provide an affirmative answer by proving the following theorem: For every class of graphs $\mathcal{C}$ that is stable and can be transduced from a class of locally bounded cliquewidth, the model-checking problems for FO is fixed-parameter tractable on $\mathcal{C}$. This would generalize several known results on efficient FO model-checking on classes of dense graphs, e.g. map graphs or interpretations of classes of bounded degree, as well as provide multiple new results.

Transducing paths from classes of unbounded shrubdepth. The emerging logically-motivated structure theory for graphs uses First-Order transductions as the main notion of embedding. It is important to understand possible duality theorems for this notion, of the following form: If a class of graphs $\mathcal{C}$ does not admit a decomposition of some form, then $\mathcal{C}$ transduces a class of specific obstacles witnessing this conclusion. The aim of this research group was to prove the most basic conjecture following this pattern: If a class of graphs $\mathcal{C}$ has unbounded shrubdepth, then $\mathcal{C}$ transduces the class of all paths.

Treewidth vs pathwidth. It is known that every graph of pathwidth $\Omega(ab)$ has treewidth at least a or contains a binary tree of depth b as a minor. It is also known that every graph of treewidth $\Omega(abc)$ has treewidth at least a , or contains a binary tree of depth b as a minor, or contains a simple path of length 2^c . This suggests the following conjecture: every graph of treewidth $\Omega(bc)$ has either pathwidth at least b or contains a simple path of length 2^c . The aim of this research group was to resolve this conjecture.

Treewidth-twin-width. The definition of the recently introduced graph parameter twin-width revolves around the mechanism of contraction sequences: simplification operations using which one can “fold” the whole graph into a single vertex. The main complexity measure of a contraction sequence is the maximum number of error edges that are adjacent to any vertex at any time. The goal of this group was to investigate the combinatorial properties of a graph parameter dubbed *treewidth-twin-width* obtained by additionally requiring that at all times, the graph composed of the error edges has bounded treewidth. Of particular interest is whether various classes known to have bounded twin-width actually have bounded treewidth-twin-width.

Integer programs equivalent to ones with bounded primal treedepth. Integer programming is known to be efficiently solvable for instances with small primal or dual treedepth. While we have a relatively good understanding of conditions when the instance can be transformed to an instance with small dual treedepth, less is known in the case of primal treedepth. The aim of this research group was to relate, for a given instance of integer programming, the smallest possible primal treedepth of an equivalent instance to invariant properties of the instance itself, in particular, to the structure of the column matroid of the constraint matrix. The ultimate goal would be to design algorithms for constructing an equivalent instance with small primal treedepth while avoiding a blow up in the entry complexity (such a blow up would prevent using the existing IP algorithms to solve the constructed instance).

Acknowledgements

Schloss Dagstuhl provided an excellent environment for hosting the seminar. The seminar room was appropriate to host the on-site participants and we found plenty of room for continuing discussions and socializing outside of the official program. This is particularly remarkable in these difficult times with the ongoing COVID pandemic. All participants were eager to meet and research together. According to the conducted survey, as well as the informal feedback to the organizers, the seminar was highly appreciated and can be considered a full success. On behalf of all participants, the organizers want to express their gratitude to the entire Dagstuhl staff and their outstanding support and service throughout the seminar.

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3 Overview of Talks

3.1 Twin-width and Sparsity

Édouard Bonnet (ENS – Lyon, FR)

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Main reference Édouard Bonnet, Eun Jung Kim, Stéphan Thomassé, Rémi Watrigant: “Twin-width I: tractable FO model checking”, CoRR, Vol. abs/2004.14789, 2020.

URL <https://arxiv.org/abs/2004.14789>

This is a tutorial on twin-width putting the focus on sparse classes. We sketch the characterization of bounded twin-width by so-called mixed minors. This is instrumental in bounding the twin-width of proper minor-closed classes; either directly, or via the functionally equivalent oriented twin-width. We see that all notions of sparsity collapse on hereditary classes of bounded twin-width. We showcase the algorithmic usefulness of a contraction sequence (witnessing low twin-width) by presenting an FPT algorithm for k -Independent Set on bounded twin-width graphs. We scale down the contraction sequences so that they exactly capture bounded rank-width and bounded linear rank-width, or in the sparse setting, bounded treewidth and bounded pathwidth. This way, the same algorithmic approach (the one presented for k -Independent Set) tackles FO model checking on a wide variety of classes, and reproves Courcelle’s theorems on MSO model checking.

3.2 Lacon- and Shrub-Decompositions: Characterizing Transductions of Bounded Expansion Classes

Jan Dreier (TU Wien, AT)

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Main reference Jan Dreier: “Lacon- and Shrub-Decompositions: A New Characterization of First-Order Transductions of Bounded Expansion Classes”, in Proc. of the 36th Annual ACM/IEEE Symposium on Logic in Computer Science, LICS 2021, Rome, Italy, June 29 – July 2, 2021, pp. 1–13, IEEE, 2021.

URL <https://doi.org/10.1109/LICS52264.2021.9470680>

This talk introduces lacon- and shrub-decompositions of graphs. We show that a graph class is a transduction of a class with bounded expansion iff it admits lacon- or shrub-decompositions with bounded expansion. This shows that on sparse graph classes, transductions are no more expressive than boolean combinations of purely existential transductions.

3.3 Combinatorial toolbox of sparsity and approximation algorithms

Zdenek Dvorák (Charles University – Prague, CZ)

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We give an overview of several basic tools from the sparsity theory and their applications in the design of approximation algorithms:

- The bounds on the expansion of shallow covers and the link between polynomial expansion and sublinear separators, used to analyse local search algorithms in graph classes with sublinear separators.

- The representations of short paths, applied to the approximation of distance domination number.
- Low-treewidth covers applied to design of constant-factor approximation algorithms and PTASes for problems expressible in the first-order logic.

3.4 Logics with Invariantly Used Relations

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Joint work of Kord Eickmeyer, Jan van den Heuvel, Ken-ichi Kawarabayashi, Stephan Kreutzer, Patrice Ossona de Mendez, Michał Pilipczuk, Daniel A. Quiroz, Roman Rabinovich, Sebastian Siebertz

Main reference Kord Eickmeyer, Jan van den Heuvel, Ken-ichi Kawarabayashi, Stephan Kreutzer, Patrice Ossona de Mendez, Michał Pilipczuk, Daniel A. Quiroz, Roman Rabinovich, Sebastian Siebertz: “Model-Checking on Ordered Structures”, *ACM Trans. Comput. Log.*, Vol. 21(2), pp. 11:1–11:28, 2020.

URL <https://doi.org/10.1145/3360011>

We study the expressive power and the complexity of model-checking for logics that are enriched by invariantly used relations: formulae in these logics may speak about e.g. a linear order on the set of elements of a structure, provided that the truth value of the formula be independent of the particular choice of a linear order. Invariant access to a linear order or a successor relation strictly increases the expressive power of first-order logic, but all known separating examples are structurally very complex. We review results showing a collapse in expressive power on certain trees and structures of bounded tree-depth. As for model-checking, we show how to interpret a successor relation in a structure with a k -walk or, alternatively, a spanning tree of bounded degree. This can be used to obtain fixed-parameter tractable model-checking algorithms for successor invariant first-order logic on classes of bounded expansion.

3.5 Stable graphs of bounded twin-width

Jakub Gajarský (University of Warsaw, PL), Michał Pilipczuk (University of Warsaw, PL), and Szymon Toruńczyk (University of Warsaw, PL)

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We prove that every class of graphs $\mathcal{C}$ that is monadically stable and has bounded twin-width can be transduced from some class with bounded sparse twin-width. This generalizes analogous results for classes of bounded linear cliquewidth [1] and of bounded cliquewidth [2]. It also implies that monadically stable classes of bounded twin-width are linearly χ -bounded.

References

- 1 Jaroslav Nešetřil, Patrice Ossona de Mendez, Roman Rabinovich, and Sebastian Siebertz. *Classes of graphs with low complexity: The case of classes with bounded linear rankwidth*. *Eur. J. Comb.*, 91:103223, 2021.
- 2 Jaroslav Nešetřil, Patrice Ossona de Mendez, Michał Pilipczuk, Roman Rabinovich, and Sebastian Siebertz. *Rankwidth meets stability*. In *SODA 2021*, pages 2014–2033. SIAM, 2021.

3.6 FO model checking of intersection graphs and twin-width

Petr Hliněný (Masaryk University – Brno, CZ) and Filip Pokrývka (Masaryk University – Brno, CZ)

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We survey past research on the complexity of FO model checking on classes of intersection graphs of geometric sets (e.g., interval graphs, permutation and circle graphs, map graphs), and show how this research direction has been affected by the new notion of twin-width. In particular, Bonnet et al have shown that FO model checking of a hereditary class of permutation graphs is in FPT if and only if the class excludes some permutation graph (assuming $\text{FPT} \neq \text{W}[1]$). Inspired by that, we prove that a hereditary class of circle graphs (i.e., of intersection graphs of chords of a circle) has the FO model checking in FPT if and only if (again) the class excludes some permutation graph. We also shortly comment on a recent conjecture of Rose McCarty about FO model checking of bounded perturbations of circle graphs.

3.7 Product structure of planar graphs

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Joint work of Gwenaél Joret, Vida Dujmovic, Louis Esperet, Cyril Gavoille, Piotr Micek, Pat Morin, Torsten Ueckerdt, David R. Wood

Main reference Vida Dujmovic, Gwenaél Joret, Piotr Micek, Pat Morin, Torsten Ueckerdt, David R. Wood: “Planar Graphs Have Bounded Queue-Number”, J. ACM, Vol. 67(4), pp. 22:1–22:38, 2020.

URL <https://doi.org/10.1145/3385731>

The product structure theorem for planar graphs states that every planar graph is a subgraph of the strong product of a bounded treewidth graph (treewidth at most 8) and a path. In this tutorial, I will first sketch the proof of this theorem. Then I will give an overview of its recent applications, and I will mention some recent generalizations of the theorem to other classes of graphs. I will conclude with a number of open questions.

3.8 Treedepth and Integer Programming

Martin Koutecký (Charles University – Prague, CZ)

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Main reference Friedrich Eisenbrand, Christoph Hunkenschroder, Kim-Manuel Klein, Martin Koutecký, Asaf Levin, Shmuel Onn: “An Algorithmic Theory of Integer Programming”, CoRR, Vol. abs/1904.01361, 2019.

URL <http://arxiv.org/abs/1904.01361>

A breakthrough result from 2018 by Koutecký, Levin, and Onn states that integer programming can be solved in time $f(a,d) \text{ poly}(L)$, where L is the input length, a is the “numeric measure” (largest coefficient of the constraint matrix) and d is the smaller of primal/dual treedepth of the constraint matrix. We give a high-level overview of this algorithm, survey

the progress since then (strongly polynomial and near-linear algorithms, mixed integer programming, and matroid parameters), and then highlight a refinement of treedepth, so-called d -fold treedepth, which emerges in this context.

3.9 Obstructions for bounded shrub-depth and rank-depth

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Joint work of O-joung Kwon, Rose McCarty, Sang-il Oum, Paul Wollan

Shrub-depth and rank-depth are dense analogues of the tree-depth of a graph. It is well known that a graph has large tree-depth if and only if it has a long path as a subgraph. We prove an analogous statement for shrub-depth and rank-depth, which was conjectured by Hliněný, Kwon, Obdrzalek, and Ordyniak [Tree-depth and vertex-minors, European J. Combin. 2016]. Namely, we prove that a graph has large rank-depth if and only if it has a vertex-minor isomorphic to a long path. This implies that for every integer t , the class of graphs with no vertex-minor isomorphic to the path on t vertices has bounded shrub-depth.

3.10 Graph Modification in Practice

Brian Lavallee (University of Utah – Salt Lake City, US)

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Joint work of Erik D. Demaine, Timothy D. Goodrich, Kyle Kloster, Brian Lavallee, Quanquan C. Liu, Hayley, Russell, Blair D. Sullivan, Ali Vakilian, Andrew van der Poel

Main reference Erik D. Demaine, Timothy D. Goodrich, Kyle Kloster, Brian Lavallee, Quanquan C. Liu, Blair D. Sullivan, Ali Vakilian, Andrew van der Poel: “Structural Rounding: Approximation Algorithms for Graphs Near an Algorithmically Tractable Class”, in Proc. of the 27th Annual European Symposium on Algorithms, ESA 2019, September 9–11, 2019, Munich/Garching, Germany, LIPIcs, Vol. 144, pp. 37:1–37:15, Schloss Dagstuhl – Leibniz-Zentrum für Informatik, 2019.

URL <https://doi.org/10.4230/LIPIcs.ESA.2019.37>

We describe a new framework for generalizing approximation algorithms for structural graph classes so that they apply to graphs “close” to a class (a scenario we expect is common when working with real-world networks) while still guaranteeing approximation ratios. The so-called structural rounding framework edits graphs into a nearby algorithmically tractable class, applies a class-specific approximation algorithm, then lifts the partial solution to the original graph. We give a general characterization of when an optimization problem is amenable to this approach, and show that it includes many well-studied graph problems, such as Independent Set, Vertex Cover, Minimum Maximal Matching, Chromatic Number, r -Dominating Set, and Connected Dominating Set. Guided by an experimental evaluation of the framework’s efficacy for Vertex Cover on near-bipartite graphs, we propose studying bicriteria approximation hardness for editing to parameterized graph classes.

3.11 Conjectures on vertex-minors

Rose McCarty (University of Waterloo, CA)

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Joint work of Rose McCarty, Jim Geelen, Paul Wollan

We survey various conjectures about proper vertex-minor-closed classes. The focus is on a conjecture of Geelen which would describe the structure of graphs in a proper vertex-minor-closed class. This conjecture is analogous to the Graph Minors Structure Theorem of Robertson and Seymour, but for vertex-minors instead of minors. We discuss the main pieces of this conjecture and its relationship with other areas of sparsity.

3.12 Posets, planarity, and sparsity

Piotr Micek (Jagiellonian University – Kraków, PL)

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This talk was an overview on connections between poset's dimension and the combinatorial side of the sparsity world. Already Dilworth has proved that for a poset to have large dimension, the poset must be wide. It does not have to be tall though. Indeed, so-called standard examples have height 2 and unbounded dimension. A remarkable feature of planar posets is that if they have large dimension then they are also tall. In other words, we can bound dimension of planar posets from above by a function of the height. This result by Strein and Trotter, proved in 2014, was a spark that kicked off a new line of research. We discussed the latest results in this line including how to bound the dimension of a poset in terms of the $(3h)$ -th weak coloring number of its cover graph. This implies bounds for the planar case and far beyond in the hierarchy of sparse classes of graphs. We also discussed the current state-of-art and pointed the key open problems in the area: including a thirty year old problem by Nešetřil and Pudlák (with essentially no progress over the years) on Boolean dimension of planar posets. Another exciting line of research is dim-boundedness which is an analogue of chi-boundedness for graphs. Here we still do not know if the class of planar posets is dim-bounded.

3.13 Empirical Evaluation of Approximation Algorithms for Generalized Graph Coloring and Uniform Quasi-Wideness

Wojciech Nadara (University of Warsaw, PL)

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Main reference Wojciech Nadara, Marcin Pilipczuk, Roman Rabinovich, Felix Reidl, Sebastian Siebertz: “Empirical Evaluation of Approximation Algorithms for Generalized Graph Coloring and Uniform Quasi-wideness”, ACM J. Exp. Algorithmics, Vol. 24(1), pp. 2.6:1–2.6:34, 2019.

URL <https://doi.org/10.1145/3368630>

The notions of bounded expansion and nowhere denseness not only offer robust and general definitions of uniform sparseness of graphs, they also describe the tractability boundary for several important algorithmic questions. In this paper we study two structural properties of

these graph classes that are of particular importance in this context, namely the property of having bounded generalized coloring numbers and the property of being uniformly quasi-wide. We provide experimental evaluations of several algorithms that approximate these parameters on real-world graphs. On the theoretical side, we provide a new algorithm for uniform quasi-wideness with polynomial size guarantees in graph classes of bounded expansion and show a lower bound indicating that the guarantees of this algorithm are close to optimal in graph classes with fixed excluded minor.

3.14 Sparsity in Practice – a bit of introspection

Felix Reidl (Birkbeck, University of London, GB)

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Joint work of Felix Reidl, Blair D. Sullivan

After several years of working towards making sparsity-based graph algorithms feasible in practice, I present my thoughts on why certain projects seem to run on their own while others stagnate. And the reasons do not seem to lie in the algorithms themselves, rather, it is a matter of whether we develop a solver (a general-purpose machinery aimed at a broad and fuzzy problem landscape) or a solution (a software that solves a specific, tangible problem for a collaborator). In either case, however, there are many small lessons to be learned about the engineering process of sparseness-based algorithms.

3.15 Improved Bounds for Centered Coloring

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Joint work of Felix Schröder, Michał Debski, Stefan Felsner, Piotr Micek

Main reference Michał Debski, Stefan Felsner, Piotr Micek, Felix Schröder: “Improved bounds for centered colorings”, in Proc. of the 2020 ACM-SIAM Symposium on Discrete Algorithms, SODA 2020, Salt Lake City, UT, USA, January 5-8, 2020, pp. 2212–2226, SIAM, 2020.

URL <https://doi.org/10.1137/1.9781611975994.136>

A vertex coloring ϕ of a graph G is p -centered if for every connected subgraph H of G either ϕ uses more than p colors on H or there is a color that appears exactly once on H . Centered colorings form one of the families of parameters that allow to capture notions of sparsity of graphs: A class of graphs has bounded expansion if and only if there is a function f such that for every $p \geq 1$, every graph in the class admits a p -centered coloring using at most $f(p)$ colors.

In the talk, we surveyed recent upper bounds for the maximum number of colors needed in a p -centered coloring of graphs from several widely studied graph classes:

1. planar graphs admit p -centered colorings with $O(p^3 \log p)$ colors;
2. bounded degree graphs admit p -centered colorings with $O(p)$ colors while it was conjectured that they may require exponential number of colors in p ;
3. graphs avoiding a fixed graph as a topological minor admit p -centered colorings with a polynomial in p number of colors. All these upper bounds imply polynomial algorithms for computing the colorings. We also discussed some non-trivial lower bounds:

4. there are graphs of treewidth t that require $\binom{p+t}{t}$ colors in any p -centered coloring and this bound matches the upper bound;
5. there are planar graphs that require $\Omega(p^2 \log p)$ colors in any p -centered coloring. We briefly talked about the proof methods for the other results before diving a little deeper into how to obtain result (2) with the entropy compression method.

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