Perceptions of Qualitative Spatial Arrangements of Three Objects

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Abstract

Cognitive grounding of formal models of qualitative spatial relations is important to bridge between spatial data and human perceptions of spatial arrangements. Here, we report on an experimental verification of the cognitive alignment of the recently proposed Ray Intersection Model (RIM) capturing qualitative relationships between three spatial objects, and human perceptions of spatial arrangements through a grouping task. Further, we explore arrangements with an object positioned "between" two other objects. We show that RIM has sufficient expressive power and aligns well with human perceptions of ternary spatial relationships.

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1 Introduction

Computational representations of qualitative spatial relations need to be computationally tractable and formalised, but they also should be cognitively valid, capturing perceptual distinctions important to people. People's perception of spatial arrangements are often subjectively impacted by their perception of form, of arrangement, and other contextual and possibly dynamic aspects [16]. One could argue that subjective differences may be even more apparent when the complexity of spatial relations increases, i.e., by increasing the number of objects in the arrangement.

Take the spatial relation "between" as an example. Some may consider the object O in Figure 1 to be positioned between objects A and B, while others may disagree, because O is not colinear with A and B but offset above their line of colinearity. Common topological relation models are binary (i.e., consider two objects), and they cannot adequately distinguish the ternary relation *"between"* from other, e.g., *disjoint* cases. Considering the complexity of

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human perception then, is it then feasible and practical to capture consensus expressions for ternary qualitative spatial relations? In this paper, we answer the research question of how do people distinguish between the relations between objects A, B, and O such as shown in Figure 1?

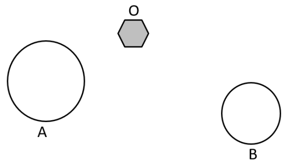


Figure 1 How do people distinguish the relations between the object O and objects A and B?

This paper investigates the human perception of different scenarios of three spatial objects that are distinguished by the recently proposed Ray Intersection Model (RIM) [12]. In an experiment, participants have been presented with image stimuli of spatial scenarios and tasked with grouping stimuli perceived as similar. Groups of stimuli common across participants may be considered as perceptually identical ternary spatial relations. The results of this study indicate that the expressive power of RIM aligns with human perception, and is therefore sufficient to facilitate reasoning about ternary spatial relations in a cognitively ergonomic manner. We further provide a breakdown of results by participant's age group, and provide insights in the verbal descriptions of the identified groupings, in particular with respect to the perception of the arrangements as "between".

The remainder of this paper is structured as follows: Section 2 briefly reviews related work on spatial relation models and ensuing experimental spatial cognition studies. Section 3 demonstrates the interactive survey designed with basic RIM scenarios. Section 4 shows the survey results and their analysis. The discussion of this study, its limitations, and concluding remarks are given in Sections 5 and 6.

2 Related works

2.1 Topological relation models

Qualitative models of spatial relations mostly focus on topological relations between objects which may be defined as relations that are invariant to topological transformations of the reference objects [5]. Most of the existing topological relation models for objects in a 2-dimensional plane are binary and based either on the intersection of point sets, such as the 4 and 9 Intersection Models (4IM and 9IM) [5], or the Region Connection Calculus [17] such as the well-known RCC-8 model [4]. However, binary relations are limited when describing complex relationships that need to consider more objects simultaneously.

Bloch et al. [2] discussed the ternary topological relations and the challenges in modelling them, but did not present a formal model. Clementini and Billen [3] proposed a ternary model for projective relations between regions that divides space around two regions into five parts (i.e., Before, Between, After, Leftside, Rightside). The third object is then described relative to the first two by its intersections with the four space divisions. Their model was named 5-intersection model and it distinguishes 31 projective relations between three regions.

2.2 Ray Intersection Model (RIM)

In this study we test the recently proposed Ray Intersection Model (RIM) introduced by Majic et al. [12] to validate its ergonomic and perceptual ability to distinguish situations differentiated by human subjects. RIM defines rays between two peripheral objects in a 2-dimensional plane (e.g., A and B in Figure 2) as straight lines that share exactly one end point with each of the peripheral objects. There may be infinite number of rays and the area covered by all possible rays is called the *ray area*, and the rays that coincide with the borders of the ray area are called *extreme rays*. The position of the third – core – object (O) in relation to peripheral objects is then represented by its topological relations with the rays, and since there may be many rays that have the same topological relation with the core object, only the distinct relations are considered (e.g., rays r_{e_1}/r_{e_2}, r_2, and r_3 in Figure 2 represent all distinct topological relations any ray between A and B can have with O as any other ray drawn between A and B will have an identical relation to O as one of these rays).

Figure 2 shows peripheral objects A and B, the core object O, and their representation with RIM. The first three matrices (3x3) show distinct 9IM relations between extreme (r_{e1}, r_{e2}) and two distinct non-extreme rays (r_2, r_3) and the core object O. These relations are then combined into a RIM matrix (4x3) that shows whether none (□), some (◪), or all (■) rays' boundaries (∂) and interiors (°) intersect the core object's interior, boundary, and exterior ($^-$). In the RIM matrix, R stands for the ray set which consists of all rays between A and B.

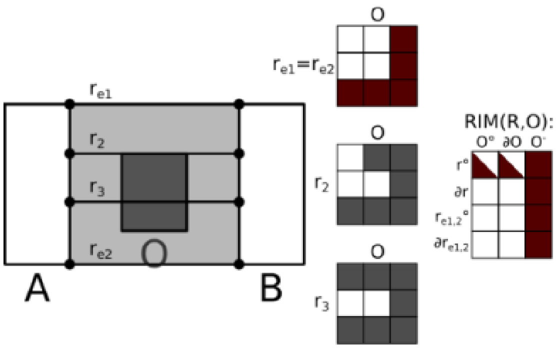


Figure 2 Basic RIM example with peripheral objects A and B, core object O, extreme rays r_{e_1} and r_{e_2}, and non-extreme rays r_2 and r_3 [12].

In [12], 23 basic RIM arrangement of three rectangles have been discussed, followed by examples of more complex scenarios discovered in a case study of betweenness of campus buildings. In theory, the expressiveness of a RIM matrix can capture 2070 different arrangements of three simple planar geometries. In [13], RIM was applied to detect missing data in OpenStreetMap, by identifying buildings with obstacles between buildings and the nearest road.

2.3 Experimental testing of perceptual categorisations

Spatial cognition research has a long history of experimental testing of the perceptual grounding of formal models of qualitative spatial relationships. Our research applies the card sorting technique [18, 7], a grouping exercise developed in human-computer interaction and usability studies where users sort cards (i.e., items) into groups that they perceive as similar in some way (e.g., design of information architecture of Websites). While all individuals categorise objects or situations somewhat subjectively, the success of our day-to-day communication and interactions affirms a level of shared perception of membership in groups. This is indeed what the card sorting technique enables to identify, based on synthesizing outcomes of individual groupings.

The card sorting technique has been previously adapted and applied in an influential series of research into spatial and linguistic conceptualizations and their interplays by Klippel and co-authors [11, 10, 9, 8]. Beyond spatial conceptualizations and their linguistic reflections, Bianchetti et al. [1] applied this technique to map symbol representations for similar concepts, but across two distinct national cartographic symbol standards for emergency maps.

Here, we equally apply a hierarchical clustering technique to understand how the coarse-grained commonalities in groups manifest across individuals. We also elicit descriptions of the groups by participants, to understand the verbal reflections of these perceptual groupings.

3 Experiment

We present the results of a survey designed to collect answers from a perceptual test of similarity in spatial arrangements. Participants were shown different spatial configurations of two peripheral and one core object. Their task was to group the depictions and provide textual descriptions for the groups they have created, based on their own judgment of similarity of arrangements. Participants' grouping patterns were analysed to gain insights about how people perceive ternary spatial relations, with particular interest if they have recognized and grouped the spatial relation "between".

A key criterion for the grouping task design was to let participants group depictions according to their perception of spatial relations between objects instead of other object properties, such as size and color. Survey instructions were designed to be clear, but excluded any spatial relation terms or hints to avoid influencing participants' answers. Thus, any spatial relation terms used in group descriptions come solely from participants and indicate their perception of spatial relations.

All depictions used in the experiment were generated from the 23 basic RIM cases shown in [12] by varying the rotation and placement of the objects as these parameters change the alignment of objects which could affect participants' perception of spatial relations. The shape and colour of the two peripheral objects remain constant in all depictions because they do not affect the spatial relations but could tempt participants to use them as criteria for grouping. The core object is colored red to stand out. Size was controlled too, however, for some RIM scenarios, it is impossible to keep the core object exactly the same. Figure 3(a), (b) and (c) shows examples of three such cases, where the core objects have different sizes to be able to express the RIM scenarios. The rotation of the depictions is another issue we do not want the participant to concentrate on. We chose random directions of the objects from a set of 8 options in the range of $0 - 360^o$ with 45^o increments.

Figure 4 shows the web interface of the grouping task, design of which was inspired by experiments of Klippel and Li [9]. Participants start the experiment with 77 ungrouped, depictions shown on the left side of the interface in a random order to mitigate the ordering effect [14]. The number on top of the interface indicates how many depictions remain ungrouped. On the right side of the interface, participants can add an unlimited number of groups, or delete any groups they have made. The main task is to sort each depiction from the left into one of the groups on the right by drag and drop. In a second step, participants label each group and describe their reasons for grouping the depictions together. A final

(a) (b) (c)

Figure 3 Examples of RIM scenarios.

submission is only possible when all depictions are sorted and all groups have been labeled and described. Groups, labels, and descriptions can be modified at any time until participants submit their answers.

To ensure that participants are familiar with the interface operations before the task starts, the survey first presents a warm-up trial task where they use the interface to sort random pictures.

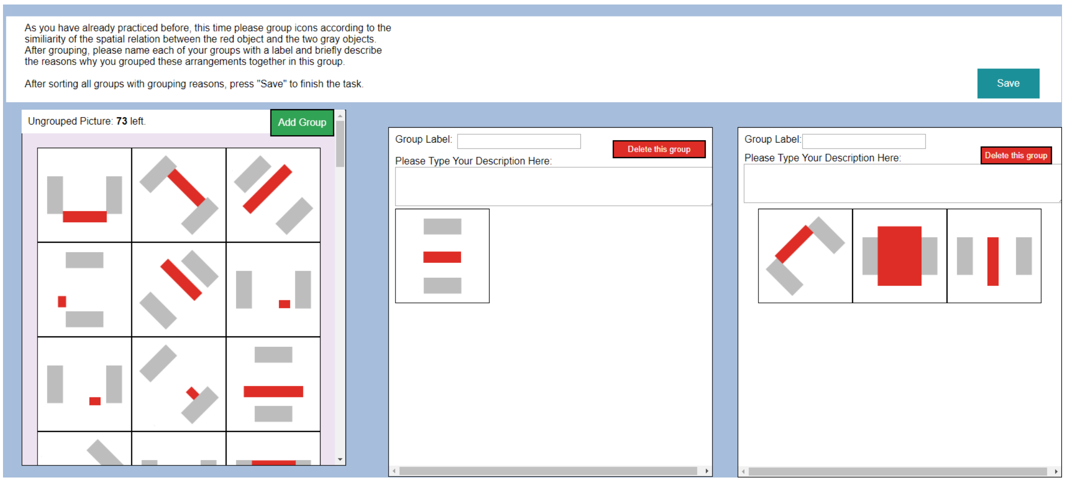


Figure 4 Grouping task interface.

The survey was administered through Amazon Mechanical Turk (Mturk) [2] and controlled the distribution of participants with regards to their primary language and age. We have divided participants into five age groups, with at least 25 participants per group: *18-25, 26-35, 36-45, 46-55, above 55* years old. In addition to their age, participants were asked to state their first language and country of residence which would provide insight into the geographical and linguistic distributions of participants. Figure 5 shows the overall flow of the survey.

[2] `https://www.mturk.com/`

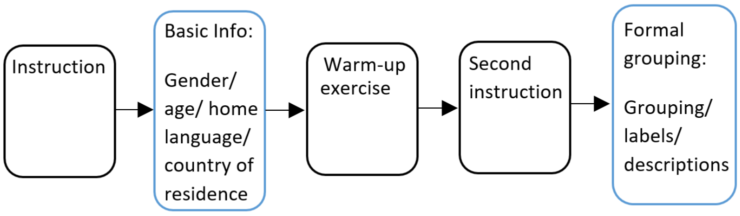


Figure 5 Overall flow of the survey.

Table 1 Summary of grouping by age group.

		Number of groups set by each participant					
Participant group	Participants	Mean	Median	Mode	Min	Max	Std. Dev.
Overall	75 (100.0%)	4.72	4	3	3	13	2.17
Age 18-25	12 (16.00%)	4.58	3.5	3	3	13	2.88
Age 25-35	13 (17.33%)	4.77	4	4	3	13	2.62
Age 35-45	25 (33.33%)	4.56	4	4	3	11	1.80
Age 45-55	10 (13.33%)	4.70	5	5	3	7	1.42
Age above 55	15 (20.00%)	5.07	4	3	3	11	2.34

4 Results

4.1 Participants overview

In a two-day period, the survey received 106 submissions from Mturk workers who were relatively evenly spread across the five age categories we have defined (Table 1). In the data cleaning process, the depiction grouping results were filtered into three classes:

1. Grouping based on spatial relations with the spatial relation terms (e.g. touching, in the middle of, between) mentioned in the group label or description.
2. Grouping based on spatial relations but no exact spatial relation terms mentioned (e.g. table-like, sandwich, pong game).
3. Grouping based solely on the directions of objects (e.g. 12 o'clock, 2 o'clock).

Although the grouping reasoning of the third class belongs to the area of spatial cognition, it is beyond the scope of this study, as the core research question is aimed at ternary spatial relations using RIM as a reference and not directional relations. Therefore, the results belonging to the third class were discarded from further analysis. After data cleaning, 75 answer sets remained. The final distributions of the age groups are shown in Tables 1 and 2.

4.2 Similarity analysis

Similarity matrices were generated to observe patterns of depictions that were frequently grouped together. Firstly, for every group from each participant, a square binary similarity matrix S_i of dimensions $m \times m$ ($m = 77$) contains entries of 1 when two depictions are grouped together (i.e., judged similar) and 0 for depictions that belong into different groups. By aggregating across matrices for all participants $S_1, S_2, S_3, \ldots, S_N$ ($N = 75$), we obtain an overall similarity matrix with element values ranging between 0 and 75. A higher number

Table 2 Descriptive statistics of all similarity matrices.

Participant group	Participants	Similarity parameter summary Mean	Median	Mode	Min	Max	Std. Dev.
Overall	75 (100.0%)	22.01	13	4	1	75	21.14
Age 18-25	12 (16.00%)	3.37	3	1	1	12	3.11
Age 25-35	13 (17.33%)	3.91	3	1	1	13	3.73
Age 35-45	25 (33.33%)	7.91	3	1	1	25	7.92
Age 45-55	10 (13.33%)	2.67	2	1	1	10	2.96
Age above 55	15 (20.00%)	4.16	3	1	1	15	4.19

in the matrix means that the two depictions were more frequently grouped together by participants, while a smaller number means that the two depictions were more likely placed into different groups. A 0 entry means that two depictions were never grouped together.

Partial similarity matrices were created for each age group to compare their grouping behaviours. In table 2, the descriptive statistics are listed for all generated similarity matrices. Additionally, we normalise the data in each participant group to the range $[0, 1]$, so that each element is divided by the total number of participants in the group (as shown in Figure 6).

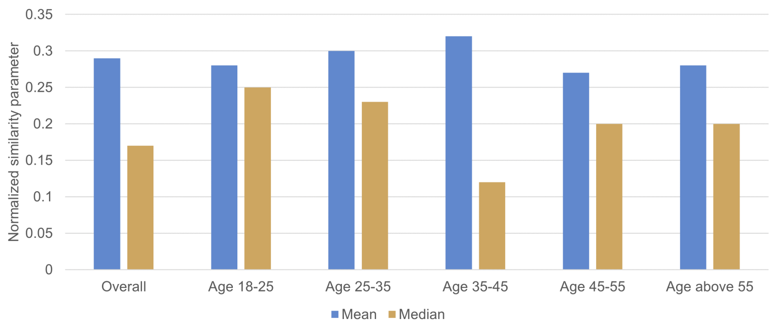


Figure 6 Normalised summary statistics for participant groups.

4.3 Hierarchical clustering

The agglomerative hierarchical cluster analysis is used to further demonstrate the patterns of participants' spatial relation groupings. All depictions are first recognized as single-object clusters, and then they may be merged into bigger clusters based on rules of the distance between each cluster [6]. Here we use Ward's method which performs better than other methods when clustering the non-multivariate data [15], which fits the requirements and the purpose of this experiment.

Therefore, we first calculate the pairwise dissimilarity matrix DS_i using the similarity matrix discussed in the previous section:

$$DS_i = 1 - \frac{S_i}{range(S_i)} \tag{1}$$

The dissimilarity matrix also contains normalised values in range [0,1] where a higher value represents less similarity between depictions. In the Ward's method, the principle of combining two clusters is based on the comparison between the squared deviations (Sq. Dev.) of all possible merges and performing the merge with minimal deviation:

$$Sq.Dev. = \sum(x_i - \bar{x}) \tag{2}$$

The result of the Ward's clustering method for all 75 selected survey answers is shown in Figure 7 as a dendogram. From the highest level of the dendogram, branches are divided in hierarchical structure and the summarized cluster levels of Figure 7 are shown in Figure 8.

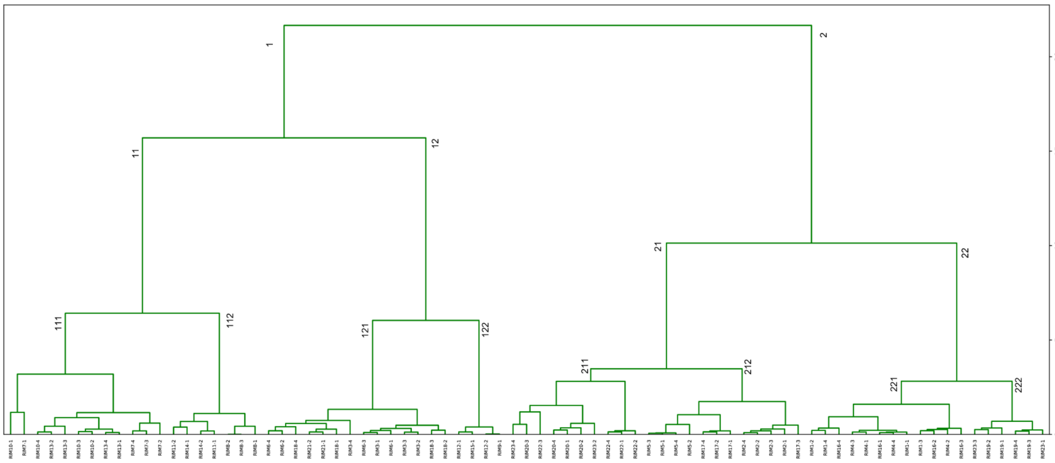


Figure 7 Dendrogram for all 75 selected answers using Ward's method.

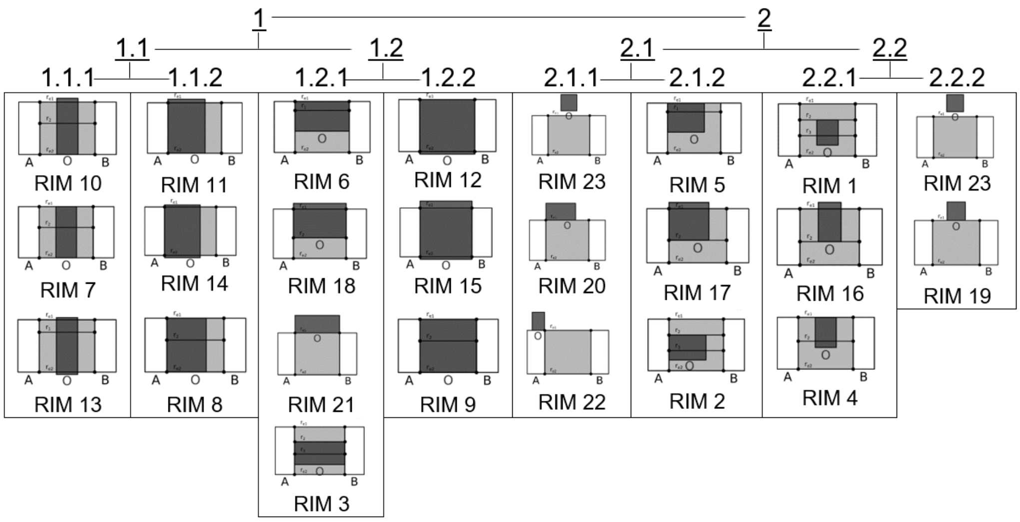


Figure 8 Hierarchical cluster structure of the overall selected answers.

5 Discussion and Limitations

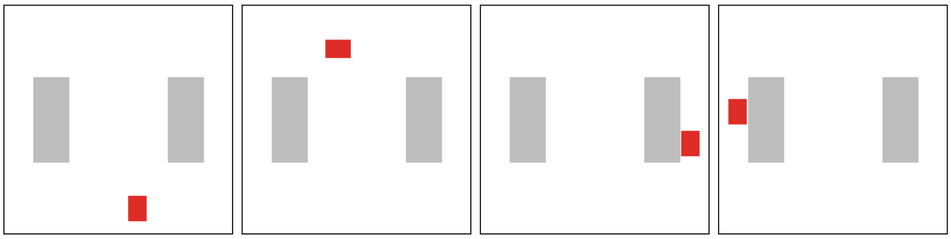
5.1 Survey depiction design evaluation

We aimed to generate four depictions for each RIM scenario (where possible/meaningful) by randomly varying the rotation of objects and translating the core object. To see whether participants perceive the depictions generated from the same RIM scenario as similar, we calculate pairwise similarities between scenarios. Table 3 shows the summary statistics for the pairwise similarities of depictions generated from the same RIM scenarios. Most

Table 3 Summary statistics of the pairwise similarity of depictions within the same RIM scenario.

Mean	Median	Mode	Maximum	Minimum	Std. Dev.
0.86	0.88	0.89	1	0.51	0.09

depictions belonging to the same RIM scenario are grouped together, hence judged similar. The mean, median, and mode similarities are all above 86%. The maximum similarity of 100% is achieved for two depictions generated from the RIM 8 (Figure 8) scenario which may be due to the limited options for the placement of the core object. The lowest similarity of 51% is achieved between two depictions generated from RIM 23. A possible explanation for this is that because the core object is not touching any rays between peripheral objects, there are more possibilities for the placement of the core object than perhaps in other RIM scenarios, as shown in Figure 9.



(a) (b) (c) (d)

Figure 9 Four depictions randomly generated from the RIM 23 scenario.

5.2 Spatial relation clustering analysis

Figure 8 shows how the 23 RIM scenarios have been clustered in the survey results. Clusters indicate that participants differentiate depictions in a manner that can be explained through analysis of the relationship between the core object and the peripheral objects, and the relationship between the core object and the rays cast between the peripheral objects. Table 4 shows the patterns in these parameters that can be observed in each cluster.

Table 4 Relationship clustering summary (in the third root level).

Cluster	Core object - peripheral objects	Core object - rays
1.1.1	Disjoint	Intersects all rays
1.1.2	Touches one boundary	Intersects all rays
1.2.1	Touches boundaries on both sides	Intersects some rays (or extreme rays)
1.2.2	Touches boundaries on both sides	Covers all rays
2.1.1	Touches one boundary	Disjoint or only touches one extreme ray
2.1.2	Touches one boundary	Intersects some rays (or extreme rays)
2.2.1	Disjoint	Intersects some rays (or extreme rays)
2.2.2	Disjoint	Disjoint or only touches one extreme ray

Table 5 Examples of group descriptions.

Group type	Examples
Explicit spatial relation terms	• This is a small red stripe of varying orientations between two grey identical stripes. • Little red block attached to the inside of grey blocks. • There is a long red block in the middle of two grey blocks.
Metaphors for spatial relations	• In all of these, the red bar is in between the two grey ones forming an almost bench-like shape. • When two grey blocks play pong with a red ball. • Shapes that remind me of a big, red table with some grey seats on the sides.

From Table 4, three distinct perceptual situations capturing the core object's relationship with the extreme rays can be extracted:

1. The core object touches/intersects all extreme rays
2. The core object touches/intersects only one extreme ray and some other rays
3. The core object touches/intersects one extreme ray but is disjoint from other rays

The results indicate that there is no difference between cases 2 and 3 in participants' perceptions of ternary spatial relations. When the core object intersects both extreme rays (case 1), then this happens in all depictions in the cluster (e.g., clusters 1.1.1, 1.1.2, and 1.2.2). But when this is not the case (cases 2 and 3), then the intersection of the extreme ray is not the determining factor as it only happens in some depictions in the cluster (e.g., clusters 1.2.1, 2.1.2, and 2.2.1). In these cases the relationship between the core object and the peripheral objects is the determining factor and consistent in all depictions within a cluster. For example, in cluster 1.2.1 the core object touches both peripheral objects, in cluster 2.1.2 one, and in cluster 2.2.1 none.

RIM 23 is the only scenario that is grouped into two different clusters (cluster 2.1.1 and cluster 2.2.2). As discussed in Section 5.1, this may be due to the large variation in placement options for the core object in situations describable as RIM 23.

5.3 Overview of group descriptions

As presented in Section 4.1, participants' descriptions of spatial relations can be divided into those that explicitly mention spatial relation terms, and those that use metaphors to express spatial relations. Typical examples are shown in Table 5. Spatial relation terms like "between", "outside", and "in the middle of" are used to describe the RIM scenarios based on whether the core object is perceived to be "between" peripheral objects or not. Metaphoric expressions like "table", "pong" and "sandwich" indicate not only the perception of spatial relations, but also other aspects of spatial cognition such as the size and shape of the core object compared to the peripheral objects. For example, participants tend to describe a relation as "sandwich" if all three objects in the depiction are parallel, mostly aligned, and have equal length.

5.4 Spatial relation "between"

In the 75 accepted survey answer sets, 35 mention the word "between" as a spatial relation in at least one of the created groups. One example of such survey answer is shown in Figure 10.

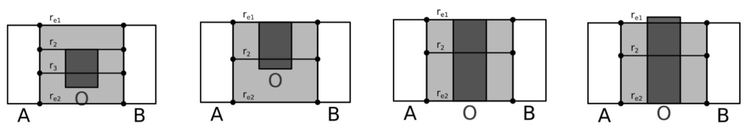
Participant ID: *removed*; **Gender:** Female; **Age:** 35-45
Home Language: English; **Country of residence:** United States of America
Group Label 1: Between touching both
Group Description 1: The red bar is between the grey bars and they are all touching.
Group Label 2: Between touching one
Group Description 2: The red bar is between the grey bars and is only touching one.
Group Label 3: Between no touching
Group Description 3: The red bar is between the grey bars but none of the bars are touching.
Group Label 4: Outside no touching
Group Description 4: The red bar is outside of the grey bars and none of the bars are touching.
Group Label 5: Outside touching
Group Description 5: The red bar is outside the grey bars but is touching one of them.

Figure 10 One participant's group labels and descriptions using the term *between*.

Different people group in different levels of detail. In the example above, the participant clearly separates the relation "between" into three classes depending on whether the core object touches the boundaries of the two peripheral objects. Other participants sometimes create the same groups that would correspond to "between", but use different parameters in their descriptions such as the size of the core object, the direction of the depiction, and whether the core object lies completely within the ray area.

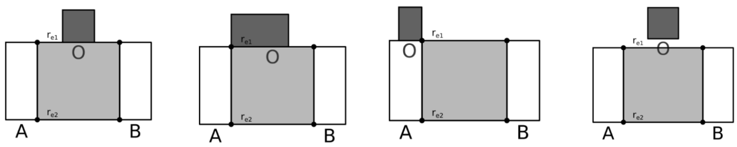
The first 4 RIM scenarios that are most frequently recognized as a relation of "between" are displayed in Figure 11, while the 4 RIM cases that are least likely to be identified as "between" are shown in Figure 12. All core objects in Figure 11 have some intersections with the rays cast between peripheral objects. Furthermore, the results indicate that these core objects should intersect some non-extreme rays. This ensures that the intersection happens mostly inside the ray area, regardless whether the core object extends outside (i.e., RIM 10) or not.

On the other hand, all core objects in Figure 12 are disjoint from the non-extreme rays. In these cases it does not matter whether they intersect one of the extreme rays as in RIM 19, RIM 20, and RIM 22, or if they are disjoint from all rays as in RIM 23. All of these core object will mostly be perceived as "not between" the peripheral objects by participants.



(a) RIM1. **(b)** RIM4. **(c)** RIM7. **(d)** RIM10.

Figure 11 RIM scenarios that can be grouped as a spatial relation "between".



(a) RIM19. **(b)** RIM20. **(c)** RIM22. **(d)** RIM23.

Figure 12 RIM scenarios that cannot be grouped as a relation "between".

5.5 Limitations of the experiment

There are several limitations regarding the human perception of ternary spatial relations that are not fully controlled in this study. Firstly, the country of origin of the majority of our participants may bias the results. 73% of the participants were from the US and 86% of the participants declared English as their first language. These biases may affect the diversity of samples collected from the survey, as language and cultural differences may affect the perception of spatial relations.

The second limitation is that only the spatial configurations that are generated from the 23 basic RIM scenarios were tested in this experiment. This covers only scenarios limited to spatial configurations of three rectangles, while perceptions of arrangements of other (circular, linear, point-like, 3D objects, complex polygons such as campus buildings in [12]) could be different.

Thirdly, in some scenes properties of spatial arrangements other than spatial relations have influenced participants answers, e.g, shape, color, size, and orientation of objects, as is visible from the depiction clusters and group descriptions. Although each cluster in Figure 8 can be justified with spatial relations, the core objects in each cluster are very similar in size. In this case, further validation may be needed to investigate whether the size of the object is tied to human perception of spatial relations.

6 Conclusion

This study has assessed the human perception of ternary spatial relations by conducting a survey with 75 participants. Their task was to sort 77 depictions showing different configuration of three spatial objects (i.e., two peripheral and one core object) into groups based on their perception of the spatial relations between these objects. These depictions were generated from 23 basic RIM scenarios by randomly translating and rotating the objects. The participants were also asked to describe each group with a label and a description. Survey answers were then analyzed with hierarchical clustering and similarity analysis to quantify participants' qualitative reasoning.

Two types of group descriptions were found in survey answers: those that explicitly use spatial relation terms and those that use metaphors to express spatial relations. We found that the participants who used metaphors were more likely to be influenced by size, shape, and rotation of objects, contrary to the survey instructions. There are no other noticeable differences between participants' grouping patterns. This is true for different age groups as well, where among five different age groups analyzed hardly any differences are found.

We note a number of patterns in the collected data. Firstly, participants perceive depictions generated from the same RIM scenario similarly, as reflected in the groupings (Table 3). This indicates that each distinct scenario captured by the RIM model [12] indeed captures ternary spatial relations that humans uniquely recognize and distinguish qualitatively, even when challenged with changes in objects' rotation, position, and size. Secondly, participants group different RIM scenarios highly similarly and consistently (Figures 7 and 8). The RIM scenarios within a cluster are conceptually closer to each other and can be hierarchically aggregated if a cognitively aligned generalization based on spatial relations is needed. The hierarchical cluster dendrogram shows how conceptually more similar scenarios group together, revealing this cognitive hierarchy in the arrangements.

RIM distinguishes scenarios that align with linguistic descriptions. The four RIM scenarios that were most frequently described with the term "between" are RIM 1, RIM 4, RIM 7, and RIM 10 (Figure 11). The core object in all of these scenarios intersects non-extreme rays, and interestingly, does not touch any of the peripheral objects. It also does not seem to matter whether the core object extends outside of the ray area or not. The four RIM scenarios that were the least associated with the term "between" all have the core object that is outside of the ray area (i.e., has no intersection with non-extreme rays) and may intersect extreme rays (Figure 12). This shows that RIM can differentiate ternary relations that people perceive as "between" or "not between".

The main limitation of this study and the motivation for the future work is the lack of cultural and language diversity in participants. This is something that can be controlled in future experiments to also investigate if there are differences in the human perception of spatial relations based on culture and language. Another possible improvement is to better communicate the goal of the study to minimize the influence of aspects other than spatial relations such as size, rotation, and color of objects on their answers. Lastly, this experiment was based on the RIM model which is currently limited to polygons [12]. It would be interesting to test human perception with objects of a different type such as points and lines, or dimensionality such as 3D objects.

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