

Finite and Algorithmic Model Theory

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Abstract

Finite and algorithmic model theory (FAMT) studies the expressive power of logical languages on finite structures or, more generally, structures that can be finitely presented. These are the structures that serve as input to computation, and for this reason the study of FAMT is intimately connected with computer science. Over the last four decades, the subject has developed through a close interaction between theoretical computer science and related areas of mathematics, including logic and combinatorics. This report documents the program and the outcomes of Dagstuhl Seminar 22051 “Finite and Algorithmic Model Theory”.

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1 Executive Summary

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Finite and Algorithmic Model Theory research revolves around the study of the expressive power of various logics on finite and finitely presented structures, and the connections between this and different computational models and mathematical formalisms.

The methods and tools of finite and algorithmic model theory (FAMT) have played an active role in the development of several areas of computer science. Finite or finitely representable structures are those that serve as inputs to computation, and the study of the expressive power of logical languages on such structures has led to fundamental insights in diverse areas, including database theory, computational complexity, random structures and combinatorics, verification, automata theory, proof complexity, and algorithmic game theory.

Over the past four decades FAMT has established itself as a rich research field with a strong and evolving community of researchers with a shared agenda. Much of the progress can be traced to the regular meetings of the community: the last such meeting was at Dagstuhl in 2017, and before that at Les Houches in 2012.

* Editor / Organizer



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The principal goals of this seminar included:

1. To identify fresh challenges in FAMT arising from some of the main application areas as well as newly emerging ones.
2. To make connections between core research in FAMT and other subfields of theoretical computer science, such as the theory of combinatorial and continuous optimization algorithms, and the theory of homomorphism counts and limit structures.
3. To transfer knowledge from emerging techniques in core FAMT back to the connected subfields and application areas.
4. To strengthen the research community in FAMT, especially by integrating younger members into it.
5. To provide continuity for what has been a successful model of regular seminars for building and consolidating the productive research community in FAMT.

One of the main goals of this Dagstuhl Seminar was to capitalize on the progress and the potential impact of some of the latest developments in FAMT and related areas. Such developments include:

a) The recently established connections between symmetric models of classical computation and bounded-variable counting logics. The symmetric counterparts of classical models of computations include threshold circuits, linear and semidefinite programs, and algebraic circuits. These new results have already been used to establish new upper and lower bounds for large families of algorithms by FAMT tools.

b) The theories of homomorphism counts and limit structures in combinatorics. A recent trend of work establishes that distinguishing the structures by the number homomorphisms they admit from certain classes of patterns, or to certain classes of patterns, is a fruitful alternative to distinguishing them by the logic formulas they satisfy.

c) Enumeration and counting methods including their use in database query processing, among others. One of the goals of this line of research is to understand and classify the logical queries for which it is possible to compute a compact representation of the output from which the query results can be obtained, efficiently, and on demand.

Organization and Activities

The organizers developed a schedule consisting of a number of invited survey talks, a number of talks focused on regular contributions proposed by participants, and an open problem session.

The seminar took place in person at Dagstuhl, with essentially all talks (except one) delivered by speakers attending Dagstuhl in person. The timing of the seminar coincided with the height of the Covid Omicron wave in Germany and Europe. This resulted in a number of late covid-related cancellations. Some talks, including invited talks, had to be cancelled, and the total number of participants (24) was fewer than originally planned.

Outcomes

Despite the many organizational challenges presented by the covid surge around the time of the workshop, the seminar was highly stimulating and surprisingly successful for those who were able to attend, and achieved many of our goals. (And thankfully there were no cases of covid during the seminar among those who did attend in person.)

	Monday	Tuesday	Wednesday	Thursday	Friday
09:00–09:15	Welcome				
09:15–09:30	Intros		Group photo		
09:30–10:00	Martin Grohe	Albert Atserias	Anuj Dawar	Mikolaj Bojanczyk	Christoph Berkholz
10:00–10:30	Martin Grohe	Albert Atserias	Anuj Dawar	Mikolaj Bojanczyk	Wrap-up
10:30–11:00	<i>Coffee break</i>				
11:00–12:00	David Roberson I	David Roberson II	Erich Graedel	Thomas Colcombet	
12:00–14:00	<i>Lunch Break</i>	<i>Lunch Break</i>	<i>Lunch Break</i>	<i>Lunch Break</i>	<i>Lunch</i>
14:00–14:30	Andrei Bulatov	Szymon Torunczyk	EXCURSION/OUTING	Marius Tritschler	
14:30–15:00	Andrei Bulatov	Szymon Torunczyk		Jerzy Marcinkowski	
15:00–15:30	Thomas Zeume			Sandra Kiefer	
15:30–16:00	<i>Coffee/Cake</i>	<i>Coffee/Cake</i>		<i>Coffee/Cake</i>	<i>Coffee/Cake</i>
16:00–16:30		Isolde Adler		Wei-Lin Wu	
16:30–17:00		Sebastian Siebertz		Open problems	
17:00–17:30		Sebastian Siebertz		Open problems	
	<i>Dinner</i>	<i>Dinner</i>	<i>Dinner</i>	<i>Dinner</i>	

The final program included invited tutorial talks by Martin Grohe on homomorphism counts (delivered online, to an in person audience at the workshop, due to a last minute cancellation for Grohe caused by COVID), David Roberson I and II on quantum isomorphism and its connection to homomorphism counting. These and other related talks at the seminar highlighted the exciting ongoing work aimed at delineating the power of homomorphism counting on various classes of graphs and structures, with surprising connections to other areas of mathematics.

Another invited talk was by Albert Atserias on symmetric computation and descriptive complexity (replacing a last minute cancellation), highlighted the exciting recent developments at the intersection of FAMT and combinatorial optimization. Another invited talk on query enumeration also had to be cancelled due to COVID. The full schedule including all the other talks of the Seminar can be found in the adjoined table.

One of the traditions of the series of workshops on Finite and Algorithmic Model Theory is to have a session in which some of the attendants present open problems and directions for further research. In this occasion, an hour of the afternoon of Thursday was devoted to such a session. A couple of days earlier we made a public call for presentations of open problems. Volunteers would write down their name on an easel pad. By Thursday, three volunteers came forward who gave 10 minute presentations (aprox.) of their proposals.

First, Erich Graedel presented an open problem on the topic of his earlier talk on Wednesday. Shortly put, the open problem asks to develop a proof theory for the emerging field of semiring semantics for logic formulas. The motivation comes from its potential applications in database theory and game analysis. Second, Isolde Adler presented an open problem on a topic covered in an earlier talk by Torunczyk. In brief, the open problem asks to study the relationships between the various notions of model-theoretic stability for classes of hypergraphs and more general relational structures. Third, Kousha Etessami gave a short talk on his recent work on applications of Tarski's Fixed-Point Theorem to economic game theory. In a nutshell, the question asks how much faster can one compute an arbitrary fixed-point of a monotone operator on a grid lattice than it takes to compute the least or greatest fixed-point by the standard iteration method. A detailed exposition of this open problem and its motivations can be found in a later section of this report. Finally, during his talk on Tuesday, Albert Atserias announced an open problem related to the optimal hardness

of approximating the minimum vertex cover on graphs. Since it was presented earlier during a talk, this open problem was not presented during the session. A detailed description of this open problem appears also in a later section of this report.

Overall, the organizers regard the seminar as a resounding success despite the difficult circumstances, and judging by the very positive feedback from participants, they agreed. We look forward to the next meeting of the FAMT community, hopefully within a few years, whether at Dagstuhl or elsewhere.

The organizers are grateful to the Scientific Directorate of the Center for its support of this workshop and the staff of Schloss Dagstuhl for their organisation of our stay (including regular covid testing) and their hospitality, despite the many challenges posed by covid.

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3 Overview of Talks

3.1 First-order property testing

Isolde Adler (University of Leeds, GB)

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 Isolde Adler

Joint work of Isolde Adler, Polly Fahey, Frederik Harwath, Noleen Köhler, Pan Peng

Main reference Isolde Adler, Noleen Köhler, Pan Peng: “On Testability of First-Order Properties in Bounded-Degree Graphs”, in Proc. of the 2021 ACM-SIAM Symposium on Discrete Algorithms, SODA 2021, Virtual Conference, January 10 – 13, 2021, pp. 1578–1597, SIAM, 2021.

URL <http://dx.doi.org/10.1137/1.9781611976465.96>

Property testing (for a property P) asks for a given graph, whether it has property P , or is “structurally far” from having that property. A “testing algorithm” is a probabilistic algorithm that answers this question with high probability correctly, by only looking at small parts of the input. Testing algorithms are thought of as “extremely efficient”, making them relevant in the context of large data sets.

In this talk I will present recent positive and negative results about testability of properties definable in first-order logic and monadic second-order logic on classes of bounded-degree graphs.

3.2 Symmetric computation, symmetric LP-lifts, and more...

Albert Atserias (UPC Barcelona Tech, ES)

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 Albert Atserias

Joint work of Albert Atserias, Anuj Dawar, Joanna Ochremiak

Main reference Albert Atserias, Anuj Dawar, Joanna Ochremiak: “On the Power of Symmetric Linear Programs”, J. ACM, Vol. 68(4), pp. 26:1–26:35, 2021.

URL <http://dx.doi.org/10.1145/3456297>

Main reference Albert Atserias, Joanna Ochremiak: “Definable Ellipsoid Method, Sums-of-Squares Proofs, and the Isomorphism Problem”, in Proc. of the 33rd Annual ACM/IEEE Symposium on Logic in Computer Science, LICS 2018, Oxford, UK, July 09-12, 2018, pp. 66–75, ACM, 2018.

URL <http://dx.doi.org/10.1145/3209108.3209186>

Descriptive complexity can be thought of as addressing the same fundamental questions as computational complexity theory but in a different model of computation. Unlike Turing machines or Boolean circuits, that get their inputs as strings, the machines and circuits of this model get their inputs as unordered sets of tuples of atomic objects; e.g., graphs. The computation is then supposed to develop in a symmetry-preserving way. In the talk, we discuss the power of fixed-point logic with counting, FPC, and of linear programming lifts, LP-lifts, as models of symmetric polynomial-time computation. We also report on the status of the program that asks to settle some of the most relevant questions of complexity theory in this restricted model, and do so unconditionally.

3.3 Number of variables vs. formula size in existential-positive FO

Christoph Berkholz (HU Berlin, DE)

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Joint work of Christoph Berkholz, Hubie Chen

Main reference Christoph Berkholz, Hubie Chen: “Compiling Existential Positive Queries to Bounded-Variable Fragments”, in Proc. of the 38th ACM SIGMOD-SIGACT-SIGAI Symposium on Principles of Database Systems, PODS 2019, Amsterdam, The Netherlands, June 30 – July 5, 2019, pp. 353–364, ACM, 2019.

URL <http://dx.doi.org/10.1145/3294052.3319693>

A crucial property of bounded-variable fragments of first-order logic is that they can be evaluated in polynomial time. It is therefore a useful preprocessing step to rewrite, if possible, a first-order query to a logically equivalent one with a minimum number of variables. However, it may occur that reducing the number of variables causes an increase in formula size. We investigate this trade-off for the existential-positive fragment of first-order queries, where variable minimisation is decidable in general. In particular, we study the blow-up in the formula size when compiling existential-positive queries to the bounded variable fragment of positive first-order logic. While the increase of the formula size is always at most exponential, we identify situations (based on the signature and the number of variables) where only a polynomial blow-up is needed. In all other cases, we show that an exponential lower bound on the formula size of the compiled formula that matches the general upper bound. This exponential lower bound is unconditional, and is the first unconditional lower bound for formula size with respect to the studied compilation; it is proved via establishing a novel interface with circuit complexity which may be of future interest.

3.4 Polyregular Functions

Mikolaj Bojanczyk (University of Warsaw, PL)

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Transducers are like automata, but instead of accepting/rejecting they produce an output, such as a string or a tree. In my talk, I will discuss some recent results transducers, mainly about the class of polyregular transducers, which can be seen as a candidate for the notion of “regular” string-to-string transducers of polynomial growth. I will discuss how this class can be characterised in many different ways, including logic, automata, and λ -calculus. From a finite model theory point of view, the most interesting fact is that it makes sense to consider mso interpretations where positions in the output string are represented by k -tuples of positions in the input string. It turns out that, somewhat surprisingly, such functions are closed under composition if the inputs and outputs are strings.

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3.5 Complexity classification of counting graph homomorphisms modulo a prime number

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Joint work of Amirhossein Kazemini, Andrei A. Bulatov

Main reference Amirhossein Kazemini, Andrei A. Bulatov: “Counting Homomorphisms Modulo a Prime Number”, in Proc. of the 44th International Symposium on Mathematical Foundations of Computer Science, MFCS 2019, August 26-30, 2019, Aachen, Germany, LIPIcs, Vol. 138, pp. 59:1–59:13, Schloss Dagstuhl – Leibniz-Zentrum für Informatik, 2019.

URL <http://dx.doi.org/10.4230/LIPIcs.MFCS.2019.59>

Counting graph homomorphisms and its generalizations such as the Counting Constraint Satisfaction Problem (CSP), its variations, and counting problems in general have been intensively studied since the pioneering work of Valiant. While the complexity of exact counting of graph homomorphisms (Dyer and Greenhill, 2000) and the counting CSP (Bulatov, 2013, and Dyer and Richerby, 2013) is well understood, counting modulo some natural number has attracted considerable interest as well. In their 2015 paper Faben and Jerrum suggested a conjecture stating that counting homomorphisms to a fixed graph H modulo a prime number is hard whenever it is hard to count exactly, unless H has automorphisms of certain kind. In this paper we confirm this conjecture. As a part of this investigation we develop techniques that widen the spectrum of reductions available for modular counting and apply to the general CSP rather than being limited to graph homomorphisms.

3.6 On Uniformisation

Thomas Colcombet (CNRS – Paris, FR)

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Joint work of Thomas Colcombet, Alex Rabinovich

In this talk, I will present several questions and results related to uniformisation questions for first-order and monadic second-order theory (MSO). Uniformisation consists in, given a formula $\phi(X, \bar{p})$, where \bar{p} is a tuple of parameters, to find a formula $\psi(X, \bar{p})$ such that: (1) If $\psi(X, \bar{p})$ holds, then $\phi(X, \bar{p})$ holds, and (2) if $\phi(X, \bar{p})$ holds for some X , then there exists a unique X such that $\psi(X, \bar{p})$ holds. In other words, if a formula has a solution, it is possible to define a unique solution. Of course these questions are parametrised by the logic and the family of models under consideration.

Questions pertaining to uniformisation are various. Some logics may admit uniformisation on some models. Some logics may require a more expressive logic for uniformisation. And uniformisability can be seen as a decision procedure.

In this talk, I will survey some results of these forms on finite and infinite structure, for first-order or monadic second-order logic. In particular, I will explain how the algebraic approach can be used to answer some uniformisability questions for MSO logic over countable ordinals. More precisely, it is known since 1998 by Shelah and Lifshes that MSO is uniformisable in itself over all ordinals shorter than ω^ω (non-uniformly), and that it is not true beyond. I will present recent unpublished new results in collaboration with Alex Rabinovich: (a) there is a single construct that can be added to MSO in order to uniformise MSO over all countable ordinals (the ability to choose a unique cofinal set of order-type ω); and (2) that it is possible to decide, given an MSO formula, whether it can be uniformised in MSO itself over all countable ordinals.

3.7 Cohomology and Finite Model Theory

Anuj Dawar (*University of Cambridge, GB*)

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 Anuj Dawar

Joint work of Samson Abramsky, Adam Ó Conghaile, Anuj Dawar

In this talk I gave a brief introduction to algebraic topology intended for the finite model theory audience, leading up to a definition of cohomology. With this in hand, I discussed some potential applications of the ideas in finite model theory. In particular, we look at a collection of partial solutions to a constraint satisfaction problem as a presheaf and identify cohomological obstructions to satisfiability. This allows us to draw conclusions about the expressive power of fixed-point logic extended with operators for solving systems of linear Diophantine equations.

3.8 Homomorphism Tensors and Linear Equations

Martin Grohe (*RWTH Aachen University, DE*)

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 Martin Grohe

Joint work of Martin Grohe, Gaurav Rattan, Tim Seppelt

Main reference Martin Grohe, Gaurav Rattan, Tim Seppelt: “Homomorphism Tensors and Linear Equations”, CoRR, Vol. abs/2111.11313, 2021.

URL <https://doi.org/10.48550/arXiv.2111.11313>

Lovász (1967) showed that two graphs G and H are isomorphic if and only if they are homomorphism indistinguishable over the class of all graphs, i.e. for every graph F , the number of homomorphisms from F to G equals the number of homomorphisms from F to H . Recently, homomorphism indistinguishability over restricted classes of graphs such as bounded treewidth, bounded treedepth and planar graphs, has emerged as a surprisingly powerful framework for capturing diverse equivalence relations on graphs arising from logical equivalence and algebraic equation systems. In this paper, we provide a unified algebraic framework for such results by examining the linear-algebraic and representation-theoretic structure of tensors counting homomorphisms from labelled graphs. The existence of certain linear transformations between such homomorphism tensor subspaces can be interpreted both as homomorphism indistinguishability over a graph class and as feasibility of an equational system. Following this framework, we obtain characterisations of homomorphism indistinguishability over two natural graph classes, namely trees of bounded degree and graphs of bounded pathwidth, answering a question of Dell et al. (2018).

3.9 Semiring Semantics and Strategy Analysis

Erich Grädel (RWTH Aachen, DE)

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Joint work of Erich Grädel, Niels Lücking, Matthias Naaf

Main reference Erich Grädel, Niels Lücking, Matthias Naaf: “Semiring Provenance for Büchi Games: Strategy Analysis with Absorptive Polynomials”, CoRR, Vol. abs/2106.12892, 2021.

URL <https://arxiv.org/abs/2106.12892>

This paper presents a case study for the application of semiring semantics for fixed-point formulae to the analysis of strategies in Büchi games. Semiring semantics generalizes the classical Boolean semantics by permitting multiple truth values from certain semirings. Evaluating the fixed-point formula that defines the winning region in a given game in an appropriate semiring of polynomials provides not only the Boolean information on who wins, but also tells us how they win and which strategies they might use. This is well-understood for reachability games, where the winning region is definable as a least fixed point. The case of Büchi games is of special interest, not only due to their practical importance, but also because it is the simplest case where the fixed-point definition involves a genuine alternation of a greatest and a least fixed point. We show that, in a precise sense, semiring semantics provide information about all absorption-dominant strategies – strategies that win with minimal effort, and we discuss how these relate to positional and the more general persistent strategies. This information enables further applications such as game synthesis or determining minimal modifications to the game needed to change its outcome. Lastly, we discuss limitations of our approach and present questions that cannot be immediately answered by semiring semantics.

3.10 Logarithmic Weisfeiler-Leman Identifies All Planar Graphs

Sandra Kiefer (RWTH Aachen, DE)

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Joint work of Martin Grohe, Sandra Kiefer

Main reference Martin Grohe, Sandra Kiefer: “Logarithmic Weisfeiler-Leman Identifies All Planar Graphs”, in Proc. of the 48th International Colloquium on Automata, Languages, and Programming, ICALP 2021, July 12-16, 2021, Glasgow, Scotland (Virtual Conference), LIPIcs, Vol. 198, pp. 134:1–134:20, Schloss Dagstuhl – Leibniz-Zentrum für Informatik, 2021.

URL <http://dx.doi.org/10.4230/LIPIcs.ICALP.2021.134>

The Weisfeiler-Leman (WL) algorithm is a well-known combinatorial procedure for detecting symmetries in graphs and it is widely used in graph-isomorphism tests. It proceeds by iteratively refining a colouring of vertex tuples. The number of iterations needed to obtain the final output is crucial for the parallelisability of the algorithm.

In my talk, I presented recent work concerning the number of iterations of the WL algorithm on planar graphs. To be more precise, we found that there is a constant k such that every planar graph can be identified (that is, distinguished from every non-isomorphic graph) by the k -dimensional WL algorithm within a logarithmic number of iterations. This generalises a result due to Verbitsky (STACS 2007), who proved the same for 3-connected planar graphs.

The number of iterations needed by the k -dimensional WL algorithm to identify a graph corresponds to the quantifier depth of a sentence that defines the graph in the $(k+1)$ -variable fragment C^{k+1} of first-order logic with counting quantifiers. Thus, our result implies that every planar graph is definable with a C^{k+1} -sentence of logarithmic quantifier depth.

3.11 Determinacy of Real Conjunctive Queries. The Boolean Case

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Joint work of Jarosław Kwiecień, Jerzy Marcinkowski, Piotr Ostropolski-Nalewaja
Main reference Jarosław Kwiecień, Jerzy Marcinkowski, Piotr Ostropolski-Nalewaja: “Determinacy of Real Conjunctive Queries. The Boolean Case”, CoRR, Vol. abs/2112.12742, 2021.
URL <https://arxiv.org/abs/2112.12742>

In their classical 1993 paper [CV93] Chaudhuri and Vardi notice that some fundamental database theory results and techniques fail to survive when we try to see query answers as bags (multisets) of tuples rather than as sets of tuples. But disappointingly, almost 30 years after [CV93], the bag-semantics based database theory is still in its infancy. We do not even know whether conjunctive query containment is decidable. And this is not due to lack of interest, but because, in the multiset world, everything suddenly gets discouragingly complicated. In this paper, we try to re-examine, in the bag semantics scenario, the query determinacy problem, which has recently been intensively studied in the set semantics scenario. We show that query determinacy (under bag semantics) is decidable for boolean conjunctive queries and undecidable for unions of such queries (in contrast to the set semantics scenario, where the UCQ case remains decidable even for unary queries). We also show that – surprisingly – for path queries determinacy under bag semantics coincides with determinacy under set semantics (and thus it is decidable).

3.12 Quantum isomorphism is equivalent to equality of homomorphism counts from planar graphs I & II

David Earl Roberson (Technical University of Denmark – Lyngby, DK)

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Joint work of Laura Mancinska, David E. Roberson
Main reference Laura Mancinska, David E. Roberson: “Quantum isomorphism is equivalent to equality of homomorphism counts from planar graphs”, in Proc. of the 61st IEEE Annual Symposium on Foundations of Computer Science, FOCS 2020, Durham, NC, USA, November 16-19, 2020, pp. 661–672, IEEE, 2020.
URL <http://dx.doi.org/10.1109/FOCS46700.2020.00067>

Over 50 years ago, Lovász proved that two graphs are isomorphic if and only if they admit the same number of homomorphisms from any graph. Other equivalence relations on graphs, such as cospectrality or fractional isomorphism, can be characterized by equality of homomorphism counts from an appropriately chosen class of graphs. Dvořák [J. Graph Theory 2010] showed that taking this class to be the graphs of treewidth at most k yields a tractable relaxation of graph isomorphism known as k -dimensional Weisfeiler-Leman equivalence. Together with a famous result of Cai, Fürer, and Immerman [FOCS 1989], this shows that homomorphism counts from graphs of bounded treewidth do not determine a graph up to isomorphism. Dell, Grohe, and Rattan [ICALP 2018] raised the questions of whether homomorphism counts from planar graphs determine a graph up to isomorphism, and what is the complexity of the resulting relation. We answer the former in the negative by showing that the resulting relation is equivalent to the so-called quantum isomorphism [Maňčinska et al, ICALP 2017]. Using this equivalence, we further resolve the latter question, showing that testing whether two graphs have the same number of homomorphisms from any planar graph is, surprisingly, an undecidable problem, and moreover is complete for the class coRE (the complement

of recursively enumerable problems). Quantum isomorphism is defined in terms of a one-round, two-prover interactive proof system in which quantum provers, who are allowed to share entanglement, attempt to convince the verifier that the graphs are isomorphic. Our combinatorial proof leverages the quantum automorphism group of a graph, a notion from noncommutative mathematics.

3.13 First-Order Logic with Connectivity Operators: expressiveness and model-checking

Sebastian Siebertz

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Main reference Nicole Schirrmacher, Sebastian Siebertz, Alexandre Vigny: “First-Order Logic with Connectivity Operators”, in Proc. of the 30th EACSL Annual Conference on Computer Science Logic, CSL 2022, February 14-19, 2022, Göttingen, Germany (Virtual Conference), LIPIcs, Vol. 216, pp. 34:1–34:17, Schloss Dagstuhl – Leibniz-Zentrum für Informatik, 2022.
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Main reference Michał Pilipczuk, Nicole Schirrmacher, Sebastian Siebertz, Szymon Toruńczyk, Alexandre Vigny: “Algorithms and data structures for first-order logic with connectivity under vertex failures”, CoRR, Vol. abs/2111.03725, 2021.
URL <https://arxiv.org/abs/2111.03725>

First-order logic (FO) can express many algorithmic problems on graphs, such as the independent set and dominating set problem parameterized by solution size. On the other hand, FO cannot express the very simple algorithmic question whether two vertices are connected. We enrich FO with connectivity predicates that are tailored to express algorithmic graph properties that are commonly studied in parameterized algorithmics. By adding the atomic predicates $conn_k(x, y, z_1, \dots, z_k)$ that hold true in a graph if there exists a path between (the valuations of) x and y after (the valuations of) z_1, \dots, z_k have been deleted, we obtain *separator logic* $FO+conn$. We thereby obtain a logic that can express many interesting problems such as the feedback vertex set problem and elimination distance problems to first-order definable classes.

We first study the expressive power of the new logic. We then study the model-checking problem and prove that from the point of view of parameterized complexity, under standard complexity theoretical assumptions, the frontier of tractability of separator logic is almost exactly delimited by classes excluding a fixed topological minor. From the atomic case of connectivity predicates we obtain the first deterministic data structure for connectivity under batched vertex failures where for every fixed number of failures, all operations can be performed in constant time.

This talk is based on the results presented in [2] and [1].

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3.14 Model Checking on Interpretations of Classes of Bounded Local Cliquewidth

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Joint work of Édouard Bonnet, Jan Dreier, Jakub Gajarský, Stephan Kreutzer, Nikolas Mählmann, Pierre Simon, Szymon Torunczyk

Main reference Michal Pilipczuk, Nicole Schirrmacher, Sebastian Siebertz, Szymon Torunczyk, Alexandre Vigny: “Algorithms and data structures for first-order logic with connectivity under vertex failures”, CoRR, Vol. abs/2111.03725, 2021.

URL <https://arxiv.org/abs/2111.03725>

We present a fixed-parameter tractable algorithm for first-order model checking on interpretations of graph classes with bounded local cliquewidth. Notably, this includes interpretations of planar graphs, and more generally, of classes of bounded genus. To obtain this result we develop a new tool which works in a very general setting of dependent classes and which we believe can be an important ingredient in achieving similar results in the future.

3.15 On guarded team logics

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Guarded logics and team logics are logics of imperfect information, in different ways. When combined, some fragments have nice model theoretic properties while still being reasonably expressive. In particular, a technique called “strategy tree transfer” can be applied to show a finite model property for these fragments.

3.16 On capturing some equivalence relations by homomorphism counts

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Joint work of Albert Atserias, Phokion G. Kolaitis, Wei-Lin Wu

Main reference Albert Atserias, Phokion G. Kolaitis, Wei-Lin Wu: “On the Expressive Power of Homomorphism Counts”, in Proc. of the 36th Annual ACM/IEEE Symposium on Logic in Computer Science, LICS 2021, Rome, Italy, June 29 – July 2, 2021, pp. 1–13, IEEE, 2021.

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A classical result by Lovasz states that two graphs are isomorphic if and only if they have the same left profile, i.e., they have the same homomorphism counts “from” all graphs. A similar result by Chaudhuri and Vardi states that two graphs are isomorphic if and only if they have the same right profile, in other words, they have the same homomorphism counts “to” all graphs. By restricting the left or right profile to a class of graphs, we have a relaxation of isomorphism. In this talk, I will share some results about what relaxations of isomorphism do/don’t coincide with a restricted left or right profile.

3.17 Iltis: Learning logic in the web

Thomas Zeume (Ruhr-Universität Bochum, DE)

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Joint work of Thomas Zeume, Gaetano Geck, Christine Quenkert, Marko Schmellenkamp, Jonas Schmidt, Felix Tschirbs, Fabian Vehlken

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The Iltis project provides an interactive, web-based system for teaching the foundations of formal methods. It is designed with the objective to allow for simple inclusion of new educational tasks; to pipeline such tasks into more complex exercises; and to allow simple inclusion and cascading of feedback mechanisms. Currently, exercises for many typical automated reasoning workflows for propositional logic, modal logic, and some parts of first-order logic are covered.

In this talk I will address (algorithmic) challenges and solution approaches for building such systems, but also show how the system can be used in logic instruction.

4 Open problems

4.1 Find C^3 -equivalent graphs with a factor-2 gap in the size of their minimum vertex covers

Albert Atserias (UPC Barcelona Tech, ES)

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The minimum vertex cover of a graph G , denoted by $k(G)$, is the least cardinality of a set of vertices that touches every edge. By rounding the straightforward linear programming relaxation of the problem, $k(G)$ can be approximated within a factor of two in polynomial time. It is conjectured that, for every positive real $\epsilon \in (0, 1]$, the problem of approximating $k(G)$ by a factor better than $(2 - \epsilon)$ is NP-hard. Is it also hard for k -variable counting logic C^k for every fixed k ? A precise statement of the problem is the following:

Question: Is it true that for every natural number k and every real number $\epsilon \in (0, 1]$ there exist C^k -equivalent graphs G and H with $k(G) > (2 - \epsilon)k(H)$? One way to ensure $k(G) > (2 - \epsilon)k(H)$ is by having G have independence number at most $o(n)$ and H have independence number at least $(1/2 - o(1))n$, where $n = |V(G)| = |V(H)|$ and $n \rightarrow \infty$.

For $k = 2$ and arbitrary $\epsilon \in (0, 1]$, graphs as required in the question can be easily found (see [1]). For $k = 3$ and $\epsilon = 0$ it follows from the results in [2] that such graphs do not exist. For arbitrary positive integer k and $\epsilon \in [0.73, 1]$, graphs as required are shown to exist in [1]. For $k \geq 3$ and $\epsilon \in (0, 0.74]$, the problem is open.

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4.2 Can the current bounds for computing a Tarski fixed point on a finite (grid) lattice be improved?

Kousha Etessami (University of Edinburgh, GB)

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Let $[N] = \{1, \dots, N\}$.

Consider a function

$$f : [N]^d \rightarrow [N]^d$$

mapping the finite d -dimensional euclidean grid lattice with sides of length N to itself.

Suppose the function f is *monotone* with respect to the standard coordinate-wise partial order on vectors in $[N]^d$, meaning that for all $x, y \in [N]^d$, if $x \leq y$ then $f(x) \leq f(y)$. By Tarski's (1953) fixed point theorem, such a monotone function f must have a fixed point, i.e., there must exist a point

$$x' \in [N]^d$$

such that $f(x') = x'$.

How hard is it to compute such a fixed point when (a) the function f is given to us as a black box and we wish to find a fixed point with a minimum number of queries? or, (b) the function f is given to us explicitly but succinctly using a boolean circuit, with $d \cdot \log(N)$ input gates and $d \cdot \log(N)$ output gates.

For (a), it is easy to see that standard (Kleene) value iteration, starting from the bottom $\bar{1}$ (or, respectively, top \bar{N}) of the lattice $[N]^d$ requires at most $d \cdot N$ queries to find the least (respectively, greatest) fixed point of f . On the other hand, suppose we don't care which fixed point we compute. Suppose any fixed point will do. Can we do better?

This problem has a number of important applications. In particular, efficient algorithms for this problem (polynomial in both d and $\log(N)$) would imply polynomial time algorithms for supermodular games (a very well studied class of games in economic theory), as well as for both Condon's simple stochastic games and Shapley's stochastic games, which are long standing open questions (see [1].)

There is a black-box algorithm due to Dang, Qi, and Ye [2], which requires at most $\log^d(N)$ queries to find a fixed point, and uses a combination of recursion and binary search. This algorithm has recently been improved by Fearnley, Palvolgyi, and Savani [3], who give an upper bound of $\log^{\lceil \frac{2d}{3} \rceil}(N)$ queries for finding a fixed point. There has also been some even more recent (unpublished) results which improve the exponent further, but it remains exponential in d .

What is the best upper bound we can obtain for computing a Tarski fixed point? This is very much an open question. Etessami, Papadimitriou, Rubinstein, and Yannakakis [1] provide a lower bound of $\Omega(\log^2(N))$ queries in the black-box model for (a), already for 2-dimensional monotone functions $f : [N]^2 \rightarrow [N]^2$. They also show that a total search version of the white-box Tarski problem (b) is in both the complexity classes PPAD and PLS. (It thus follows from recent results that the Tarski problem is also in the total search complexity classes CLS and EOPL.)

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