A Bilevel Model for the Frequency Setting Problem

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Abstract

Based on a partnership between IMT Atlantique and the French company Lumiplan, this work is part of a process of strengthening the Heurès software currently offered by Lumiplan to public transport operators to support their bus and driver scheduling operations. This work addresses the frequency setting problem which aims at defining the frequencies of the bus lines of a network for different time periods of a day. This operation complements a study on line planning with more accurate estimations of the demand, necessary bus types and passengers behaviors. In this paper, the operator's exploitation costs are minimized while respecting service-levels constraints, based on the predictions of the path choice made by the passengers. The problem is solved by an easily implementable process and a case study based on a real network is presented to show the efficiency of our method.

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1 Introduction

Adapting line frequencies to demand is a key element for an efficient network design. Known as the *Frequency Setting Problem*, this problem can be executed after redefining the lines of a network or seasonally to take into account some changes in passenger demand. This problem can thus be seen as a strategic as well as a tactical problem. According to the public transportation system first introduced by Ceder and Wilson in 1986 [1], the Frequency Setting Problem takes place after the Bus Network Design and before the Timetable Design. It consists in determining, for a given time period, the number of times buses pass on the lines, to satisfy a range of travelers. This problem is necessary to consider additional parameters into account such as heterogeneous bus fleet, authorized frequencies and capacity constraints for the fleet and the network. Regarding the satisfaction of travelers, [8] analyzed the perception of potential users about existing bus services in Delhi, India, and concluded most of people avoid using buses due to overloading, excessive travel time compared with a personal vehicle, the need to make a transfer, and lack of punctuality. The line frequencies have impact both on the operational costs of the operator and on the service-levels offered to passengers. Hence, the estimation of waiting times as a function of line frequencies is a crucial point to model service quality. [9] introduces a distinction between *short-headway lines* and *long-headway lines* with ten minutes as the bound and proposes an expected waiting

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time depending on this distinction. To do this, they propose an expected waiting time for short-headway lines equal to half the headway interval and an expected waiting time set at an arbitrary value for long-headway lines.

According to [4], there are two major types of approaches to solve the frequency setting problem. The first one consists in solving the problem without taking into account the choice of path made by the passengers according to the line frequencies. Among the relative works, we retain those of [6] where they assume a fixed demand-line assignment, as well as the work of [5] who determine the frequency and demand on each line considering discrete frequencies, non-captive vehicles, limited fleet size and with an objective of minimizing the travel time of all passengers. The second type of approach is based on a bilevel approach. The first level sets the bus line frequencies, which impact expected waiting times on passenger paths. At the second level, passengers decide on which path they prefer to reach their destination. Indeed, the frequency of a bus line influences the traveler's perception and choice of whether to use it or not. Thus, the use of a bilevel model makes it possible to determine line frequencies while taking into account passenger choice. Among the first papers dealing with a bilevel approach for the Frequency Setting Problem, we find [2] who define their upper-level as determining the frequencies which minimize the total expected travel and waiting times while their lower-level consists in a transit assignment problem.

In this paper, after having previously defined a set of bus lines to operate [3], we focus on determining the frequency of these bus lines, with the objective of minimizing operating costs considering service-level constraints and operational constraints. Furthermore, in order to better model traveler behavior, we introduce constraints that ensure that each traveler takes his fastest path according to the defined line frequencies. To solve this problem, we propose a Mixed Integer Linear Program based on a path formulation of passenger paths. This formulation captures the bilevel problem in a single stage but it has many variables and constraints. To make this model tractable, we propose a *Path Selection Process* denoted PSP. This process integrates several steps which dynamically select the passenger paths that are integrated in the model. Experiments show that PSP leads to qualitative solutions within a short solving time.

2 The Frequency Setting Problem

The Frequency Setting Problem (FSP) consists in determining the number of buses of each possible type on each line for a given operating period, in order to minimize the operating cost while satisfying passenger demand. In this problem, this demand is modeled by a time-dependent origin-destination matrix containing the number of passengers willing to travel from a station to another, for each time period. We study this problem at a tactical level, where operator expenses are detailed in terms of cumulative kilometric costs and cumulative driving times.

We consider a transportation network based on a graph $G = (V, A, \mathcal{L})$, composed of bus lines contained in a set $\mathcal{L}$, running on sequences of road sections represented by arcs contained in a set A. Each arc (i, j) connects a pair of stations in V. Each of these bus lines can be associated with different types of buses, all of them contained in a set $\mathcal{B}$. Each bus type is associated with an operating cost and a capacity. We assume that each arc $(i, j) \in A$ is associated to a distance and a travel time depending on the time period of the day. For each period, we consider capacity constraints on the overall number of buses of each type available at that period as well as on the number of buses traveling on each arc. To travel in the network, passengers use paths defined in a set $\mathcal{P}$. A passenger path is associated with a single OD pair and consists of a sequence of arcs, each associated to a line, on the network.

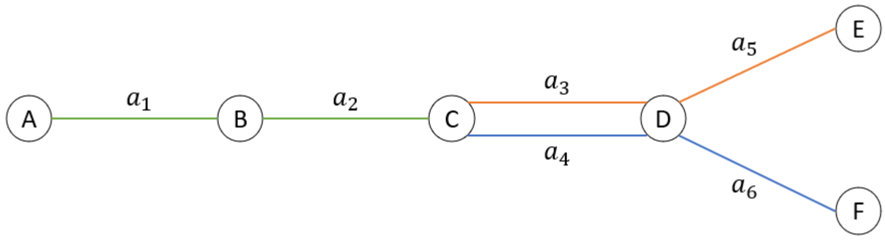


Figure 1 Network example with three bus lines $l_1 : (A, B, C)$, $l_2 : (C, D, E)$ and $l_3 : (C, D, F)$.

We consider passenger paths with 0 or 1 transfer. An exception is made for travelers who do not have a path with 0 or 1 transfer. In this case, a path with 2 transfers is proposed. Let us illustrate this on the small network of Figure (1). On this network, a passenger from B to D has two paths p and p' defined as : $p = [a_2, a_3]$ and $p' = [a_2, a_4]$. The path p takes lines l_1 and l_2 and the path p' takes lines l_1 and l_3.

We define the *traveling time* of a passenger path as the sum of the *riding times* on this path and the *expected waiting times* induced by boarding at the first stop of the path or at transfers. To estimate expected waiting times on passenger paths, we follow [9] and propose a different calculation mode depending on the line frequency: We set an expected waiting time equal to half the time between two buses for high frequency lines (more than six buses per hour). For low frequency lines (five buses per hour or less), the expected waiting time is set to five minutes. This constant expected waiting time for low-frequency lines has been chosen under two assumptions: (1) when the frequency of a line is low, each passenger selects carefully its bus departure time and arrives five minutes in advance at the station. (2) The synchronization of arrival and departure times for connections with a low-frequency line is usually done in a later step. Thus, setting a maximum expected waiting time of 5 minutes allows to anticipate this future synchronization. Passengers are supposed to always chose the path which has the minimum traveling time.

We assume that 100% of the demand must be satisfied. Passenger satisfaction is modeled with two criteria: (1) traveling time and (2) comfort of the trip. The traveling time criterion is based on the notion of *reference traveling time* of a passenger. This reference traveling time is typically estimated by the operator based on what should be expected by passengers according to their shortest possible riding time on the network or based on the actual performance of the existing network. A first service level constraint specify that a passenger path cannot be longer than a given percentage α^{max} ($> 100\%$) of the reference traveling time for the path OD. To model the comfort offered to passengers, we define a maximum bus filling percentage (τ_b) that set the operational bus capacity used in the model.

3 Solving the frequency setting problem

3.1 Bilevel model

To model the presented FSP, we propose a MILP which extends the model of [5] on four major aspects: (1) we consider the sum of operating costs as the objective and not as a constraint, (2) we integrate capacity constraints for the network and for an heterogeneous fleet, (3) we use a path-based model to represent the path chosen by passengers on the network, (4) we introduce a bi-level formulation in order to model the passenger assignment by enforcing each path assigned to an OD to be a shortest path for this OD in term of waiting and traveling times. This model is described in details in Appendix (A).

Table 1 Main notation used.

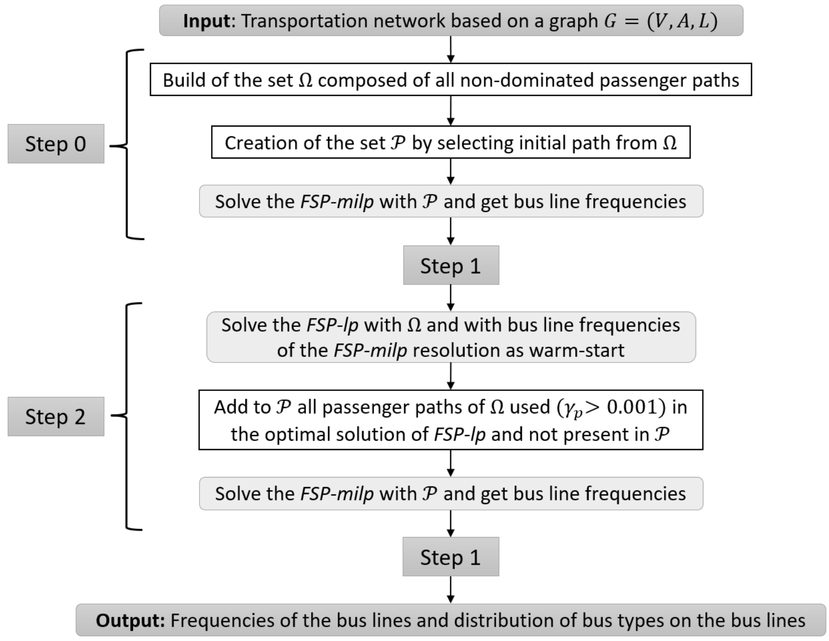
$G = (V, A, \mathcal{L})$	Graph representing the infrastructure of the network
V	Main stations
A	Road sections
$\mathcal{L}$	Set of bus lines that are operational
$\mathcal{B}$	All types of buses being operational
$\mathcal{P}$	Set of passenger paths
$\mathcal{L}_p$	Set of lines associated to passenger path $p \in \mathcal{P}$
$\mathcal{L}(a)$	Set of lines passing through the arc $a \in A$
$\mathcal{F}$	Set of frequencies authorized to be operated
$q^{\tau}_{[s,t]}$	Quantity of passenger demand from s to t during time period τ
$\Delta(\tau)$	Duration of the time period τ considered
o_b	Number of operable buses of type $b \in \mathcal{B}$
κ_b	Passenger capacity of buses of type $b \in \mathcal{B}$
$s_{a,b}$	Arc a saturation threshold for b-type buses
d_l	Round trip distance of the line $l \in \mathcal{L}$
t_l	Round trip time of the line $l \in \mathcal{L}$
t_p	Time of the passenger path $p \in \mathcal{P}$
$cost_b$	Operational cost per kilometer for buses of type $b \in \mathcal{B}$
$wage$	Hourly wage for bus drivers

For passenger paths, we let x_p be a binary variable being equal to one if a path $p \in \mathcal{P}$ is used. The variable γ_p is a continuous variable representing the percentage of the OD pair using path $p \in \mathcal{P}$ that satisfies $x_p = 1$, while w_p models the expected waiting time on path $p \in \mathcal{P}$ if it is used. For bus line variables, we let $y_{l,f}$ be a binary variable equal to one if bus line $l \in \mathcal{L}$ is assigned to frequency $f \in \mathcal{F}$ and $\psi_{l,b}$, an integer variable equal to the number of buses of type $b \in \mathcal{B}$ on line $l \in \mathcal{L}$.

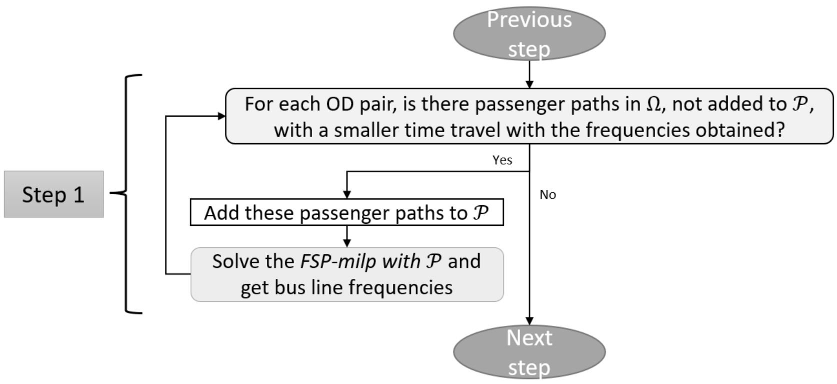
The bilevel problem is modeled with a single level reformulation using the optimal value function introduced by [7]. In our problem, this results in Constraint (13), called *shortest path constraint*. This constraint states that if a path $p \in \mathcal{P}$ is selected for an OD $[s, t]$, then it has to be at least as short as any other path from s to t.

3.2 General process

Given the number of paths and constraints to be added to the model, we propose an iterative method of path selection to accelerate the resolution of the model. Hence, we propose the *Path Selection Process (PSP)* summarized in Figure 2a. Step 0 generates an exhaustive set of passenger paths Ω, all compatible with the targeted service levels. The generation of a set Ω of passenger paths is based on two steps: a dominance step and the second being a filtering step. For this purpose, for each passenger path we introduce two notions: minimum traveling time and maximum traveling time. The minimum traveling time is defined as the sum of the riding time and the lowest possible expected waiting time at departure and at each transfer. The maximum traveling time is defined as the sum of the riding time and the highest possible expected waiting time at departure and at each transfer. The dominance step test if the minimum traveling time of a path p is lower than the maximum traveling time of all other paths p' relying the same origin-destination and with at most the same number of transfers. The filtering step removes path p whose minimum traveling time is



(a) Overview of the Path Selection Process (PSP) for solving the FSP.



(b) Zoom on the step 1 of the Path Selection Process (PSP).

strictly greater than $\alpha_{max}\times$ the reference traveling time of the OD pair. Then, the selection of initial paths among this set Ω to create a set $\mathcal{P}$ is performed by selecting for each OD pair, all direct paths and the path associated with the minimum traveling time. The *FSP-milp* is then solved with $\mathcal{P}$ by using CPLEX. Step 1 aims at building a feasible solution in which each OD is assigned to its shortest path. This step ensures that there is no shorter path that an OD should have taken. This is done by iteratively adding paths from Ω to $\mathcal{P}$ when these paths are shorter than those used in the solution of the *FSP-milp*. This results in a dynamic addition of shortest path constraints as soon as a passenger path from Ω is added to $\mathcal{P}$. Finally, Step 2 consists in solving the *FSP-lp*, a relaxed version of the *FSP-milp*, with the set of passenger paths Ω as input to select additional paths from Ω to be added to $\mathcal{P}$. The model is then solved again with a warm-start procedure based on the last integer solution obtained in Step 2. Finally, Step 1 is executed one more time to build a feasible solution, from the integer solution obtained in step 2, in which each OD can be assigned a path.

4 Numerical Results

Our experiences are based on a case study of the agglomeration of Poitiers, France, which has more than 130,000 inhabitants and is itself part of the ”Grand Poitiers” urban area (200,000 inhabitants). The data has been produced in collaboration with RTP (Régie des Transports Poitevin) the public transport operator of the ”Grand Poitiers” urban area. To carry out this study, a graph composed of 78 nodes and 106 edges has been defined, based on the existing network. Furthermore, based on a study of travelers' trips conducted by the RTP operator, we generate 15 one-hour OD matrices covering a typical operating day from 6am to 9pm. This case study is based on the optimization of the frequency of the 23 bus lines currently operated on four time periods. For each of these time periods we use PSP and evaluate the quality of the solution obtained compared to upper and lower bounds. To obtain upper and lower bounds, we run our FSP model for each time period with all passenger paths in the ω set generated in step 0 and solve it with CPLEX with a time limit of 10 hours. The models are implemented with Julia and solved by Cplex 20.1 through the JuMP interface on a DELL R440 1U server with a 2.1 GHz Intel Xeon 6230 CPU and 192GB of RAM. For our experiments, the set of bus line frequencies $\mathcal{F}$ is defined from 0 to 12 and the bus filling percentage τ_b is set to 20% for all type of buses. Furthermore, the α^{max} service-level parameter is set to 100%, enforcing each passenger to use a path with a traveling time at most equal to the reference traveling time for the path OD.

Table 2 The columns show the time period, the number of OD pairs and the number of passengers considered in the period, the upper and lower bounds and optimality gap obtained when solving FSP model with all passenger paths and finally the objective value, the gap to the best upper bound ub^* found and the solving time with PSP. All solving times are in seconds and (tl) means that the wall time has reached the maximum solving time.

Instance			FSP-milp with all passenger paths				PSP		
Time period	# OD	# Pass.	Up. b. ub^*	Lower bound	Opt. gap	Solv. time	Obj value	Gap to ub^*	Solv. time
6am-7am	1301	1503	17942	17047	4.99%	36000 (tl)	17942	0%	642
7am-8am	1770	4993	139553	32113	76.99%	36000 (tl)	39079	−72%	12632
8am-9am	1728	3640	172876	22025	87.26%	36000 (tl)	27229	−84%	2147
9am-10am	1526	2337	88922	17003	80.88%	36000 (tl)	18880	−79%	748
10am-11am	1442	1938	137585	15774	88.54%	36000 (tl)	17012	−88%	518
11am-12am	1562	2532	150290	17638	88.26%	36000 (tl)	20372	−86%	993
12am-1pm	1530	2917	143699	22825	84.12%	36000 (tl)	24435	−83%	908
1pm-2pm	1540	3018	113038	21299	81.16%	36000 (tl)	25434	−78%	1029
2pm-3pm	1614	2703	141884	17126	87.93%	36000 (tl)	19010	−87%	1274
3pm-4pm	1705	3392	51539	20362	60.49%	36000 (tl)	25224	−51%	3595
4pm-5pm	1929	4047	118728	22975	80.65%	36000 (tl)	26330	−78%	952
5pm-6pm	1941	5205	167713	25498	84.80%	36000 (tl)	33169	−80%	1812
6pm-7pm	1681	2596	19591	19591	0%	23973	19591	0%	700
7pm-8pm	1078	1123	16112	16112	0%	22217	16112	0%	223
8pm-9pm	818	617	12962	12962	0%	16467	12962	0%	71

From results in Table (2), we make several observations:

- Our PSP method stops in less than 1 hour for all time periods except $7am - 8am$ (3 hours and 30 minutes).

- For all time periods, the objective value obtained with our PSP method is less than or equal to the best upper bound ub^* of the FSP-milp found with all passenger paths.
- The mean deviation of the objective value obtained with our PSP method from the best upper bound ub^* found is equal to -58% and the median deviation is equal to -78%.
- For $6pm - 7pm$, $7pm - 8pm$ and $8pm - 9pm$ time periods, the solution obtained with PSP method is proven optimal by the resolution of the FSP-milp with all passenger paths.

5 Conclusion

We proposed a model and a heuristic for solving a practical application of the frequency setting problem. In this model, we minimize the operator cost while respecting service-levels for passengers. The problem is solved efficiently by a simple path selection process. Computational results were obtained on a case study and showed the relevance of our heuristic for public transport companies. These results can be used to refine frequencies on a network or to produce better cost and fleet estimations in a bus network design perspective.

References

1 Avishai Ceder and Nigel H.M. Wilson. Bus network design. *Transportation Research Part B: Methodological*, 20(4):331–344, August 1986. doi:10.1016/0191-2615(86)90047-0.

2 Isabelle Constantin and Michael Florian. Optimizing frequencies in a transit network: a nonlinear bi-level programming approach. *International Transactions in Operational Research*, 2(2):149–164, 1995. doi:10.1016/0969-6016(94)00023-M.

3 Hector Gatt, Jean-Marie Freche, Fabien Lehuédé, and Thomas G Yeung. A Column Generation-Based Heuristic for the Line Planning Problem with Service Levels. In *ATMOS 2021: International Symposium on Algorithmic Approaches for Transportation Modelling, Optimization, and Systems*, Lisbon, Portugal, September 2021. URL: https://hal.archives-ouvertes.fr/hal-03356433.

4 O.J. Ibarra-Rojas, F. Delgado, R. Giesen, and J.C. Muñoz. Planning, operation, and control of bus transport systems: A literature review. *Transportation Research Part B: Methodological*, 77:38–75, July 2015. doi:10.1016/j.trb.2015.03.002.

5 Héctor Martínez, Antonio Mauttone, and María E. Urquhart. Frequency optimization in public transportation systems: Formulation and metaheuristic approach. *European Journal of Operational Research*, 236(1):27–36, July 2014. doi:10.1016/j.ejor.2013.11.007.

6 G. F. Newell. Dispatching Policies for a Transportation Route. *Transportation Science*, 5(1):91–105, February 1971. Publisher: INFORMS. doi:10.1287/trsc.5.1.91.

7 M. Schmidt and Beck Y. A gentle and incomplete introduction to bilevel optimization, 2021. URL: www.optimization-online.org/DB_FILE/2021/06/8450.pdf.

8 Hemant Kumar Suman, Nomesh B. Bolia, and Geetam Tiwari. Perception of potential bus users and impact of feasible interventions to improve quality of bus services in Delhi. *Case Studies on Transport Policy*, 6(4):591–602, December 2018. doi:10.1016/j.cstp.2018.07.009.

9 Bin Yu, Zhongzhen Yang, and Jinbao Yao. Genetic Algorithm for Bus Frequency Optimization. *Journal of Transportation Engineering*, 136(6):576–583, June 2010. doi:10.1061/(ASCE)TE.1943-5436.0000119.

A FSP model

$$\min \quad \sum_{l\in\mathcal{L}}\sum_{b\in\mathcal{B}} \psi_{l,b} \times c_{l,b} \tag{1}$$

$$\text{s.t.} \quad \sum_{f\in\mathcal{F}} y_{l,f} = 1 \qquad \forall l \in \mathcal{L} \tag{2}$$

$$\sum_{b\in\mathcal{B}} \psi_{l,b} \geq \theta_f \times y_{l,f} \qquad \forall l \in \mathcal{L},\ f \in \mathcal{F} \tag{3}$$

$$\psi_{l,b} \leq \sum_{f\in\mathcal{F}} \theta_f \times y_{l,f} \qquad \forall l \in \mathcal{L},\ b \in \mathcal{B} \tag{4}$$

$$\sum_{p\in\mathcal{P}_{[s,t]}} \gamma_p = 1 \qquad \forall [s,t] \in OD^\tau \tag{5}$$

$$o_b \geq \sum_{l\in\mathcal{L}} \frac{t_l}{\Delta(\tau)} \times \psi_{l,b} \qquad \forall b \in \mathcal{B} \tag{6}$$

$$s_{a,b} \geq \sum_{l\in\mathcal{L}:a\in l} \psi_{l,b} \qquad \forall a \in A,\ b \in \mathcal{B} \tag{7}$$

$$\sum_{f\in\mathcal{F}} \theta_f \times y_{l,f} \geq \gamma_p \qquad \forall p \in \mathcal{P},\ l \in \mathcal{L}_p \tag{8}$$

$$\sum_{b\in\mathcal{B}} \tau_b \times \kappa_b \times \psi_{l,b} \geq \sum_{[s,t]\in OD^\tau} \sum_{\substack{p\in\mathcal{P}_{[s,t]}: \\ l\in\mathcal{L}_p(a)}} \gamma_p \times q^\tau_{[s,t]} \qquad \forall l \in \mathcal{L},\ a \in l \tag{9}$$

$$x_p \geq \gamma_p \qquad \forall p \in \mathcal{P} \tag{10}$$

$$rtt_{o(p),d(p)} \times \alpha^{max} \geq t_p \times x_p + w_p \qquad \forall p \in \mathcal{P} \tag{11}$$

$$w_p \geq \sum_{l\in\mathcal{L}_p}\sum_{f\in\mathcal{F}} wait_f \times y_{l,f} - (1 - x_p) \times 5 \times |\mathcal{L}_p| \qquad \forall p \in \mathcal{P} \tag{12}$$

$$t_{p'} + \sum_{l\in\mathcal{L}_{p'}}\sum_{f\in\mathcal{F}} wait_f \times y_{l,f} \geq x_p \times t_p + w_p \qquad \forall [s,t] \in OD^\tau,\ (p,p') \in \mathcal{P}_{[s,t]} \tag{13}$$

$$y_{l,f} \in \{0,1\} \qquad \forall l \in \mathcal{L},\ f \in \mathcal{F}$$

$$x_p \in \{0,1\} \qquad \forall p \in \mathcal{P}$$

$$\gamma_p \in [0,1] \qquad \forall p \in \mathcal{P}$$

$$w_p \in \mathbb{R}_+ \qquad \forall p \in \mathcal{P}$$

- The objective function (1), is defined as the sum of total operating costs. To do this, $c_{l,b}$ is defined equal to the sum of cumulative kilometric costs and cumulative driving times. Hence, $c_{l,b} = d_l \times cost_b + t_l \times \frac{wage}{60}$.
- Constraint (3) enforces each line to be associated to a exactly one frequency.
- Constraints (3) and (4) rely the frequencies of bus lines and the buses operating on them.
- Constraint (5) forces the totality of each OD pair demand to be dispatched on passenger paths satisfying them.
- Constraint (6) ensures the number of buses used for operation is available.
- Constraint (7) ensures the number of buses of each type driving on a road section during the period is lower than the saturation limit.
- Constraint (8) forces a passenger path used to have each of the associated lines being operated.
- Constraint (9) is used to integrate the bus filling percentage service-level parameter.
- Constraints (10), (11), (12) are used to integrate the α^{max} service-level parameter.
- Constraint (13) enforces a traveler to take its fastest possible path with chosen frequencies.