Brief Announcement: Distributed Algorithms for Minimum Dominating Set Problem and Beyond, a New Approach

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Abstract

In this paper, we study the minimum dominating set (MDS) problem and the minimum total dominating set (MTDS) problem. We propose a new idea to compute approximate MDS and MTDS. This new approach can be implemented in a distributed model or parallel model. We also show how to use this new approach in other related problems such as set cover problem and k-distance dominating set problem.

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1 Introduction

Let $G = (V, E)$ be an undirected graph with the vertex set V and the edge set E and without isolated vertex. We denote the set of adjacent vertices to a vertex v, neighbors of v, by $N(v)$. A set $S \subseteq V$ is a dominating set of G if each node $v \in V$ is either in S or has a neighbor in S. Also, $S \subseteq V$ is a total dominating set of G if each node $v \in V$ has a neighbor in S. Let $\gamma(G)$ and $\gamma_t(G)$ be the size of a minimum dominating set (MDS) and a minimum total dominating set (MTDS) of G, respectively. It is easy to prove that

$$\gamma(G) \leq \gamma_t(G) \leq 2\gamma(G).$$

Also, a subset of vertices such that each edge of the graph G is incident to at least one vertex of the subset is a vertex cover. A minimum vertex cover (MVC) of G is a vertex cover with the smallest possible number of vertices. The size of MVC is denoted by $\beta(G)$. A subset of the vertices such that no two vertices in the subset represent an edge of G is an independent set of G. An independent set with the largest possible number of vertices is called a maximum independent set (MIS). The size of MIS is denoted by $\alpha(G)$.

An interesting problem is computing the MDS in the distributed model, which we will consider in this paper. In a distributed model the network is abstracted as a simple n-node undirected graph $G = (V, E)$. There is a processor on each node $v \in V$, with a unique $\Theta(\log n)$-bit identifier $ID(v)$, who initially knows only its neighbors in G. Communication happens in synchronous rounds. Per round, each node can send one, possibly different, $O(\log n)$-bit message to each of its neighbors. Ultimately, each node should know its own

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part of the output. When computing the dominating set, each node knows whether it is in the dominating set or has a neighbor in the dominating set [2]. When the size of the messages is restricted to be $O(\log n)$, then the algorithm is CONGEST.

2 Theoretical result and the algorithm

For a given graph G, with no isolated vertex, we construct a graph G' with the same set of vertices as in G as follows. For each vertex v of degree at least two, we choose two of its neighbors arbitrarily and add an edge between them in G'. If v is of degree one, we add a loop edge on its neighbor. We call this edge the corresponding edge of v in G' and denote it by e_v. Note that if the graph G has a cycle of length 4, with vertices a, b, c, d then the edge bd can be the corresponding edge of both a and c in G'. Let $\alpha(G)$ and $\beta(G)$ be the size of a maximum independent set and the size of a minimum vertex cover of G, respectively (as defined previously). Then, we have the following theorem.

▶ **Theorem 1.** $\gamma_t(G) \leq n - \alpha(G') = \beta(G')$.

Proof. Suppose that D is a maximum independent set of G', so $|D| = \alpha(G')$. We show that $V \backslash D$ is a total dominating set for G. For each vertex v, we choose two of its neighbors, for example, u and w, and add an edge between them in G'. Since there is an edge between u and w, at most, one of them can be in D, which means at least one of them is in $V \backslash D$. The same argument applies when an edge is a loop. Thus, for each vertex v, at least one of its neighbors in G is in $V \backslash D$, so $V \backslash D$ is a total dominating set for G. The size of $|V \backslash D|$ equals $n - \alpha(G')$ and we have $\gamma_t(G) \leq n - \alpha(G') = \beta(G')$. ◀

Note that the graph G' can be constructed is a constant number of rounds in the CONGEST model. According to our time and space constraints, we can use the known distributed algorithms for computing a vertex cover for G' (See [3, 4]).

3 Extension to the other problems

Set cover problem

In the set cover problem we are given a set $A = \{a_1, a_2, \dots a_n\}$ of n elements and m subsets, $A_1, A_2, \dots, A_m$ of A. The goal is to choose the minimum number of subsets that cover all the elements of A.

Our algorithm is as follows. Each element a_i chooses a subset A_j with maximum size such that $a_i \in A_j$. Let x_i be the number of times that A_i is chosen by the elements of A. We construct a graph G' that its vertices are the subsets $A_1, A_2, \dots, A_m$. For each $a \in A$ we choose two subsets A_i and A_j with maximum values of x_i's such that $a \in A_i$ and $a \in A_j$ and add an edge between them. Similar to the proof of Theorem 1, it can be shown that a vertex cover for G' is a set cover for A.

k-distance dominating set

An extension of the MDS problem is the minimum k-distance dominating set problem where the goal is to choose a subset $S \subseteq V$ with the minimum cardinality such that for every vertex $v \in V \backslash S$, there is a vertex $u \in S$ where the shortest path between them at most k. The minimum total k-distance dominating set is defined similarly. A k-observer Ob of a network N is a set of nodes in N such that each message, that travels at least k hops in N, is handled (and so observed) by at least one node in Ob. A k-observer Ob of a network N is minimum iff the number of nodes in Ob is less than or equal to the number of nodes in every k-observer of N (See [1]). This problem is equivalent to the k-distance dominating set problem. In this problem for each node v, the neighbors of v, is the set of nodes whose distance from v is less than $k+1$. Then we apply the proposed algorithms as before.

Note that computing a minimum k-distance dominating set for a graph G is equivalent to computing a minimum dominating set for G^k, where G^k is a graph with the same vertex set as G and we put an edge between two vertices in G^k if the distance between them in G is less than $k+1$.

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