Brief Announcement: Minimizing Congestion in Hybrid Demand-Aware Network Topologies

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Abstract

Emerging reconfigurable optical communication technologies enable demand-aware networks: networks whose static topology can be enhanced with demand-aware links optimized towards the traffic pattern the network serves. This paper studies the algorithmic problem of how to jointly optimize the topology and the routing in such demand-aware networks, to minimize congestion. We investigate this problem along two dimensions: (1) whether flows are splittable or unsplittable, and (2) whether routing on the hybrid topology is segregated or not, i.e., whether or not flows either have to use exclusively either the static network or the demand-aware connections. For splittable and segregated routing, we show that the problem is 2-approximable in general, but APX-hard even for uniform demands induced by a bipartite demand graph. For unsplittable and segregated routing, we show an upper bound of $O(\log m/\log\log m)$ and a lower bound of $\Omega(\log m/\log\log m)$ for polynomial-time approximation algorithms, where m is the number of static links. Under splittable (resp., unsplittable) and non-segregated routing, even for demands of a single source (resp., destination), the problem cannot be approximated better than $\Omega(c_{\max}/c_{\min})$ unless P=NP, where $c_{\max}$ (resp., $c_{\min}$) denotes the maximum (resp., minimum) capacity. It is still NP-hard for uniform capacities, but can be solved efficiently for a single commodity and uniform capacities.

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1 Introduction

Emerging demand-aware networks, whose topologies are typically hybrid, in that a static (and demand-oblivious) network is enhanced with reconfigurable (and demand-aware) links, introduce unprecedented flexibility in adapting the network topology towards current traffic demands. In such hybrid networks, the reconfigurable links are usually enabled by optical circuit switches [1, 8, 13], and particularly, each optical circuit switch provides reconfigurable links by establishing connections between pairs of its ports, i.e., a matching.

Extensive past works studied the question of how to jointly optimize topology and routing of such hybrid (reconfigurable) networks [17] for different networking performance metrics, e.g., latency [11], throughput [4, 7], routing length [14, 15, 16], flow times [3] etc. Interestingly, *min-congestion*, a most central performance metric in traditional networks, is still not well-understood in hybrid networks. Avin et al. [6] and Pacut et al. [12] study optimal

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Table 1 Summary of our approximation upper and lower bounds on the MCHN problem (Def. 1).

Approximation Upper & Lower Bounds (Complexity)	Splittable Flow	Segregated Routing	Restrictions On Demands
2-approximation	yes	yes	
APX-complete	yes	yes	uniform and bipartite demands
$O(\log m / \log\log m)$-approximation	no	yes	
Lower Bound: $\Omega(\log m / \log\log m)$	no	both	
$(2 \cdot c_{\max}/c_{\min})$-approximation	yes	no	single source (resp., dest.)
Lower Bound: $\Omega(c_{\max}/c_{\min})$	both	no	single source (resp., dest.)

bounded-degree topology designs to minimize both the route length and the congestion. Dai et al. [2] worked on the same network model as us, showing that the problem is already NP-hard for *splittable* (resp., *unsplittable*) and *segregated* (resp., *non-segregated*) routing models when the static network is a tree of height at least two, but tractable for static networks of star topologies. Zheng et al. [10] introduced a greedy-based heuristic algorithm for our *segregated* model but on specific topologies of datacenters. However, not much more is known w.r.t. corresponding approximation bounds, which motivates our study, summarized in Table 1.

2 Model

Network Model. We consider a *hybrid* network [5, 9] $N = (V, E, \mathcal{E}, c)$, where a *static* network (V, E) is represented by an bidirected (simple) graph of nodes V, any two distinct nodes $v_i, v_j \in V$ imply a possible *reconfigurable link* denoted by a bidirected edge $\{i, j\}$ in $\mathcal{E}$, and a function $c : \vec{E} \cup \vec{\mathcal{E}} \mapsto \mathbb{R}_{\geq 0}$ defines *capacities* for both directions of each bidirected link in $E \cup \mathcal{E}$ with the *maximum* (resp., *minimum*) *capacity* denoted by $c_{\max}$ (resp., $c_{\min}$). The hybrid network N must decide a matching $M \subseteq \mathcal{E}$ to obtain an enhanced graph $N(M) = (V, E \cup M, c)$, which determines the actual topology of the communicating network.

Traffic Demands. A certain communication pattern (*demands*) on nodes V is represented by a matrix $D := (d_{i,j})_{|V|\times|V|}$, where an entry $d_{i,j} \in \mathbb{R}_{\geq 0}$ denotes the traffic load (frequency) or a demand from the node $v_i \in V$ to the node $v_j \in V$.

Routing Models. The *unsplittable* routing requires that flows of each demand must be sent along a single (directed) path, otherwise the routing is called *splittable*. For a hybrid network, *segregated* routing requires that each demand $d_{i,j}$ is either sent on the reconfigurable link $\{i, j\}$, if it exists, or purely on the static network, otherwise it is *unsegregated* routing. Hence, we consider *four different routing models*: *Unsplittable & Segregated (US)*, *Unsplittable & Non-segregated (UN)*, *Splittable & Segregated (SS)*, and *Splittable & Non-segregated (SN)*.

▶ **Definition 1** (Min-Congestion Hybrid Network Problem (MCHN)). *Given a hybrid network $N = (V, E, \mathcal{E}, c)$, a routing model $\tau \in \{US, UN, SS, SN\}$, and a demand matrix D, find a matching $M \subseteq \mathcal{E}$, s.t., the congestion λ, i.e., the maximum load on $\vec{E} \cup \vec{M}$, to serve D in $N(M)$ is minimized.*

3 Our Contributions

We initiate the study of approximation algorithms for minimizing congestion in hybrid demand-aware networks (for a given matrix of demands). Our results include an overview of approximation results and complexity characterizations in general settings. We also provide a fine-grained algorithmic analysis for restricted cases:

Segregated Routing. We can give a mixed-integer programming formulation for segregated and un-/splittable flow models whose LP relaxation can be solved efficiently. For splittable flows, we provide a 2-approximation algorithm by a novel deterministic rounding approach, and also prove APX-hardness even if demands are uniform and bipartite. However, we also show that the problem becomes tractable for demands with a single source (resp., dest.). For unsplittable flows, we show that the hybrid network problem cannot be approximated better than the min-congestion multi-commodity unsplittable flow problem (MCMF) [18], but any ρ-approximation algorithm based on rounding techniques for the MCMF problem can be utilized to give a 2ρ-approximation. This implies an approximability of $\Theta(\log m / \log\log m)$ for segregated and unsplittable routing, where $m = |E|$.

Non-Segregated Routing. Under the splittable (resp., unsplittable) flow model, even for demands of a single source (resp., destination), the problem cannot be approximated better than $\Omega(c_{\max}/c_{\min})$ unless P=NP, but still $(2 \cdot c_{\max}/c_{\min})$-approximable for the splittable flow, where $c_{\max}$ (resp., $c_{\min}$) denotes the maximum (resp., minimum) capacity on all links, and it still remains NP-hard for *uniform capacities*, i.e., $c : \vec{E} \cup \vec{\mathcal{E}} \mapsto \{a\}$ for $a \in \mathbb{R}_{>0}$. However, the problem with uniform capacities becomes efficiently solvable for demands of a single commodity under un-/splittable flow.

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