Gathering of Mobile Robots with Defected Views*

Yonghwan Kim
Nagoya Institute of Technology, Aichi, Japan

Masahiro Shibata
Kyushu Institute of Technology, Fukuoka, Japan

Yuichi Sudo
Hosei University, Tokyo, Japan

Junya Nakamura
Toyohashi University of Technology, Aichi, Japan

Yoshiaki Katayama
Nagoya Institute of Technology, Aichi, Japan

Toshimitsu Masuzawa
Osaka University, Japan

Abstract

An autonomous mobile robot system consisting of many mobile computational entities (called *robots*) attracts much attention of researchers, and it is an emerging issue for a recent couple of decades to clarify the relation between the capabilities of robots and solvability of the problems.

Generally, each robot can observe all other robots as long as there are no restrictions on visibility range or obstructions, regardless of the number of robots. In this paper, we provide a new perspective on the observation by robots; a robot cannot necessarily observe all other robots regardless of distances to them. We call this new computational model the *defected view model*. Under this model, in this paper, we consider the *gathering* problem that requires all the robots to gather at the same non-predetermined point and propose two algorithms to solve the gathering problem in the adversarial (N,$N-2$)-defected model for $N \geq 5$ (where each robot observes at most $N-2$ robots chosen adversarially) and the distance-based (4,2)-defected model (where each robot observes at most two robots closest to itself), respectively, where N is the number of robots. Moreover, we present an impossibility result showing that there is no (deterministic) gathering algorithm in the adversarial or distance-based (3,1)-defected model, and we also show an impossibility result for the gathering in a relaxed (N, $N-2$)-defected model.

2012 ACM Subject Classification Theory of computation → Self-organization

Keywords and phrases mobile robot, gathering, defected view model

Digital Object Identifier 10.4230/LIPIcs.OPODIS.2022.14

Related Version *Previous Version*: `https://doi.org/10.4230/LIPIcs.DISC.2022.46`

Funding This work was supported in part by JSPS KAKENHI Grant Numbers 18K18031, 19H04085, 19K11823, 20H04140, 20KK0232, 21K17706, 22K11971, and Foundation of Public Interest of Tatematsu.

1 Introduction

An autonomous mobile robot system is a distributed system consisting of many mobile computational entities (called *robots*) with limited capabilities, e.g., robots cannot distinguish other robots, or cannot remember their any past actions. The robots operate autonomously

* A part of this paper was presented in *the 36th International Symposium on DIStributed Computing (DISC 2022)* as a brief announcement.

26th International Conference on Principles of Distributed Systems (OPODIS 2022).
Editors: Eshcar Hillel, Roberto Palmieri, and Etienne Rivière; Article No. 14; pp. 14:1–14:18
Leibniz International Proceedings in Informatics
LIPICS Schloss Dagstuhl – Leibniz-Zentrum für Informatik, Dagstuhl Publishing, Germany

and cooperatively; each robot observes the other robots (*Look*), computes the destination (*Compute*), and moves to the destination (*Move*). Each robot autonomously and cyclically performs the above operations to achieve the given common goal. Since an autonomous mobile robot system has been firstly introduced in [21], the literature [17, 18, 19, 21] provides a formal discussion on the capabilities of the robots for the distributed coordination (e.g., gathering, scattering, or pattern formation), and many researchers are interested in clarifying the relationship between the capabilities of the robots and solvability of the problems.

Generally, in *Look* operation, each robot can observe all other robots (within its visibility range if the range is limited). In other words, each robot can take a snapshot consisting of all other robots' (relative) positions in its *Look* operation, i.e., each robot temporarily remembers the positions of up to $N-1$ robots, where N is the total number of robots. From the practical viewpoint, we claim that a robot with low functionality may not have such large working memory. That is, the main question we address in this paper is "*what occurs if a robot cannot observe some of the other robots?*". More precisely, "*how many other robots should be observed to achieve the goals of the problems?*".

Related works. The gathering problem [16], which requires all the robots to move to a common (non-predetermined) position, is a fundamental problem for autonomous mobile robot systems. There are many studies about the gathering of autonomous mobile robots; Cieliebak et al. presented the first algorithm to achieve the gathering from any arbitrary configuration [4], Klasing et al. studied the gathering of mobile robots in one node of an anonymous unoriented ring [13], D'Angelo et al. introduced a gathering algorithm of robots without multiplicity detection on grids and trees [5], and many works for the gathering of robots with dynamic (or inaccurate) compasses are also introduced [9, 10, 11, 20]. The capability of the robots deeply affects the solvability of the gathering problem, thus some investigations about the required capability or impossibility are introduced [16, 17]. However, all of these works assume that each robot can observe all other robots within its visibility range if there is no obstruction (e.g., any opaque robot) between the robots.

The works most related to this paper are those with the limited visibility range [1, 3, 7, 12, 14]. The robots with the limited visibility cannot necessarily observe all robots, which is similar to the defected view model we propose. But visibility is limited by distance in the limited visibility model and thus all robots can be observed when they gather closely enough. On the other hand, the defected view model cannot guarantee such a full view of the robots. As another well-related work, Heriban et al. studied some problems of robots with uncertain visibility sensors [8]: if the distance between two robots is longer than the visibility range, the two robots adversarially observe each other. However, also in this study, every robot can observe all other robots within the visibility range regardless of the number of robots. The works for fault-tolerance [2, 6, 15] are also closely related to this paper. The defected view can be considered as a new type of fault in autonomous mobile robot systems.

Contribution. To provide some answers for the above research questions, we propose a new computational model with restriction on the number of robots that each robot can observe, named the *defected view model*, where each robot observes only k other robots for $1 \le k < N-1$. This assumption naturally arises by considering some issues for robots with low functionality such as (1) each robot does not have enough working memory to store the entire observation result, (2) each robot may miss some of observation results due to memory failure, or (3) each robot fails to observe some of other robots by sensing failure. It is obvious that when k becomes the lower, the problem becomes the harder (possibly impossible) to solve. We consider two different defected view models regarding which k robots are observed: the adversarial (N,k)-defected model and the distance-based (N,k)-defected model. In the former, each robot observes the other k robots determined adversarially, and in the latter, each robot observes the other k robots closest to its current position.

More precisely, the k robots that each robot r can observe are chosen from the robots located at points different from r's current position. Concerning r's current position, r can detect only whether another robot exists at the point or not (so called the *weak multiplicity detection*). Such an assumption that the robots at r's current position are excluded from the candidates of the observed k robots is motivated by the following observation: each robot r observes the robots at remote points and those at r's current position by different ways usually. Each robot observes the remote robots by, for example, a radar sensor or a vision sensor, but senses the other robots at the same point by, for example, a contact sensor.

As the first step of the gathering in the defected view model, we investigate only the case of $k = N - 2$. The main contributions of this paper are as follows: (1) we propose a gathering algorithm in the adversarial (N,$N - 2$)-defected model for any $N \geq 5$, (2) we present another algorithm to solve the gathering problem in the distance-based (4,2)-defected model, and (3) we provide the impossibility result showing that there is no (deterministic) algorithm to solve the gathering problem in the adversarial or distance-based (3,1)-defected model. Moreover, we present another impossibility result in a naturally relaxed (N,$N - 2$)-defected model where the observed k robots can contain the robots at the observer's current position. This impossibility result shows the necessity of the assumption that the observed $N - 2$ robots should be chosen from robots other than those located at the observer's current position.

The rest of this paper is organized as follows: Section 2 presents the system model (including two defected view models) and problem definition; Section 3 introduces an algorithm to solve the gathering problem in the adversarial (N, $N - 2$)-defected model for any $N \geq 5$; Section 4 gives a gathering algorithm in the distance-based (4,2)-defected model; Section 5 shows two impossibility results showing that there is no algorithm in the adversarial or distance-based (3,1)-defected model and the relaxed adversarial (N,$N - 2$)-defected model; and Section 6 concludes the paper and provides some open problems.

2 Model and Problem Definition

2.1 Robots

Let $R = \{r_1, r_2, ..., r_N\}$ be the set of N autonomous mobile robots deployed in a plane. Robots are indistinguishable by their appearance (i.e., identical), execute the same algorithm (i.e., uniform or homogeneous), and have no memory (i.e., oblivious). There is no geometrical agreement; robots do not agree on any axis, the unit distance, or chirality. A point in the plane is called an *occupied point* if there exists a robot at the point. We allow two or more robots to occupy the same point at the same time. We call a robot a *single robot* if the point occupied by the robot has no other robot. Otherwise, we call it an *accompanied robot*.

Each robot cyclically performs the three operations, *Look*, *Compute*, and *Move*: (***Look***) a robot obtains the positions (based on its local coordinate system centered on itself) of all other observed robots, (***Compute***) a robot determines the destination according to the given algorithm based on the result of *Look* operation. Since each robot has no memory, the result of *Compute* is determined only by the result of *Look* operation, and (***Move***) a robot moves to the destination computed in *Compute* operation. We assume *rigid movement* which ensures each robot can reach the destination during its *Move* operation, i.e., a robot never stops before it reaches its destination.

2.2 Schedule and Configuration

We assume a fully-synchronous scheduler (FSYNC): all robots fully-synchronously perform their operations (*Look*, *Compute*, and *Move*). This means that all robots perform the same operation at the same time instant and duration. We call the time duration in which all robots perform the three operations (*Look*, *Compute*, and *Move*) once *a round*.

Let *configuration* C_t be the set of the (global) coordinates of all robots at a given time t: $C_t = \{(r^t_{1.x}, r^t_{1.y}), (r^t_{2.x}, r^t_{2.y}) \dots (r^t_{N.x}, r^t_{N.y})\}$, where $r^t_{i.x}$ (resp. $r^t_{i.y}$) is the X-coordinate (resp. Y-coordinate) of robot r_i at time t. Note that no robot knows its global coordinate. Configuration C_t is changed into another configuration C_{t+1} after one round (i.e., all robots execute the three operations once).

2.3 Observation: Visibility Range and Multiplicity Detection

We basically assume that every robot has unlimited visibility range, i.e., any two robots can observe each other regardless of their distance, while we introduce in Definition 1 the defect in the information obtained by *Look* operation. Moreover, we assume a *weak multiplicity detection*, i.e., each robot cannot get the exact number of robots occupying the same point but can distinguish whether the point is occupied by one robot or by multiple robots. This implies that when each robot observes any point, it can distinguish the three cases: there is *no robot*, *one robot*, or *two or more robots* at the point.

We consider a *defected view* such that each robot may not observe all other robots. We define the (N,k)-defected model, where $1 \le k < N$ as follows:

▶ **Definition 1** ((N,k)-defected model)**.** *Each robot r can get from* Look *operation the set of occupied points (in its coordinate system) where k robots not accompanied with r are located (i.e., the k robots contains no robot located at r's current point). When the number of robots not accompanied with r is k or less, all such robots are observed. The weak multiplicity detection concerning the k robots is assumed: a point occupied by only one of the k robots can be distinguished from that occupied by two or more of the k robots.*

Note that the $(N,N-1)$-defected model is equivalent to the commonly used model (with the weak multiplicity detection) where each robot can observe all robots. The (N,k)-defected model has options depending on how the observed k robots are chosen. We consider the two options in this paper, named *adversarial (N,k)-defected model* and *distance-based (N,k)-defected model*. In the *adversarial (N,k)-defected model*, k robots observed by each robot are determined adversarially. In the *distance-based (N,k)-defected model*, each robot r observes the k closest robots to the r's current point. Tie break among the robots the same distance apart is determined in an arbitrary way.

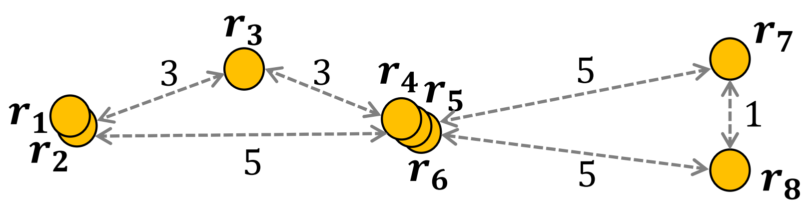


Figure 1 An example configuration by 8 robots.

To help to understand, we explain the model using examples. Figure 1 illustrates an example configuration by 8 robots; $R = \{r_1, r_2, \ldots, r_8\}$. Robots r_1 and r_2 (resp. r_4, r_5 and r_6) are accompanied, and the other robots are single. The dotted arrow between robots represents the distance between the points occupied by the robots. Let p_i denote the point occupied by robot r_i. Now we explain the models as follows:

- **The adversarial (8,4)-defected model.** In this model, each robot observes 4 other robots chosen adversarially. Assume that robot r_3 observes 4 robots, r_1, r_2, r_6, and r_8. In this case, robot r_3 gets a set of points $P^{r_3} = \{p_1^*, p_3, p_6, p_8\}$ including point p_3 occupied by r_3 itself, where p_i^* denotes that p_i is recognized to be occupied by two or more robots. Robot r_3 knows that two or more robots exist at point p_1 because both robots r_1 and r_2 are chosen, however, r_3 does not know that there is another robot at p_6 because it observes only r_6 at p_6. Robot r_4 (or r_5, r_6) observes 4 robots among 5 robots, r_1, r_2, r_3, r_7, and r_8. If robots r_1, r_3, r_7, and r_8 are chosen, $P^{r_4} = \{p_1, p_3, p_4^*, p_7, p_8\}$ holds, which means that r_4 observes all points, however, it does not know that another robot exists at p_1 (and the other points except for p_4^*). Robot r_4 can know that point p_4 is occupied by another robot other than itself. If robot r_4 observes robots r_1, r_2, r_7, and r_8, $P^{r_4} = \{p_1^*, p_4^*, p_7, p_8\}$ holds, which means that robot r_4 knows there exist two or more robots at p_1, but it cannot observe point p_3 occupied by robot r_3. Notice that r_5 and r_6 located at p_4 are allowed to observe the set of points different from those observed by r_4.
- **The distance-based (8,3)-defected model.** In this model, each robot observes 3 closest robots to itself. Robot r_7 observes 3 robots, r_8 (the closest one) and two robots among three robots at point p_4, thus $P^{r_7} = \{p_4^*, p_7, p_8\}$ always holds. Robot r_4 observes robot r_3 (the closest one) and two robots among 4 robots, r_1, r_2, r_7, and r_8, which are the same distance apart. Note that the observed robots are determined in an arbitrary way, thus in this case, P^{r_4} becomes one among $\{p_1^*, p_3, p_4^*\}$, $\{p_1, p_3, p_4^*, p_7\}$, $\{p_1, p_3, p_4^*, p_8\}$, or $\{p_3, p_4^*, p_7, p_8\}$.

It is obvious that the adversarial (N,k)-defected model is weaker[1] than the distance-based one, that is, any algorithm to achieve the gathering in the adversarial (N,k)-defected model works correctly also in the distance-based (N,k)-defected model.

2.4 Problem Definition: Gathering

We define the gathering problem as follows.

▶ **Definition 2** (The Gathering Problem). *Given a set of N robots located at arbitrary points. Algorithm $\mathcal{A}$ solves the gathering problem if $\mathcal{A}$ satisfies all the following conditions:*

(1) *algorithm $\mathcal{A}$ eventually reaches a configuration such that no robot can move, and*

(2) *when the algorithm $\mathcal{A}$ terminates, all the robots are located at the same point.*

[1] Strictly speaking, we do not know the adversarial (N,k)-defected model is properly weaker than the distance-based one yet; it is obvious that the adversarial (N,k)-defected model is NOT stronger than the distance-based one.

3 Algorithm in the Adversarial (N,$N-2$)-defected Model where $N \geq 5$

Algorithm 1 presents an algorithm for robot r_i to achieve the gathering in the adversarial (N, $N-2$)-defected model where $N \geq 5$. We use two functions defined as follows:

- OPSET() : a function that returns a set of points $\{p \,|\, p$ is occupied by r_i or by the robots that r_i observed$\}$
- isMulti(p) : a function that returns TRUE if point p is occupied by two or more robots that r_i observed (weak multiplicity), otherwise FALSE.

The algorithm adopts, as the destination of robot r_i, the center of the smallest enclosing circle (SEC) of the occupied points that r_i observed in the *Look* operation. Before proving the correctness of the algorithm, we show some fundamental properties of the SEC of points in a plane.

Algorithm 1 Gathering algorithm in the adversarial $(N, N-2)$-defected model where $N \geq 5$.

```
1: if ∀p ∈ OPSET() : isMulti(p) = TRUE then
2:     move to the center of the smallest enclosing circle of OPSET()
3: else if (r_i is single) ∧ (∃p ∈ OPSET() : isMulti(p) = TRUE) then
4:     move to an arbitrary point p ∈ OPSET() such that isMulti(p) = TRUE
5: else if ∀p ∈ OPSET() : isMulti(p) = FALSE then
6:     move to the center of the smallest enclosing circle of OPSET()
7: end if        ▷ No action if (r_i is accompanied) ∧ (∃p ∈ OPSET() : isMulti(p) = FALSE)
```

▶ **Proposition 3.** *Let P be a set of n distinct points in a plane and C be the SEC of P. The following properties hold.*

1. *The SEC of P is unique.*
2. *Let $p \in P$ be any point (if exists) properly inside C, C is the SEC of $P \setminus \{p\}$.*
3. *When there exist three points $p_1, p_2, p_3 \in P$ on the boundary of C that form an acute or right triangle, C is the SEC of $\{p_1, p_2, p_3\}$.*
4. *When three or more points in P are on the boundary of C, there exist three points $p_1, p_2, p_3 \in P$ on the boundary of C that form an acute or right triangle.* ◀

A key property of the (N, $N-2$)-defected model used in the following proofs is that *any accompanied robot can observe all the robots* (but only with the weak multiplicity detection).

▶ **Lemma 4.** *In the adversarial (N, $N-2$)-defected model ($N \geq 5$), Algorithm 1 solves the gathering problem in two rounds from any configuration where there exist three or more accompanied robots.*

Proof. When every robot is accompanied, each robot detects all the occupied points in the *Look* operation and recognizes that each of them is occupied by multiple robots. Every robot moves to the center of the SEC of all the occupied points (by lines 1 and 2 in Algorithm 1) and thus the gathering is achieved in one round.

When there exists a single robot r, every accompanied robot observes r and does not move (see line 7 in Algorithm 1). Every single robot misses at most one accompanied robot in its *Look* operation and can detect at least one point occupied by multiple robots: a point occupied by three or more robots (if exists) or one of the points each occupied by two robots. Each single robot moves to one of such points (by lines 3 and 4 in Algorithm 1), which results in the configuration where every robot is accompanied. Thus the gathering is achieved in the next round as shown above. ◀

Notice that Lemma 4 holds for $N \geq 3$.

▶ **Lemma 5.** *In the adversarial $(N, N-2)$-defected model $(N \geq 5)$, Algorithm 1 solves the gathering problem in two rounds from any configuration where there exist only two accompanied robots.*

Proof. Let r_1 and r_2 be the two accompanied robots. Robots r_1 and r_2 observe all robots and recognize that single robots exist, which makes r_1 and r_2 stay at the current point.

Now consider actions of single robots. A single robot r misses one robot in its *Look* operation, which implies that r observes (a) both r_1 and r_2 or (b) only one of r_1 and r_2. In case (a), r moves to the point, say p_a, occupied by r_1 and r_2. In case (b), r moves to the center, say p_b, of the SEC of all the occupied points. Thus after one round, all the robots are located at p_a or p_b. Note that p_a is occupied by multiple robots including r_1 and r_2.

When p_b is not occupied by any robot, the gathering is already achieved. When p_b is occupied by multiple robots, the robots at p_b observe all the robots. Thus, all the robots move to the center of the SEC of p_a and p_b (or the midpoint of p_a and p_b) in the next round (by lines 1 and 2 in Algorithm 1), which achieves the gathering. When p_b is occupied by only one robot r, r detects that p_a is occupied by multiple robots and moves to p_a in the next round (by lines 3 and 4 in Algorithm 1) while the robots at p_a recognize that p_b is occupied by only one robot and does not move (see line 7 in Algorithm 1). Thus, the gathering is achieved. ◀

Notice that Lemma 5 holds for $N \geq 4$.

▶ **Lemma 6.** *In the adversarial $(N, N-2)$-defected model $(N \geq 5)$, Algorithm 1 solves the gathering problem in three rounds from any configuration where all robots are single.*

Proof. Each robot misses one robot in its *Look* operation. When there exist two robots r_1 and r_2 that miss the same robot, r_1 and r_2 get the same point set $\mathsf{OPSET}()$ and moves to the center of the SEC of $\mathsf{OPSET}()$ (by lines 5 and 6 in Algorithm 1). From Lemmas 4 and 5, two additional rounds are enough to achieve the gathering.

When no two robots miss the same robot, for any pair of two distinct robots r_1 and r_2, the robot missing r_1 is different from the robot missing r_2. Let C be the SEC of all the occupied N points. First, consider the case that two (or more) robots r_a and r_b are located properly inside C. The SEC of $R \setminus \{r_a\}$ is equal to the SEC of $R \setminus \{r_b\}$ (that is C from the second property of Proposition 3), which implies that the two robots observing $R \setminus \{r_a\}$ and $R \setminus \{r_b\}$ move to the same point (or the center of the SEC). From Lemmas 4 and 5, two additional rounds are enough to achieve the gathering.

Second, consider the case that $N-1$ or N robots are on the boundary of C. From the last property of Proposition 3, there exist three robots r_1, r_2, r_3 on the boundary of C that form an acute or right triangle. There exist two robots r_4 and r_5 other than r_1, r_2, r_3 from $N \geq 5$. Both the robots observing $R \setminus \{r_4\}$ and $R \setminus \{r_5\}$ observe all of r_1, r_2, r_3. The third property of Proposition 3 implies that the two robots find the same SEC (or the SEC of r_1, r_2, r_3), which implies that they move to the same point (or the center of the SEC) (by lines 5 and 6 in Algorithm 1). From Lemmas 4 and 5, two additional rounds are enough to achieve the gathering. ◀

From Lemmas 4, 5 and 6, the following theorem holds.

▶ **Theorem 7.** *In the adversarial $(N, N-2)$-defected model $(N \geq 5)$, Algorithm 1 solves the gathering problem in three rounds.* ◀

Algorithm 1 cannot solve the gathering problem for the case of $N = 4$. Assume that four robots, $R = \{r_0, r_1, r_2, r_3\}$. Three robots r_1, r_2 and r_3 are deployed to form an equilateral triangle as Figure 11 and r_0 is located at the center of the triangle (i.e., point p_c in Figure 11). Consider the case that r_i observes $r_{(i+1) \bmod 3}$ and $r_{(i+2) \bmod 3}$ for each i $(0 \leq i \leq 3)$. According to Algorithm 1, r_0 moves to the midpoint of r_1 and r_2, r_1 moves to p_0, r_2 moves to the midpoint of r_2 and r_3, and r_3 moves to the midpoint of r_3 and r_1. In the resultant configuration, r_0, r_2 and r_3 form an equilateral triangle and r_1 is located at the center p_1 of the triangle, which shows by repeating the argument that the gathering is never achieved.

Thus we need another gathering algorithm for the adversarial (4, 2)-defected model, however, we do not know whether the gathering problem in the adversarial (4,2)-defected model is solvable or not yet. In the next section, we present an algorithm to solve the gathering problem in the distance-based (4,2)-defected model.

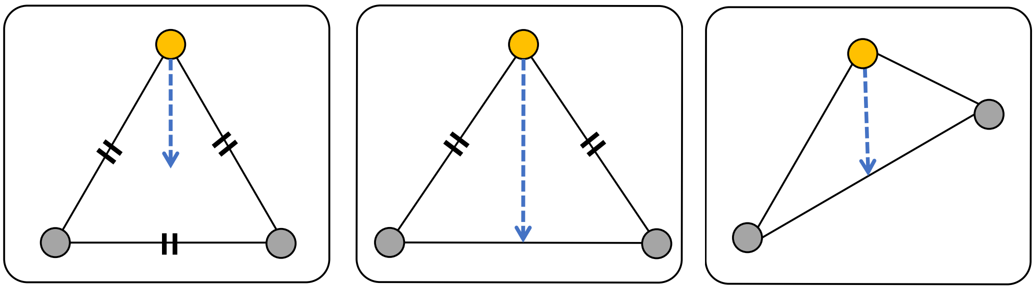
4 Algorithm in the Distance-based (4,2)-defected Model

In this model, the number of robots is 4 and each robot observes at most two occupied points other than its current location (three points in total including the one occupied by itself). In other words, the observation result of each robot forms a triangle (by three points/robots) when every robot is single. The strategy of the proposed algorithm is to determine one unique point from the formed triangle. Therefore, two robots observing the same three occupied points (including its location) move to the same point according to the proposed algorithm. If two or more robots are accompanied, the gathering can be achieved as the same manner introduced in Algorithm 1. Obviously, in this strategy, we have to consider the case so that all 4 robots observe different triangles. We resolve this problem by the geometrical property (recall that each robot cannot observe the farthest robot from itself in the distance-based defected model).

Algorithm 2 Gathering algorithm for robot r_i in the distance-based (4,2)-defected model.

1: **if** $\forall p \in \mathsf{OPSET}() : \mathsf{isMulti}(p) = \mathsf{TRUE}$ **then**
2:     move to the center of the smallest enclosing circle of $\mathsf{OPSET}()$
3: **else if** (r_i is single) $\wedge$ ($\exists p \in \mathsf{OPSET}() : \mathsf{isMulti}(p) = \mathsf{TRUE}$) **then**
4:     move to an arbitrary point $p \in \mathsf{OPSET}()$ such that $\mathsf{isMulti}(p) = \mathsf{TRUE}$
5: **else if** $\forall p \in \mathsf{OPSET}() : \mathsf{isMulti}(p) = \mathsf{FALSE}$ **then**
6:     **if** $\mathsf{OPSET}()$ forms an equilateral triangle **then**
7:         move to the center of the triangle (i.e., incenter) ▷ Rule 1
8:     **else if** $\mathsf{OPSET}()$ forms an isosceles triangle **then**
9:         move to the midpoint of the base of the triangle ▷ Rule 2
10:     **else** ▷ the other triangle or collinear three points
11:         move to the midpoint of the longest line ▷ Rule 3
12:     **end if**
13: **end if** ▷ No action if (r_i is accompanied) $\wedge$ ($\exists p \in \mathsf{OPSET}() : \mathsf{isMulti}(p) = \mathsf{FALSE}$)

Algorithm 2 presents an algorithm to achieve the gathering in the distance-based (4,2)-defected model (two functions, $\mathsf{OPSET}()$ and $\mathsf{isMulti}()$, are the same functions described in Section 3). Each robot which does not observe any accompanied robots executes one among three rules (lines from 6 to 11 in Algorithm 2). Figure 2 illustrates these three rules. If a robot observes an equilateral triangle (i.e., the points observed by the robot form an equilateral triangle), it moves to the center of the triangle (Figure 2(a)), and if it observes an isosceles triangle, it moves to the midpoint of the base of the triangle (Figure 2(b)). In the other case, it moves to the center point of the longest line of the triangle (Figure 2(c)). It is obvious that two robots observing the same set of points (i.e., the same *view*: $r_i.\mathsf{OPSET}() = r_j.\mathsf{OPSET}()$, where $i \neq j$), move to the same point according to Algorithm 2. Hence the following lemma holds.



(a) Case of an equilateral triangle. **(b)** Case of an isosceles triangle. **(c)** The other case.

Figure 2 Three rules in Algorithm 2.

▶ **Lemma 8.** *In any configuration where no robot is accompanied, if two or more robots have the same view, the robots move to the same point in one round by Algorithm 2.* ◀

In Algorithm 2, actions when a robot observes any accompanied robots (including itself) are the exactly same as Algorithm 1 (lines from 1 to 4 in both algorithms). Lemmas 4 and 5 are proved for the adversarial defected model but obviously hold for the distance-based defected model. Remind that Lemmas 4 and 5 hold for $N \geq 3$ and $N \geq 4$, respectively. Moreover, we can see from the proof that the gathering is achieved in one round (not two rounds) in Lemma 4 for $N = 4$. Thus,the following lemma holds.

▶ **Lemma 9.** *In the distance-based (4,2)-defected model, Algorithm 2 solves the gathering in one round (resp. two rounds) when there exist three or more (resp. only two) accompanied robots.* ◀

Even when all 4 robots are single, if two or more robots observe the same set of points, the robots move to the same point (by Lemma 8), thus the gathering is achieved by Lemma 9.

Now we show that the gathering is eventually achieved in any configuration where all 4 (single) robots have the different views (i.e., observe the different set of points).

▶ **Lemma 10.** *In the distance-based (4,2)-defected model, if all robots have the different views, the shape formed by the robots is a convex quadrilateral.*

Proof. We prove the contraposition of the lemma: if the robots do not form a convex quadrilateral, there exist two robots having the same view.

Assume that the 4 robots, from r_1 to r_4, form a concave quadrilateral as Figure 3 (Note that we can also assume that the robots form a triangle (i.e., three robots are collinear), it can be also proved in the same manner). A concave quadrilateral has an interior angle which is larger than 180°, so we assume robot r_1 is located at the point with such an angle as Figure 3. Let e be the line $\overline{r_1 r_2}$, either angle $\angle r_2 r_1 r_4$ or angle $\angle r_2 r_1 r_3$ is an obtuse angle (i.e., angle larger than 90°) because interior angle $\angle r_4 r_1 r_3$ is larger than 180°. Without loss of generality, we assume angle $\angle r_2 r_1 r_3$ is an obtuse angle (denoted by θ). Due to $\theta > 90°$, d is longer than c and e (see Figure 3). This implies that robot r_3 observes r_1 and robot r_2 also

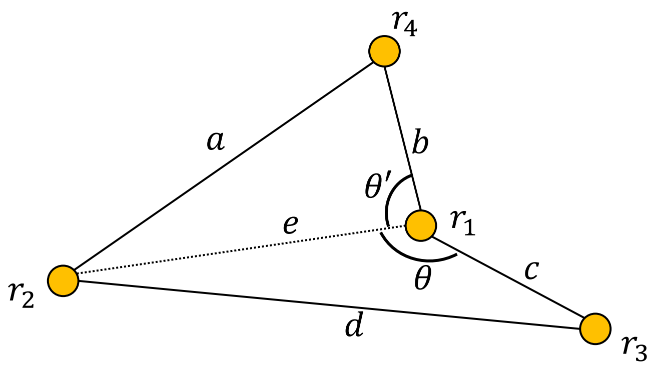


Figure 3 An example of a concave quadrilateral.

observes r_1 (because the farthest robot is missed in the distance-based defected model). If angle $\angle r_2r_1r_4$ (denoted by θ') is also an obtuse angle, robot r_4 also observes r_1 by the same reason. As a result, all robots observe r_1 (including r_1 itself) and the lemma holds because there are two or more robots which have the same view by the pigeonhole principle. If angle $\angle r_2r_1r_4$ is an acute angle (i.e., angle smaller than $90°$) or a right angle, $\theta + \theta' < 270°$ holds. This means that an exterior angle $\angle r_4r_1r_3$ (i.e., $360° - \theta - \theta'$) is an obtuse angle, thus b is shorter than $\overline{r_4r_3}$. Also in this case, robot r_4 observes r_1 and the lemma holds. ◀

▶ **Lemma 11.** *Assume that all robots have different views. If robot r_i cannot observe robot r_j (i.e., robot r_i's view does not include the point occupied by r_j), r_j cannot observe r_i neither.*

Proof. To help to explain, we introduce a directed graph $\vec{G} = (V, A)$ such that $V = \{r_1, r_2, r_3, r_4\}$ and $(r_i, r_j) \in A$ if robot r_i cannot observe r_j. If all robots have different views, there exist only two cases, as shown in Figure 4. And we show that there is no case as Figure 4(a) to prove the lemma.

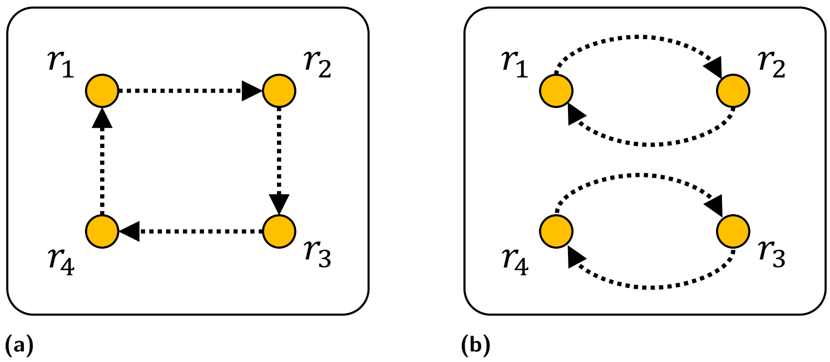


Figure 4 Directed graphs representing unobserved relation.

Assume the case as Figure 4(a): robot r_1 cannot observe r_2, robot r_2 cannot observe r_3, and so on. $\overline{r_1r_4} \le \overline{r_1r_2}$ holds because robot r_1 cannot observe r_2. For the same reason, $\overline{r_1r_2} \le \overline{r_2r_3}$, $\overline{r_2r_3} \le \overline{r_3r_4}$, and $\overline{r_3r_4} \le \overline{r_1r_2}$ also hold. Therefore, $\overline{r_1r_4} \le \overline{r_1r_2} \le \overline{r_2r_3} \le \overline{r_3r_4} \le \overline{r_1r_4}$ holds, thus $\overline{r_1r_2} = \overline{r_2r_3} = \overline{r_3r_4} = \overline{r_1r_4}$ holds. For simplicity, we assume that the length of $\overline{r_1r_2}$ is 1.

Now we consider the triangle $\triangle r_1r_2r_3$. Due to $\overline{r_1r_2} = \overline{r_2r_3}$, triangle $\triangle r_1r_2r_3$ is an isosceles triangle (the base is $\overline{r_1r_3}$). Similarly, triangle $\triangle r_1r_3r_4$ is also an isosceles triangle which has line $\overline{r_1r_3}$ as the base. Line $\overline{r_1r_3}$ is the common base of these two isosceles triangles, thus the locations of 4 robots are as Figure 5.

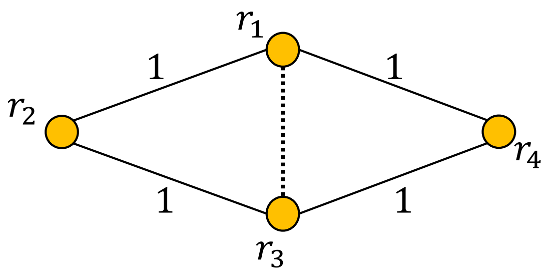


Figure 5 Two isosceles triangles.

In Figure 5, we consider the lengths of two diagonal lines, $\overline{r_1 r_3}$ and $\overline{r_2 r_4}$. By the assumption, robot r_1 cannot observe r_2, therefore, $\overline{r_1 r_3} \leq 1$ holds because robot r_1 observes r_3. As the same reason, $\overline{r_2 r_4} \leq 1$ also holds. However, both $\overline{r_1 r_3} \leq 1$ and $\overline{r_2 r_4} \leq 1$ cannot hold in this rhombus, therefore, there is no case as Figure 4(a) and the lemma holds. ◀

By Lemma 11, if all robots have different views, we have two disjoint pairs of robots such that robots in each pair cannot observe each other as in Figure 4(b). Now we discuss the location relations among the robots in this case by the following lemma.

▶ **Lemma 12.** *If all robots have different views in the distance-based (4,2)-defected model, each of two robots which cannot be observed each other are diagonally located on the formed convex quadrilateral.*

Proof. We already proved that the robots form a convex quadrilateral if all robots have different views by Lemma 10. Let r_1 and r_2 be two robots which do not observe each other, and we assume for contradiction that r_1 and r_2 are not diagonally located (i.e., line $\overline{r_1 r_2}$ is an edge of the convex quadrilateral). For simplicity, we assume the length of $\overline{r_1 r_2}$ is 1.

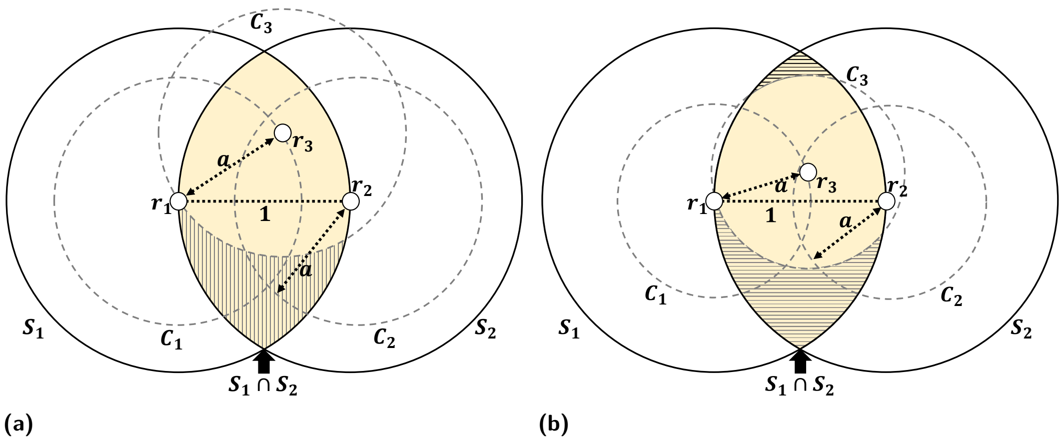


Figure 6 Possible positions of robots r_3 and r_4.

Figure 6(a) illustrates two circles, called S_1 and S_2, with radius 1 centered at r_1 and r_2 respectively. Consider the position of robot r_3: robot r_3 should be located in area $S_1 \cap S_2$, because both r_1 and r_2 observe r_3 (remind that r_1 and r_2 do not observe each other). Locate r_3 in an arbitrary point in area $S_1 \cap S_2$. Let $a = max(|\overline{r_1 r_3}|, |\overline{r_2 r_3}|)$, here we assume a is the length of $\overline{r_1 r_3}$ without loss of generality. Circles C_1, C_2, and C_3 present the circles with radius a centered at r_1, r_2, and r_3 respectively. By Lemma 11, robots r_3 and r_4 cannot observe each other, thus $|\overline{r_3r_4}| \geq a$ holds; robot r_4 should be located outside of C_3. As a result, robot r_4 should be located in $(C_1 \cap C_2) - C_3$ which is presented as the shaded area in Figure 6. In this case, robots r_1 and r_2 (resp. r_3 and r_4) are diagonally located on a convex quadrilateral, which is a contradiction.

We can consider another case where the shaded area appears on the same side as r_3 (with respect to $\overline{r_1r_2}$) if a is short enough as Figure 6(b). However, if robot r_4 is located on the same side as r_3, then robot r_3 is inside the triangle $\triangle r_1r_2r_3$. This implies that four robots form a concave quadrilateral, which is a contradiction. ◀

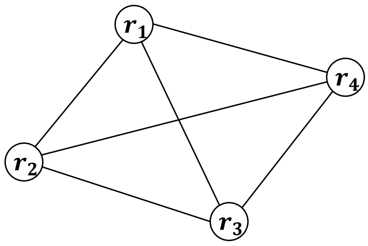


Figure 7 6 lines by 4 robots.

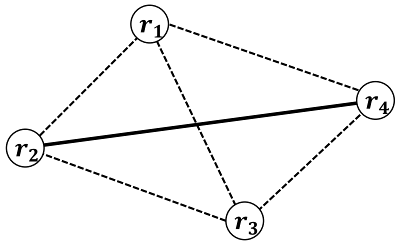


Figure 8 Configuration with one longest line.

Now we show that even when all single robots have different views, two or more robots move to the same point by Algorithm 2. We consider the 6 lines derived by the combination of 4 robots (refer to Figure 7). We focus on the lengths of these 6 lines, and the following corollary holds by Lemma 12.

▶ **Corollary 13.** *Consider the 6 lines connecting distinct pairs of two robots. If all robots are single and have different views, there is no (side) line which is longer than any diagonal line.*

It is worthwhile to mention that there can be at most 4 longest lines among 6 lines. We focus on the number of longest lines and show that the algorithm works correctly in all cases. By Corollary 13, if there exist one or two longest lines, they are diagonal lines. The following lemma holds.

▶ **Lemma 14.** *Assume that all robots are single and have different views in the distance-based (4,2)-defected model, and consider the 6 lines connecting distinct pairs of two robots. If there exist one or two longest lines, two or more robots become accompanied in one round.*

Proof. Figure 8 illustrates an example configuration including the only one longest line (as a diagonal line), where the thick solid line represents the unique longest line. Without loss of generality, we assume that line $\overline{r_2r_4}$ is the longest one. From the assumption, r_1 and r_3 do not observe each other: r_1 observes triangle $\triangle r_1r_2r_4$, and r_3 observes triangle $\triangle r_2r_3r_4$. These two triangles are not equilateral triangles because line $\overline{r_2r_4}$ is the unique longest line. Therefore, robots r_1 and r_3 move to the midpoint of line $\overline{r_2r_4}$ (by line 9 or 11). If there are two longest lines, the both lines are diagonal lines by Corollary 13 ($\overline{r_1r_3}$ and $\overline{r_2r_4}$ in Figure 8). However, this does not affect to the actions of robots r_1 and r_4; they move to the midpoint of line $\overline{r_2r_4}$. Thus the lemma holds. ◀

Now we consider the case that there is a side line whose length is the same as two diagonal lines; there are three or four longest lines.

▶ **Lemma 15.** *Assume that all robots are single and have different views in the distance-based (4,2)-defected model, and consider the 6 lines connecting distinct pairs of two robots. If there are the three longest lines, two or more robots become accompanied in two rounds.*

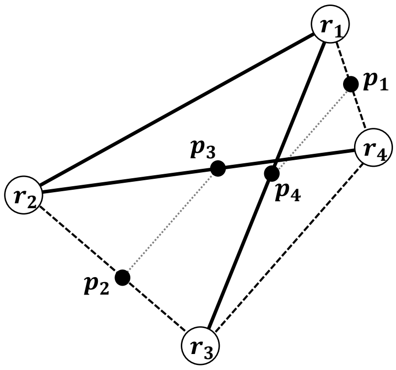


Figure 9 Case with three longest lines.

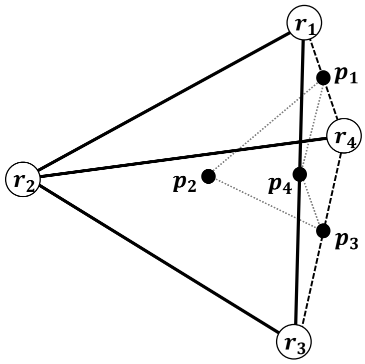


Figure 10 Case with four longest lines.

Proof. Figure 9 illustrates the only configuration including three longest lines. Three thick solid lines are the three longest lines. Remind that robots r_1 and r_3 (or r_2 and r_4) cannot observe each other. By Algorithm 2, all robots move to the different points: robot r_1 (resp. r_2) moves to the midpoint p_1 (resp. p_2) of line $\overline{r_1r_4}$ (resp. $\overline{r_2r_3}$) since r_1 (resp. r_2) observes an isosceles triangle $\triangle r_1r_2r_4$ (resp. $\triangle r_1r_2r_3$). Robot r_3 (resp. r_4) moves to the midpoint p_3 (resp. p_4) of line $\overline{r_2r_4}$ (resp. $\overline{r_1r_3}$) that is the longest line of the observed triangle $\triangle r_2r_3r_4$ (resp. $\triangle r_1r_3r_4$). In this case, triangles $\triangle r_2r_3r_4$ and $\triangle r_2p_2p_3$ are similar, the length of line $\overline{p_2p_3}$ is half of the length of line $\overline{r_3r_4}$, and line $\overline{p_2p_3}$ and line $\overline{r_3r_4}$ are parallel. Through the same argument for lines $\overline{p_1p_4}$ and $\overline{r_4r_3}$, we can show that the lengths of lines $\overline{p_1p_4}$ and $\overline{p_2p_3}$ are the same and these two lines are parallel. This means that the quadrilateral formed in the next round is a parallelogram: even if all robots have different views in this configuration, two or more robots become accompanied in the next round because diagonal line $\overline{p_1p_2}$ is the unique longest line (by Lemma 14). ◀

▶ **Lemma 16.** *Assume that all robots are single and have different views in the distance-based (4,2)-defected model, and consider the 6 lines connecting distinct pairs of two robots. If there are four longest lines, two or more robots become accompanied in two rounds.*

Proof. Figure 10 illustrates the only possible configuration including four longest lines. Four thick solid lines are the four longest lines. By Algorithm 2, all robots move to the different points: robot r_1 (resp. r_3) moves to the midpoint p_1 (resp. p_3) of line $\overline{r_1r_4}$ (resp. $\overline{r_3r_4}$) since r_1 (resp. r_3) observes an isosceles triangle $\triangle r_1r_2r_4$ (resp. $\triangle r_2r_3r_4$). Robot r_2 moves to the center p_2 of the equilateral triangle $\triangle r_1r_2r_3$ it observes, and r_4 moves to the midpoint p_4 of the unique longest line $\overline{r_1r_3}$ it observes (note that if $|\overline{r_1r_4}| = |\overline{r_3r_4}|$, triangle $\triangle r_1r_3r_4$ is an isosceles triangle, however robot r_4 moves to the midpoint p_4 of the base line also in this case). As a result, the four points, from p_1 to p_4, form a concave quadrilateral. Hence, two or more robots become accompanied in the next round by Lemma 10. ◀

From Lemmas 4, 9, 14, 15 and 16, the following theorem holds.

▶ **Theorem 17.** *In the distance-based (4, 2)-defected model, Algorithm 2 solves the gathering problem in at most four rounds.* ◀

5 Impossibility Results

In this section, we present two impossibility results for the gathering problem in the defected view model; (1) there is no (deterministic) algorithm in the distance-based (3,1)-defected model, and (2) there is no (deterministic) algorithm in the relaxed adversarial (N,$N-2$)-defected model defined in Section 5.2.

5.1 Impossibility in (3,1)-defected model

By the two gathering algorithms we introduced in the previous sections, the gathering can be achieved in the adversarial (and thus also in the distance-based) (N,$N-2$)-defected model for $N \geq 5$, and in the distance-based (4,2)-defected model. These results bring us a problem to find an algorithm to solve the gathering problem in the distance-based (or adversarial) (3,1)-defected model. Here we show that there is no such algorithm.

▶ **Theorem 18.** *There is no (deterministic) algorithm to solve the gathering problem in the distance-based (3,1)-defected model.*

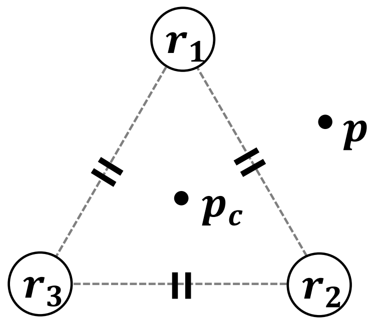


Figure 11 Example for an unsolvable configuration in the distance-based (3,1)-defected model.

Proof. We prove this theorem by showing that there is no (deterministic) algorithm even in the distance-based (3,1)-defected model. Note that the distance-based (3,1)-defected model is stronger than the adversarial one, this result implies that the gathering is also unsolvable in the adversarial one. Assume that three robots, $R = \{r_1, r_2, r_3\}$, are arranged in an equilateral triangle as Figure 11, and robot r_1 (resp. r_2 and r_3) observes r_2 (resp. r_3 and r_1). All robots do not agree on any geometrical agreement (e.g., direction, orientation, chirality, or unit distance), thus we can assume that every robot r_i considers the direction from itself to the center of the triangle (p_c) (i.e., $\overrightarrow{r_i p_c}$) as the positive direction of X-axis in its local coordinate system. Moreover, we also assume that all robots have the same chirality (e.g., clockwise) and the same unit distance. This means that all robots obtain the exactly same view from of *Look* operation.

Let $\mathcal{A}$ be an algorithm for gathering in the distance-based (3,1)-defected model. In the above configuration, all robots have the same views, thus they execute the same behavior according to $\mathcal{A}$ (i.e., all robots move to the same x and y coordinates in their local coordinate systems). This causes another configuration forming a different equilateral triangle, which shows by repeating the argument that the robots cannot gather at the same point forever. The only way to prevent the robots from forming another equilateral triangle is to move to point p_c, i.e., each robot moves to the point located at $|\overline{r_i r_j}|/\sqrt{3}$ distance in the 30°

clockwise direction of the observed robot r_j. However, if all robots agree on the opposite direction of chirality (counter-clockwise in this case), they move to the outside of triangle $\triangle r_1r_2r_3$ (i.e., robot r_1 moves to point p'_c instead of p_c). As a result, the robots form another equilateral triangle. ◀

5.2 Impossibility in the relaxed adversarial (N,$N-2$)-defected model

The (N, k)-defected model assumes that k robots observed by robot r are chosen from the robots that are located at points other than r's current position and that r can detect whether it is single or accompanied. Natural relaxation of the model is to choose the k robots other than r (i.e., robots at r's current position can be chosen) and assume the weak multiplicity detection for the k robots and r itself. We call the model with the relaxation the *relaxed adversarial* (N,k)-defected model. Notice that the key property of the (N, $N-2$)-defected model such that any accompanied robot can observe all the robots does not hold in the relaxed model.

The following theorem shows that the gathering is impossible (from some configuration) in the relaxed adversarial (N,$N-2$)-defected model.

▶ **Theorem 19.** *There is no (deterministic) algorithm to solve the gathering problem in the relaxed adversarial (N,$N-2$)-defected model.*

Proof. Let $\mathcal{A}$ be a gathering algorithm in the relaxed adversarial (N,$N-2$)-defected model. We consider only initial configurations where all robots are located at two points p_1 and p_2.

First, consider the initial configuration where $N-1$ robots are located at p_1 and one robot, say r_1, is located at p_2. When the robots at p_1 do not observe r_1, they misunderstand that the gathering is already achieved and terminate. To achieve the gathering, r_1 has to move to p_1. This implies that $\mathcal{A}$ has the following action (**Action 1**): when a single robot r observes only one occupied point other than r's current point and recognizes that the point is occupied by multiple robots, r has to move to the point.

Notice that **Action 1** is sufficient to show that $\mathcal{A}$ cannot solve the gathering in the relaxed adversarial (4,2)-defected model. Consider the initial configuration where two robots exist at both of p_1 and p_2 (four robots in total). When the robots at p_1 (resp. p_2) observe only the two robots at p_2 (resp. p_1), the robots at p_1 (resp. p_2) move to p_2 (resp. p_1) by **Action 1**. At the resultant configuration, two robots exist at both of p_1 and p_2, which shows by repeating the argument that algorithm $\mathcal{A}$ cannot solve the gathering problem.

Second, consider the initial configuration where $N \geq 5$ and all robots recognize that both p_1 and p_2 are occupied by multiple robots, which can occur when a point is occupied by three or more robots and the other is occupied by two or more robots. When all the robots at the same point observe the same set of robots (but still they recognize that both the points are occupied by multiple robots), the robots at the same point execute the same action (i.e., move to the same point). Since algorithm $\mathcal{A}$ solves the gathering problem, all robots eventually have to move to the same point (precisely the midpoint of the two points occupied by robots) to achieve the gathering. This implies that $\mathcal{A}$ has the following action (**Action 2**): when an accompanied robot r observes only one occupied point other than r's current point and recognizes that the point is occupied by multiple robots, r has to move to the midpoint of the two points.

Finally, consider the initial configuration of N (≥ 5) robots where two robots exist at p_1 and $N-2$ robots exist at p_2. When each robot r_1 at p_1 observes only $N-2$ robots at p_2 (and recognizes itself as a single robot), r_1 moves to p_2 by **Action 1**. On the other hand, when each robot r_2 at p_2 observes the two robots at p_1 and $N-4$ robots (other than r_2)

at p_2, r_2 moves to the midpoint of p_1 and p_2 by **Action 2**. At the resultant configuration, two robots exist at p_2 and $N-2$ robots exist at the midpoint of p_1 and p_2. By repeating the argument, we can show that algorithm $\mathcal{A}$ cannot solve the gathering problem although all robots converge at the same point (i.e., the distance between the two groups of robots becomes smaller and smaller but does not become zero).

Consequently, there is no gathering algorithm in the relaxed adversarial (N,$N-2$)-defected model. ◀

6 Conclusion and Open Problems

In this paper, we introduced a new computational model, the (N, k)-*defected model*, where each robot cannot necessarily observe all other robots: i.e., each robot observes at most k other robots not located at its current position (where $k < N-1$). We addressed the gathering problem, which is one of the basic problem in autonomous mobile robot systems, in the (N, $N-2$)-defected model. We proposed two gathering algorithms: (1) an algorithm in the adversarial (N,$N-2$)-defected model that achieves the gathering within three rounds, and (2) an algorithm in the distance-based (4,2)-defected model that achieves the gathering within four rounds. Moreover, we showed that there is no (deterministic) algorithm in either the adversarial or distance-based (3,1)-defected model. In the proposed model, we assume that each robot r observes k other robots among the robots located at the different points than the point occupied by r itself. The relaxation of this assumption, where k robots are chosen among all other robots other than r, can be considered, however, we proved that the gathering is unsolvable in this relaxed model.

The remaining problem we are most interested in is to clarify the solvability of the gathering problem in the adversarial (4,2)-defected model. Remind that the basic strategy of the proposed algorithm in the distance-based (4,2)-defected model is to determine one unique point from the triangle formed by the observed set of points. We call the algorithm using this strategy the *set-based algorithm*, where each robot determines the destination referring to only the set of observed points: for example, when a robot observes an isosceles triangle, it always moves to the midpoint of the base, regardless of whether it is adjacent to the base or not, i.e., we do not use the information of the (relative) position of the observing robot. It can be easily proved that *there is no (deterministic) set-based algorithm to solve the gathering problem in adversarial (4,2)-defected model*. This means that if a gathering algorithm exists in the adversarial (4,2)-defected model, each robot has to use its relative position in the set of observed points, e.g., when a robot observes an isosceles triangle, the destination point changes depending on whether the robot is at a point incident to the base of the triangle or not.

An important future work is to find the minimum k that allows a solution for the gathering problem in the adversarial or distance-based (N, k)-defected model. In this paper, we considered only the gathering problem, therefore, to challenge other problems under the (N, k)-defected model is another future work.

References

1 Hideki Ando, Yoshinobu Oasa, Ichiro Suzuki, and Masafumi Yamashita. Distributed memoryless point convergence algorithm for mobile robots with limited visibility. *IEEE Trans. Robotics Autom.*, 15(5):818–828, 1999. doi:10.1109/70.795787.

2 Zohir Bouzid, Shantanu Das, and Sébastien Tixeuil. Gathering of mobile robots tolerating multiple crash faults. In *IEEE 33rd International Conference on Distributed Computing Systems, ICDCS*, pages 337–346. IEEE Computer Society, 2013. doi:10.1109/ICDCS.2013.27.

3 Avik Chatterjee, Sruti Gan Chaudhuri, and Krishnendu Mukhopadhyaya. Gathering asynchronous swarm robots under nonuniform limited visibility. In *Distributed Computing and Internet Technology – 11th International Conference, ICDCIT*, volume 8956 of *Lecture Notes in Computer Science*, pages 174–180. Springer, 2015. doi:10.1007/978-3-319-14977-6_11.

4 Mark Cieliebak, Paola Flocchini, Giuseppe Prencipe, and Nicola Santoro. Solving the Robots Gathering Problem. In *Proceedings of the 30th International Colloquium on Automata, Languages and Programming, ICALP*, pages 1181–1196. Springer, 2003. doi:10.1007/3-540-45061-0_90.

5 Gianlorenzo D'Angelo, Gabriele Di Stefano, Ralf Klasing, and Alfredo Navarra. Gathering of robots on anonymous grids and trees without multiplicity detection. *Theor. Comput. Sci.*, 610:158–168, 2016. doi:10.1016/j.tcs.2014.06.045.

6 Xavier Défago, Maria Gradinariu, Stéphane Messika, and Philippe Raipin Parvédy. Fault-Tolerant and Self-stabilizing Mobile Robots Gathering. In *Proceedings of the 20th International Symposium on Distributed Computing, DISC*, pages 46–60. Springer, 2006. doi:10.1007/11864219_4.

7 Paola Flocchini, Giuseppe Prencipe, Nicola Santoro, and Peter Widmayer. Gathering of Asynchronous Oblivious Robots with Limited Visibility. In *Proceedings of the 18th Annual Symposium on Theoretical Aspects of Computer Science, STACS*, pages 247–258. Springer, 2001. doi:10.1007/3-540-44693-1_22.

8 Adam Heriban and Sébastien Tixeuil. Mobile robots with uncertain visibility sensors: Possibility results and lower bounds. *Parallel Process. Lett.*, 31(1):2150002:1–2150002:21, 2021. doi:10.1142/S012962642150002X.

9 Nobuhiro Inuzuka, Yuichi Tomida, Taisuke Izumi, Yoshiaki Katayama, and Koichi Wada. Gathering Problem of Two Asynchronous Mobile Robots with Semi-dynamic Compasses. In *Proceedings of the 15th International Colloquium on Structural Information and Communication Complexity, SIROCCO*, pages 5–19. Springer, 2008. doi:10.1007/978-3-540-69355-0_3.

10 Taisuke Izumi, Yoshiaki Katayama, Nobuhiro Inuzuka, and Koichi Wada. Gathering Autonomous Mobile Robots with Dynamic Compasses: An Optimal Result. In *Proceedings of the 21st International Symposium on Distributed Computing, DISC*, pages 298–312. Springer, 2007. doi:10.1007/978-3-540-75142-7_24.

11 Yoshiaki Katayama, Yuichi Tomida, Hiroyuki Imazu, Nobuhiro Inuzuka, and Koichi Wada. Dynamic Compass Models and Gathering Algorithms for Autonomous Mobile Robots. In *Proceedings of the 14th International Colloquium on Structural Information and Communication Complexity, SIROCCO*, pages 274–288. Springer, 2007. doi:10.1007/978-3-540-72951-8_22.

12 David G. Kirkpatrick, Irina Kostitsyna, Alfredo Navarra, Giuseppe Prencipe, and Nicola Santoro. Separating bounded and unbounded asynchrony for autonomous robots: Point convergence with limited visibility. In *PODC '21: ACM Symposium on Principles of Distributed Computing*, pages 9–19. ACM, 2021. doi:10.1145/3465084.3467910.

13 Ralf Klasing, Euripides Markou, and Andrzej Pelc. Gathering asynchronous oblivious mobile robots in a ring. *Theor. Comput. Sci.*, 390(1):27–39, 2008. doi:10.1016/j.tcs.2007.09.032.

14 Giuseppe Antonio Di Luna, Ryuhei Uehara, Giovanni Viglietta, and Yukiko Yamauchi. Gathering on a circle with limited visibility by anonymous oblivious robots. In *34th International Symposium on Distributed Computing, DISC*, volume 179 of *LIPIcs*, pages 12:1–12:17. Schloss Dagstuhl – Leibniz-Zentrum für Informatik, 2020. doi:10.4230/LIPIcs.DISC.2020.12.

15 Debasish Pattanayak, Kaushik Mondal, H. Ramesh, and Partha Sarathi Mandal. Fault-tolerant gathering of mobile robots with weak multiplicity detection. In *Proceedings of the 18th International Conference on Distributed Computing and Networking*, page 7. ACM, 2017. URL: http://dl.acm.org/citation.cfm?id=3007786.

16 Giuseppe Prencipe. On the feasibility of gathering by autonomous mobile robots. In *Proceedings of the 12th International Colloquium on Structural Information and Communication Complexity, SIROCCO*, pages 246–261. Springer, 2005. doi:10.1007/11429647_20.

17 Giuseppe Prencipe. Impossibility of gathering by a set of autonomous mobile robots. *Theor. Comput. Sci.*, 384(2-3):222–231, 2007. doi:10.1016/j.tcs.2007.04.023.

18 Giuseppe Prencipe. Autonomous Mobile Robots: A Distributed Computing Perspective. In *Proceedings of the 9th International Symposium on Algorithms and Experiments for Sensor Systems, Wireless Networks and Distributed Robotics, ALGOSENSORS*, pages 6–21. Springer, 2013. doi:10.1007/978-3-642-45346-5_2.

19 Giuseppe Prencipe and Nicola Santoro. Distributed Algorithms for Autonomous Mobile Robots. In *Proceedings of Fourth IFIP International Conference on Theoretical Computer Science, TCS*, pages 47–62. Springer, 2006. doi:10.1007/978-0-387-34735-6_8.

20 Samia Souissi, Xavier Défago, and Masafumi Yamashita. Gathering Asynchronous Mobile Robots with Inaccurate Compasses. In *Proceedings of the 10th International Conference on Principles of Distributed Systems, OPODIS*, pages 333–349. Springer, 2006. doi:10.1007/11945529_24.

21 Ichiro Suzuki and Masafumi Yamashita. Distributed Anonymous Mobile Robots: Formation of Geometric Patterns. *SIAM J. Comput.*, 28(4):1347–1363, 1999. doi:10.1137/S009753979628292X.