Fewer Trains for Better Timetables: The Price of Fixed Line Frequencies in the Passenger-Oriented Timetabling Problem

Pedro José Correia Duarte
Econometric Institute, Erasmus Center for Optimization in Public Transport (ECOPT), Erasmus University Rotterdam, The Netherlands

Marie Schmidt
Institute of Computer Science, Faculty of Mathematics and Computer Science, Universität Würzburg, Germany

Dennis Huisman
Econometric Institute, Erasmus Center for Optimization in Public Transport (ECOPT), Erasmus University Rotterdam, The Netherlands
Process quality and Innovation, Netherlands Railways, Utrecht, The Netherlands

Lucas P. Veelenturf
Department of Technology and Operations Management, Rotterdam School of Management, Erasmus University, Rotterdam, The Netherlands

Abstract

This paper introduces the Passenger-Oriented Timetabling problem with flexible frequencies (POT-flex) in the context of railway planning problems. POT-flex aims at creating feasible railway timetables minimising total perceived passenger travel time. The contribution of the POT-flex lies in its relaxation of the generally adopted assumption that line frequencies should be a fixed part of the input. Instead, we consider flexible line frequencies, encompassing a minimum and maximum frequency per line, allowing the timetabling model to decide on optimal line frequencies to obtain better solutions using fewer train services per line. We develop a mixed-integer programming formulation for POT-flex based on the Passenger-Oriented Timetabling (POT) formulation of [13] and compare the performance of the new formulation against the POT formulation on three instances. We find that POT-flex allows to find feasible timetables in instances containing bottlenecks, and show improvements of up to 2% on the largest instance tested. These improvements highlight the cost that fixed line frequencies can have on timetabling.

2012 ACM Subject Classification Applied computing → Transportation

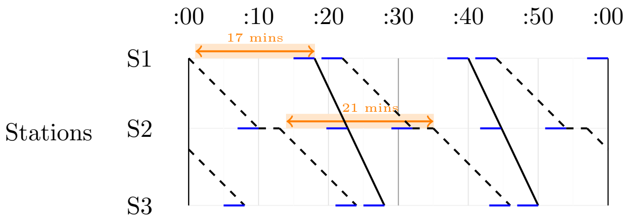
Keywords and phrases PESP, Passenger Oriented Timetabling, Perceived Travel Time

Digital Object Identifier 10.4230/OASIcs.ATMOS.2023.8

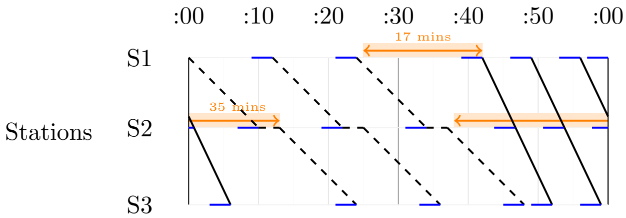
Funding This project is co-funded by Netherlands Railways (NS).

1 Introduction

Railway timetabling is part of a larger set of problems commonly referred to as railway planning problems. Because railway planning problems are generally solved sequentially [6, 2], the input of the timetabling problem relies on the output of previously solved problems. These problems include decisions regarding the infrastructure of the network, and the definition of a set of train lines and line frequencies. In this paper, we study the impact of line frequencies in the generation of periodic timetables, i.e. timetables that recur at regular intervals with a fixed time period. In particular, we evaluate the cost of *fixed* line frequencies on timetables from a passenger perspective.



(a) Timetable with 2 “fast” trains and 3 “slow” trains.



(b) Timetable with 3 “fast” trains and 3 “slow” trains.

Figure 1 Example that timetables with more trains can lead to worse perceived travel times. The blue lines at each station represent the minimum headway time.

An inherent limitation of addressing railway planning problems sequentially is that it often results in situations where solving one problem may give rise to sub-optimal or infeasible solutions in subsequent problems [2]. In the case of periodic timetabling, the line frequencies determined in earlier stages of the planning process can sometimes not be realised simultaneously, meaning that no feasible timetable exists. Furthermore, even if a timetable for the given frequencies is found, it may be sub-optimal with respect to perceived passenger travel time, defined as a weighted sum of waiting time at the origin, in-train time, and transfer time. While it is generally assumed that increased line frequencies lead to improved timetables, this might not hold due to infrastructural constraints or mismatches with passenger demand. In contrast, we argue that reduced frequencies may lead to lower perceived travel times.

▶ **Example 1.** Because the use of fewer train services (hereandafter referred to as trains) per line to obtain better timetables may appear counter-intuitive, let us examine the example presented in Figure 1. We consider a network containing 3 stations S1, S2, and S3, where the arrival of passengers at the stations is assumed to be uniformly distributed. Let us consider two lines, ℓ_1 and ℓ_2, where ℓ_1 is a fast line with stops {S1,S3} (whose trains are depicted with straight lines in Figure 1) and ℓ_2 is a slower line with stops {S1,S2,S3} (whose trains are depicted with dashed lines in Figure 1). Both lines have a maximum frequency of 3 and share the same tracks (no overtaking is allowed). In Figure 1a, we can see that a maximum of 5 trains can be scheduled by alternating trains from lines ℓ_1 and ℓ_2 without violating the headway constraints, i.e. the minimum time between two train arrivals ensuring safe operation of the timetable, depicted with the blue lines. Due to the headway constraints, we cannot add another train for ℓ_1 if we want to keep the same alternating structure. Nonetheless, a timetable containing 6 trains is feasible by arranging them as shown in Figure 1b. This results in a larger maximum waiting time between two trains for passengers going from S1 to S2 or from S2 to S3 (35 minutes instead of the 21 minutes in Figure 1a). Let the rate of passenger arrival be 1 passenger per minute. Then, for passengers going from S1 to S2 or from S2 to S3, the total waiting time is $(22 \times (22/2)) + (19 \times (19)) + (19 \times (19/2)) = 603$ in the first timetable and $(36 \times (36/2)) + (12 \times (12/2)) + (12 \times (12/2)) = 792$ in the second timetable. If the demand from stations S1 to S2 and S2 to S3 is large enough relative to the demand from S1 to S3, a timetable containing fewer trains can lead to lower total perceived passenger travel time.

Our research expands on the mathematical formulation developed by [13] for the Strategic Passenger-Oriented Timetabling (SPOT) problem. Our contribution is threefold; First, we introduce a variant of the timetabling problem to minimise perceived travel time discussed in [13, 11] that allows to choose line frequencies flexibly. We call this new problem the Passenger-Oriented Timetabling problem with flexible frequencies (POT-flex). Second, we provide a MILP formulation for the POT-flex problem. Third, we provide insights on the cost that the fixed line frequency assumption has on total perceived passenger travel time. We implement and solve the POT and the POT-flex problems on three instances to highlight the factors impacting optimal line frequency decisions.

The remainder of this paper is organised as follows. Section 2 provides an overview of the related literature regarding periodic passenger-oriented timetabling models. Furthermore, Section 3 provides a description of the POT-flex problem with flexible frequencies and Section 4 defines the Mixed Integer Linear Programming formulation of the flexible frequency model. Finally, Section 5 provides insights on the improvements of our model over one with fixed frequencies on three instances of interest, and in Section 6, a conclusion is drawn.

2 Passenger-Oriented Timetabling in the Literature

Many periodic timetabling models, including ours, use as a basis the Periodic Event Scheduling Problem (PESP) as defined by [17]. PESP is used to find feasible periodic timetables and is known to be NP-complete [17]. The addition of passenger routing aiming at creating passenger-oriented timetables makes the problem even more complex. Some papers attempt to tackle those issues and offer applicable methods minimising total passenger travel time starting from the moment that passengers leave the origin [14, 18, 4].

In this paper, we consider the importance of both line frequency decisions and adaption time, defined as time difference between the passenger desired departure time and the scheduled departure, in the passenger-oriented timetabling problem. We primarily refer to [15] for an extensive review on line planning and cite [16, 7, 10, 3] regarding the integration of line planning and timetabling. Beyond integration of line planning and timetabling, some papers also consider how to address infeasibilities stemming from the input in the timetabling problem [12]. To the best of our knowledge, only a few papers consider adaption time as part of their objective. We refer to [1, 20, 13, 11] where adaption time is included in the timetabling objective and [5] where it is included in line planning.

As aforementioned, this paper expands on the Mixed Integer Linear Programming (MILP) formulation of [13] for the Strategic Passenger-Oriented Timetabling (SPOT) problem. The objective of SPOT is to create a timetable that minimises the total perceived passenger travel time. In the SPOT problem, lines are assumed to have fixed frequencies and headway constraints are not taken into account. In [11], the authors solve the Passenger-Oriented Timetabling (POT) problem, an extension of the SPOT that considers headway constraints, using an iterative heuristic. They define a starting solution by solving the SPOT problem, then use a Lagrangian heuristic to generate feasible solutions with respect to the headway constraints. The possibility to not schedule some train services is added in the Lagrangian heuristic in order to find a feasible timetable.

Although the approach in [11] allows for reduction of line frequencies, it is only done to find feasible solutions and not better solutions. In this paper, we research an extension of the POT problem introducing the concept of *flexible* frequencies, such that a minimum and maximum frequency per line is used as part of the input. Flexible frequencies allow the model to find feasible solutions in instances where the maximum frequencies cannot be realised, and better solutions in instances where the reduction of line frequencies is beneficial for the passengers' perceived travel time. We denote this new problem as the POT-flex problem.

3 Problem Definition

This section introduces and describes the concepts necessary in the definition of the problem studied in this paper. The input of the POT-flex problem is defined in Section 3.1. Then, we describe how to define the problem on a graph in Section 3.2. Finally, we describe the perceived passenger travel time in Section 3.3.

3.1 Problem input and problem parameters

In order to solve the POT-flex problem, we consider the following input:

An Infrastructure Network: the infrastructure network contains the information about capacity of the stations, the different tracks that can be used by trains, the safety requirements (defined as the minimum time difference allowed between trains using the same infrastructure), and the minimum transfer time (the minimum amount of time required for passenger transfer between two lines at a station).

A Line Set $\mathcal{L}$: Each line $\ell \in \mathcal{L}$ is defined by the sequence of stations that the train visits, the subset $\mathcal{S}_\ell$ of stations where the train stops (altering the dwell time of trains at a station), the type of rolling stock used (altering the maximum speed and therefore the minimum travel time), and the minimum and maximum frequencies, respectively $\underline{f}_\ell$ and $\overline{f}_\ell$. We assume that lines have the same frequency in both directions. Furthermore, throughout this paper, we define a *train* as two train services following the sequence of stops related to a line ℓ (one train service per direction). Trains are not considered to be rolling stock.

An $\mathcal{OD}$-matrix: For each pair k of two stations, the passenger demand d_k to go from the first station to the second station is given in the *Origin-Destination* ($\mathcal{OD}$) matrix.

A Time Period T: the time period is the interval of time during which events, representing the arrival or departure of a train at a station, are scheduled. Each event can be scheduled at a discrete point in time $t \in \{0, \dots, T-1\}$. Those events are then repeated every T units of time.

Using the aforementioned input, the goal is to create a periodic timetable for the lines defined in the line set, subject to the constraints defined by the infrastructure network, that minimises the total perceived passenger travel time, using the $\mathcal{OD}$-matrix as an estimation of passenger demand. We define the problem on a directed graph called the event-activity network [8].

3.2 Event-Activity Network

An event-activity network (EAN) is a directed graph $G = (V, A)$ where V is the set of events to be scheduled and A is the set of activities that link the events. Each event $i \in V$ represents the arrival or departure of a train at a station. Therefore, every event is defined by its station, line, train index (denoting if it is first, second, etc... train of a line in the period), direction (forward or backward), and whether it is an arrival or departure event. An activity $(i, j) \in A$ is a directed arc that represents the time difference between two events i and j such that $(i, j) \in A$. The lower- and upper-bounds for the time duration that activities can take is defined by *activity constraints*. In our model, we consider four different type of activities;

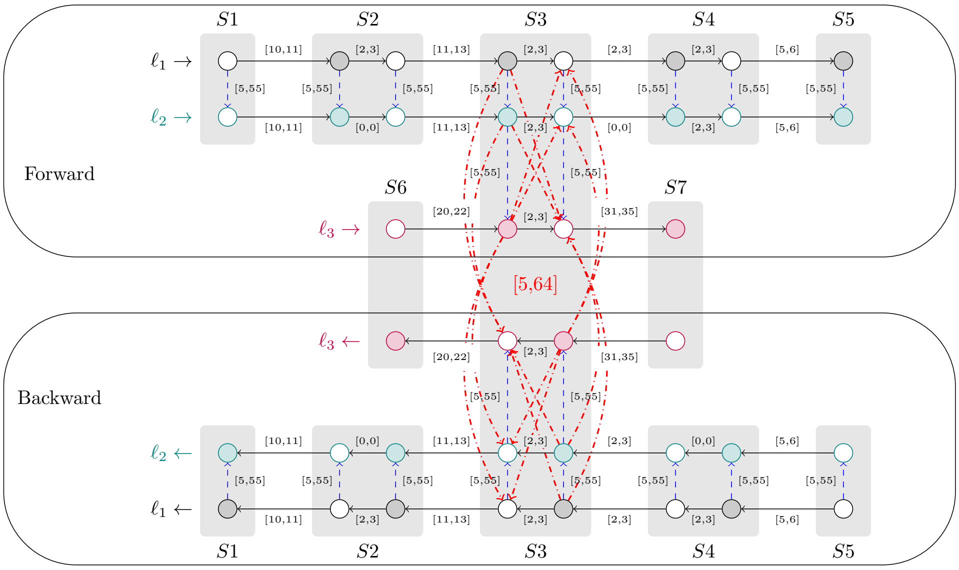


Figure 2 Event Activity Network of Instance 2; The black straight arrows represent the drive and dwell activities of a line, the dashed blue arrows represent the headway activities between trains, and the red dashed-dotted arrows represent the transfer activities.

Drive activities represent the time spent by a train travelling from one station to another. The lower-bound of a drive activity constraint is defined by the minimum travel time given the distance and the maximum speed of the train between two stations. The upper-bound is defined by the maximum allowed deviation from the minimum travel time.

Dwell activities represent the time spent by a train at a station. This time is used by passengers to either enter or leave a train. We use dwell activity constraints to impose a lower-bound to the time that a train spends at a station.

Transfer activities represent the time allocated for the transfer of a passenger from one train to another. Transfer activity constraints provide a lower bound for transfer times such that passengers have the time to go from one platform to another. A good timetable aims at reducing the time of these transfer activities while enabling passengers to make their transfers.

Safety/Headway activities represent infrastructure constraints that guarantee a safe operation. Safety activity constraints define the minimum time difference between the arrival or departure of two trains using the same tracks. This minimum headway time ensures that no collision is possible if all trains operate according to the timetable.

Activity constraints ensure the proper definition of a timetable from both an operational and passenger-oriented perspective. All activity constraints are needed to ensure the successful execution of the timetable from an operational perspective. Only the drive, dwell, and transfer activities are needed to evaluate the quality of the timetable from a passenger perspective. An example of EAN with its associated activity bounds is displayed in Figure 2.

3.3 Perceived Passenger Travel Time

Our objective is to minimise the perceived passenger travel time. In doing so, we consider two elements:

1. The travel time of a passenger is defined as the sum of the drive, dwell, and transfer activity lengths for the route taken by the passenger. However, those activities do not

weight equally in the eyes of the passenger. For instance, a route that contains a transfer does not have the same appeal to passengers as a route of similar time duration without a transfer.

2. For passengers, the amount of time spent waiting for the train at the origin station is equally important, if not more so, compared to the actual travel time. The amount of time between the arrival of the passenger at a station and the start of his travel route is called the adaption time.

Both of those points are accounted for in the objective function through the addition of penalties for in-route transfers and the addition of penalised adaption time. Our objective is to minimise the sum of passengers' perceived travel time. The perceived travel time of a passenger is defined as

$$(\gamma_w \cdot W_r + Y_r) \qquad \text{with} \qquad Y_r = \sum_{\forall a \in r : a \in A} [y_a + \gamma_t \mathbb{1}_t(a)] \tag{1}$$

where γ_w is the adaption time penalty factor, W_r is the adaption time of the passenger for his route r, and Y_r is the route's length defined by the sum of its associated drive, dwell, and transfer activity lengths y_a, with a penalty of γ_t for transfer activities ensured by the indicator function $\mathbb{1}_t(a)$ equal to 1 if a is a transfer activity.

Finally, for the purpose of our formulation, we make the following assumptions. The first assumption is that the arrival of passengers at their origin station is uniformly distributed. This ensures that the timetable is optimised for passengers arriving at any point in time during the period. The second assumption is that passengers always take the route with the lowest perceived travel time. Finally, we assume that train capacities are infinite such that the rolling stock is not taken into account in the timetabling model.

4 Formulating the Passenger-Oriented Timetabling Problem as a Mixed-Integer Linear Program

This section introduces our formulation for the POT-flex problem. This new formulation is an extension of the POT formulation of [11] where the activity constraints and the objective are modified to account for selection of the optimal line frequency. Section 4.1 describes the addition of flexible frequencies in the activity constraints. Then, Section 4.2 introduces the objective and the rest of the model. Finally, Section 4.3 describes the full formulation.

4.1 Flexible Line Frequencies in the PESP

The basis of the model is the Periodic Event Scheduling Problem (PESP) formulation as defined by [17]. For simplicity, we use the notation $[n]$ to represent a set $\{1, \ldots, n\}$. Given a set V of events, a set $A \subseteq V \times V$ of activities, intervals $[l_{ij}, u_{ij}]$ for all $(i,j) \in A$, and a period length T, the PESP is to find a feasible periodic schedule, that is, find event times $\pi : V \to \{0, \ldots, T-1\}$ and corresponding activity lengths $y_{i,j}$ satisfying

$$y_{ij} = \pi_j - \pi_i + Tp_{ij} \qquad \forall (i,j) \in A \tag{2a}$$

$$l_{ij} \leq y_{ij} \leq u_{ij} \qquad \forall (i,j) \in A \tag{2b}$$

$$p_{ij} \in \mathbb{Z} \qquad \forall (i,j) \in A \tag{2c}$$

$$\pi_i \in \{0, \ldots, T-1\} \qquad \forall i \in V \tag{2d}$$

where l_{ij} and u_{ij} are respectively the lower and upper bounds of the time an activity $(i,j) \in A$ can take. An important feature of this model is that if $u_{ij} - l_{ij} \geq T - 1$, the activity constraint no longer bounds the event times π_i and π_j.

In order to extend the model defined in (2a-2d) such that not all trains need to be scheduled, we define the following notation; for a line $\ell \in \mathcal{L}$ with minimum frequency $\underline{f}_\ell$ and maximum frequency $\overline{f}_\ell$, we assign to each train of the line in period T an index $tr \in [\overline{f}_\ell]$. As aforementioned, trains here denote train services in both directions, such that if a train is not scheduled for a line, it is not scheduled in both directions. We define the variable $\tau_{\ell,tr}$ such that:

$$\tau_{\ell,tr} = \begin{cases} 1 & \text{if the train with index } tr \text{ of line } \ell \text{ is scheduled} \\ 0 & \text{otherwise} \end{cases}$$

We set $\tau_{\ell,tr} = 1$ $\forall \ell \in \mathcal{L}$ and $\forall tr \in [\underline{f}_\ell]$ to ensure that we run at least $\underline{f}_\ell$ trains of line ℓ. Additionally, we define $A[\ell, tr] \subseteq A$ to be the set of activities related to the train tr of line ℓ and $V[\ell, tr] \subseteq V$ the set of events related to the train tr of line ℓ. We will now focus on the definition of the constraints for each type of activity.

The bounds of drive and dwell activities are defined as follows:

$$\tau_{\ell,tr} l_{ij} \leq y_{ij} \leq \tau_{\ell,tr} u_{ij} \qquad \forall \ell \in \mathcal{L}, \forall tr \in [\overline{f}_\ell], \text{ and } \forall (i,j) \in A[\ell, tr]. \tag{3}$$

This allows us to define the constraints in two possible cases:

1. If $\tau_{\ell,tr} = 1$, then this means that (3) is equal to (2b), and the train needs to be scheduled.
2. If $\tau_{\ell,tr} = 0$, then train tr of line ℓ is not scheduled and therefore all drive activities of this train will have length 0.

Now we consider the case of activities concerning two different trains (i.e. headway and transfer constraints). Two trains are considered to be different if their train index tr and/or lines ℓ are different. The goal is to make sure that the activity constraint is no longer binding if one of the trains is not scheduled. Hence, we define for each $\ell, \ell' \in \mathcal{L}$, $tr \in [\overline{f}_\ell]$, and $tr' \in [\overline{f}_{\ell'}]$ such that $(tr, \ell) \neq (tr', \ell')$ the following constraints:

$$(\tau_{\ell,tr} + \tau_{\ell',tr'} - 1) l_{ij} \leq y_{ij} \leq u_{ij} + (2 - \tau_{\ell,tr} - \tau_{\ell',tr'}) T$$
$$\forall (i,j) \in A : i \in V[tr, \ell], j \in V[tr', \ell']. \tag{4}$$

Then, we have the following possible cases for different values of $\tau_{\ell,tr}$ and $\tau_{\ell',tr'}$:

Bounds	$\tau_{\ell,tr} = 0$	$\tau_{\ell,tr} = 1$
$\tau_{\ell',tr'} = 0$	$[-l_{ij}, u_{ij} + 2T]$	$[0, u_{ij} + T]$
$\tau_{\ell',tr'} = 1$	$[0, u_{ij} + T]$	$[l_{ij}, u_{ij}]$

If one or both trains related to activity (i,j) are not scheduled in the timetable, then the difference between the new lower- and upper-bound of the activity is greater than T. The event times of scheduled trains are then no longer affected by transfer and headway activity constraints related to non-scheduled trains. It can be noted that similar methods have been applied by other authors to modify activity constraints but, to the best of our knowledge, this has only been done to consider track choices in the PESP [19, 9].

4.2 Perceived Travel Time with Flexible Frequencies

The objective of the model is to minimise the total perceived passenger travel time. In this section, we consider the formulation of the perceived passenger travel time given in the MILP model of [13] and alter it to accurately model non-scheduled trains in the objective.

Let us consider $\mathcal{R}^k$ the set of routes available for $\mathcal{OD}$-pair k. That is, any route $r \in \mathcal{R}^k$ starts at a departure event at the origin station of $\mathcal{OD}$-pair k and ends at an arrival event at its destination station. The set $\mathcal{R}^k$ is determined in a pre-processing step such as to discard excessively long routes. Then, given a timetable π, we compute for each $\mathcal{OD}$-pair k and each available route $r \in \mathcal{R}^k$ the travel time

$$Y_r(\pi) = \sum_{(i,j)\in r} \Big[y_{ij} + \gamma_t \cdot \mathbb{1}_t(i,j) \Big] + \sum_{(\ell,tr)\in r} M_k^{\tau} \cdot (1 - \tau_{\ell,tr}) \tag{5}$$

where $\mathbb{1}_t(i,j)$ is an indicator function equal to 1 if an activity (i,j) is a transfer activity, and M_k^{τ} is a large enough penalty value such that a passenger is never assigned a route r using a train that is not scheduled in the timetable. We must now ensure that the route with smallest perceived travel time is chosen by the passenger. Let $\sigma(r)$ be the first event of the route $r \in \mathcal{R}^k$, and let V^k be the set of first departure events for all routes considered for an $\mathcal{OD}$-pair k such that

$$V^k = \bigcup_{r\in\mathcal{R}^k} \{\sigma(r)\} \tag{6}$$

We define Y_v^k to be the perceived travel time for passengers of $\mathcal{OD}$-pair k from event $v \in V^k$ onwards. It is important to note two things regarding how the computation of Y_v^k is modelled in the MILP formulation:

1. v might be the starting event of multiple routes. Since multiple transfers can be possible, only once the timetable is built can we determine which route has smallest perceived length among the routes in $\mathcal{R}^k$ starting at event v.
2. There might exist another route with a different starting event $v' \neq v$ minimising the perceived travel time of the passenger arriving before v. In such a case, the passenger might not want to start his route with event v, wait longer at the origin station, and take the new route with starting event v'.

To account for these situations, Y_v^k is defined by minimising the weighted sum of two components. The first component of Y_v^k, representing the (potential) additional adaption time from event v, can be written as

$$\gamma_w \cdot (\Delta_{v,v'}), \qquad \text{with } \Delta_{v,v'} = \pi_{v'} - \pi_v + T\alpha_{v,v'} \tag{7}$$

for each starting event $v' \in V^k$. In Equation (7), $\Delta_{v,v'}$ is the difference in time between starting events v and v', and $\alpha_{v,v'}$ is a binary variable used to model the modulo operator ensuring that $\Delta_{v,v'} \in \{0, \ldots, T-1\}$. The second component is the travel time of the route $r \in \mathcal{R}^k$ with starting event $\sigma(r) = v'$. For each $\mathcal{OD}$-pair $k \in \mathcal{OD}$ and event $v \in V^k$ we define Y_v^k as

$$Y_v^k = \min_{v'\in V^k, r\in\mathcal{R}^k:\sigma(r)=v'} \left\{ \Delta_{v,v'} \cdot \gamma_w + Y_r \right\}. \tag{8}$$

The demand d_k $\forall k \in \mathcal{OD}$ is assumed to be uniformly distributed over the period. Hence, the number of passengers between an event v and its preceding event v with perceived travel time Y_v^k is equal to d_k/T multiplied by the time difference between event $v \in V^k$ and the latest preceding event $v' \in V^k$ according to timetable π. Let this time difference be denoted L_v^k, then, we have

$$L_v^k := \min_{v'\in V^k\setminus\{v\}} \{\Delta_{v',v}\}. \tag{9}$$

Furthermore, the average adaption time of the passengers arriving during this time interval, defined as W_v^k, is equal to half the length of the time interval, that is,

$$W_v^k = \frac{1}{2} L_v^k. \tag{10}$$

Using this notation, the objective function of the problem can be rewritten as

$$\sum_{k \in \mathcal{OD}} \frac{d_k}{T} \sum_{v \in V^k} L_v^k \cdot (\gamma_w \cdot W_v^k + Y_v^k). \tag{11}$$

4.3 Passenger-Oriented Timetabling Model with Flexible Frequencies

Using the constraints and objective defined in Sections 4.1 and 4.2, we can write the MILP of the Passenger-Oriented Timetabling problem with flexible frequencies as

$$
\begin{aligned}
\min \quad & \sum_{k \in \mathcal{OD}} \frac{d_k}{T} \sum_{v \in V^k} L_v^k \cdot (\gamma_w \cdot W_v^k + Y_v^k) && && \text{(12a)}\\
\text{s.t.} \quad & y_{ij} = \pi_j - \pi_i + T p_{ij} && \forall (i,j) \in A && \text{(12b)}\\
& \tau_{\ell,tr} l_{ij} \le y_{ij} \le \tau_{\ell,tr} u_{ij} && \forall \ell \in \mathcal{L}, \forall tr \in [\overline{f}_\ell], && \text{(12c)}\\
& && \text{and } \forall (i,j) \in A[\ell, tr], && \\
& y_{ij} \ge (\tau_{\ell,tr} + \tau_{\ell',tr'} - 1) l_{ij} && \forall (i,j) \in A : i \in V[tr,\ell], && \text{(12d)}\\
& && j \in V[tr',\ell'], && \\
& && \text{and } (\ell,tr) \ne (\ell',tr'), && \\
& y_{ij} \le u_{ij} + (2 - \tau_{\ell,tr} - \tau_{\ell',tr'}) T && \forall (i,j) \in A : i \in V[tr,\ell], && \text{(12e)}\\
& && j \in V[tr',\ell'], && \\
& && \text{and } (\ell,tr) \ne (\ell',tr'), && \\
& \tau_{\ell,tr} = 1 && \forall \ell \in \mathcal{L}, \forall tr \in [\underline{f}_\ell], && \text{(12f)}\\
& Y_r = \sum_{(i,j) \in r} \Big[y_{ij} + \gamma_t \cdot \mathbb{1}_t(i,j) \Big] && \forall k \in \mathcal{OD}, \forall r \in \mathcal{R}^k && \text{(12g)}\\
& \qquad + \sum_{(\ell,tr) \in r} M_k^\tau \cdot (1 - \tau_{\ell,tr}) && && \\
& \Delta_{v,v'} = \pi_{v'} - \pi_v + T \alpha_{v,v'} && \forall k \in \mathcal{OD}, v \in V^k, && \text{(12h)}\\
& && v' \in V^k \setminus \{v\} && \\
& L_v^k = \min\{T, \min_{v' \in V^k \setminus \{v\}} \{\Delta_{v',v}\}\} && \forall k \in \mathcal{OD}, v \in V^k, && \text{(12i)}\\
& \alpha_{v,v'} = 1 - \alpha_{v',v} && \forall k \in \mathcal{OD}, v \in V^k, && \text{(12j)}\\
& && v' \in V^k \setminus \{v\} && \\
& Y_v^k = \min_{v' \in V^k, r \in \mathcal{R}^k : \sigma(r) = v'} \{\Delta_{v,v'} \cdot \gamma_w + Y_r\} && \forall k \in \mathcal{OD}, v \in V^k, && \text{(12k)}\\
& W_v^k = \frac{1}{2} L_v^k && \forall k \in \mathcal{OD}, v \in V^k, && \text{(12l)}\\
& \textit{variable domains} && && \text{(12m)}
\end{aligned}
$$

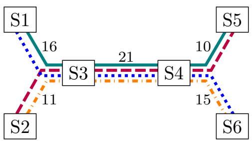
The objective function (12a) represents the sum of perceived passenger travel time for all $\mathcal{OD}$-pairs. Constraints (12b) define the time duration of an activity $(i,j) \in A$. Constraints (12c) define the lower- and upper-bounds on the duration of drive and dwell activities, and

Constraints (12d-12e) represent respectively the lower- and upper-bounds on the duration of transfer and headway activities. Constraints (12f) ensure that the minimum train frequencies are met. Constraints (12g) define the perceived duration of a route r. Constraints (12h) measure the time difference between two starting events of an $\mathcal{OD}$-pair k and Constraints (12i) measure the time difference between an event v and its closest predecessor. Constraints (12j) ensure that for each pair $(v, v') : v, v' \in V^k$ and $v \neq v'$, either $\alpha_{v,v'}$ or $\alpha_{v',v}$ (but not both) is equal to 1. Constraints (12k) measure the minimum perceived travel time of a passenger who arrived at the station between event v and its predecessor. Finally, Constraints (12l) measure the average waiting time of a passenger before an event v. The variable domains (12m) are available in Appendix A.

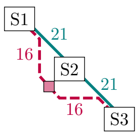
Note that the Objective (12a) and Constraints (12i) and (12k) are not yet linear in this formulation. Further detail about the linearisation of the constraints and objective is available in Appendix C for the interested reader.

5 Experiments

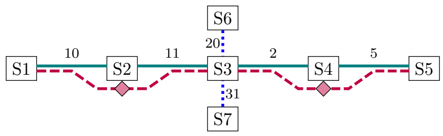
The model defined in Section 4 is implemented in Java 13.0.3 and solved using CPLEX 22.1.0 for three instances. All experiments are run on the Dutch National Supercomputer Snellius with 32 cores and 240 Gb of RAM per experiment. Each experiment is run until optimality, or until the memory limit is exceeded. The solution of the POT formulation (also implemented in Java 13.0.3 solved using CPLEX 22.1.0) from [11] using the maximum frequency $\overline{f}_\ell$ as fixed frequency is used as a benchmark for solution quality. We consider for each instance the time period to be $T = 60$ (minutes), the penalty values to be $\gamma_t = 20$ and $\gamma_w = 2$, and double-track railway segments (i.e. no overtaking).



(a) Instance 1.



(b) Instance 2.



(c) Instance 3.

Figure 3 Test instances. Coloured squares next to stations are used to indicate when a line goes thought the station but does not stop at the station (transit station). The straight (green) lines represent line ℓ_1, the dashed (purple) lines represent line ℓ_2, the dotted (blue) lines represent line ℓ_3, and the dash-dotted (orange) lines represent line ℓ_4.

5.1 Instances

The model is tested on three instances visualised in Figure 3. Each instance provides a different insight regarding the price of fixed frequencies in the creation of timetables. The complexity of both formulations only allows us to prove the optimality of the solutions of Instance 2, thereby limiting the maximum size of instances that can be studied.

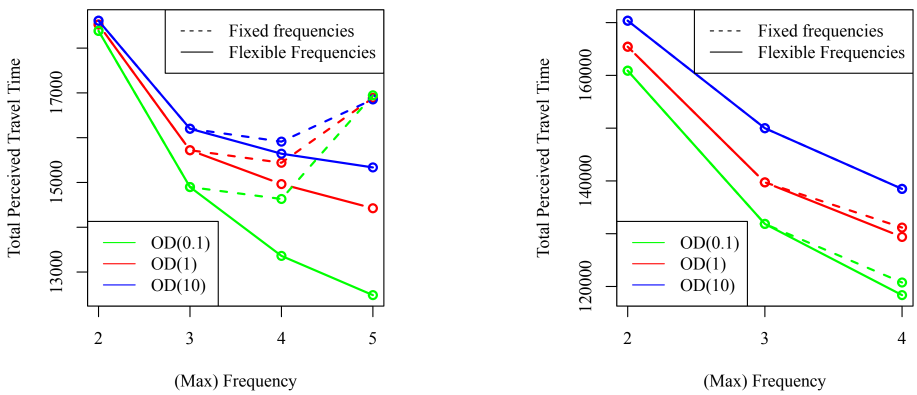
In Instance 1, we consider a network using a central connection (S3-S4) extensively, leading to a bottleneck. For this instance, a fixed line frequency of 2 for all stations is infeasible for POT formulation. In comparison, the POT-flex formulation with $(\underline{f}_\ell, \overline{f}_\ell) = (1, 2)$ provides insight into which lines can be increased to obtain a better feasible timetable.

In Instances 2 and 3, we consider, similar to the example in the introduction, a "slow" line that stops at all stations and a "fast" line only stopping at the main stations. We call stations where some lines go through but do not stop *transit stations*. These instances provide insights in situations where there is a large time difference between the departure from the first station and the arrival at the last station between two lines. These situations are common in real life instances such as in the Dutch Railway Network. Further, Instance 3 contains an additional line and additional stations to study the performance of the formulation for a larger instance that also includes transfer decisions.

The $\mathcal{OD}$-matrices used to simulate the demand in the instances are defined as follows. We define $\mathcal{OD}(n)$ as the $\mathcal{OD}$-matrix such that the demand d_k of any $\mathcal{OD}$-pair k where the origin and the destination are main stations (i.e. not transit stations) is equal to n times the demand $d_{k'}$ where k' is an $\mathcal{OD}$-pair such that either the origin or the destination is a transit station. This allows us to evaluate Instances 2 and 3 for varying demand from the transit stations in comparison to the rest of the network. Instance 1 is tested with $\mathcal{OD}(1)$ (uniform demand), and Instances 2 and 3 are tested with $\mathcal{OD}(0.1)$ (high demand at transit stations), $\mathcal{OD}(1)$, and $\mathcal{OD}(10)$ (low demand at transit stations). For consistency, we keep the same total number of passengers for each OD-matrix of each instance. The matrices are available in Appendix D.2.

Table 1 Results of the experiments for Instance 1.

			Objective		Optimality Gap		Optimal Frequencies			
	$\underline{f}_\ell$	$\overline{f}_\ell$	POT	POT-flex	POT	POT-flex	ℓ_1	ℓ_2	ℓ_3	ℓ_4
Instance 1	1	1	127,000	127,000	6.47%	6.53%	1	1	1	1
	1	2	infeasible	107,964	-	16.1%	1	2	2	1



(a) Results Instance 2. **(b)** Results Instance 3.

Figure 4 Objective values of the POT and POT-flex formulations for Instances 2 and 3.

5.2 Results

As aforementioned, only Instance 2 can be solved to optimality under the memory restrictions using both formulations. We therefore provide the optimality gaps for the best found solutions of Instances 1 and 3 respectively in Tables 1 and 3.

Table 1 summarises the solutions found for Instance 1. As the (maximum) frequency increases to 2, the POT problem fails to find a feasible solution. In contrast, the POT-flex model not only finds a feasible solution, but also selects the best combination of trains such as to minimise the total perceived travel time. This highlights the feasibility repair advantage of POT-flex over POT to find solutions minimising perceived passenger travel time.

Table 2 Line frequencies of the solution of the POT-flex problem and objective difference with the solution of the POT problem with fixed frequency $\overline{f}_\ell$ for Instances 2 and 3. **The best found solutions for Instance 3 are not proven by the solver to be optimal. Optimality gaps are presented in Table 3.*

			$\mathcal{OD}(0.1)$				$\mathcal{OD}(1)$				$\mathcal{OD}(10)$			
	$\underline{f}_\ell$	$\overline{f}_\ell$	ℓ_1	ℓ_2	ℓ_3	Δ Obj	ℓ_1	ℓ_2	ℓ_1	Δ Obj	ℓ_1	ℓ_2	ℓ_3	Δ Obj
Instance 2	1	3	3	3	-	0%	3	3	-	0%	3	3	-	0%
	1	4	4	2	-	8.7%	4	3	-	3%	4	3	-	1.7%
	1	5	5	1	-	26.3%	5	2	-	14.5%	5	3	-	9%
Instance 3*	1	3	3	3	3	0%	3	3	3	0%	3	3	3	0%
	1	4	4	2	4	2%	4	3	4	1.3%	4	4	4	0%

Table 3 Optimality gaps of the best solutions of the POT and POT-flex for Instance 3.

			$\mathcal{OD}(0.1)$		$\mathcal{OD}(1)$		$\mathcal{OD}(10)$	
	$\underline{f}_\ell$	$\overline{f}_\ell$	POT	POT-flex	POT	POT-flex	POT	POT-flex
Instance 3	1	3	4.15%	11.05%	3.78%	11.36%	4.60%	9.89%
	1	4	5.88%	18.01%	6.34%	19.08%	3.98%	18.18%

Figure 4 shows the objective values of the POT and POT-flex formulations for Instance 2 and 3. Table 2 reports on the line frequencies selected in the optimal solution of POT-flex and the improvements in % between the found solutions of POT-flex and POT with fixed frequencies $\overline{f}_\ell$. Note that the solutions found for Instance 3 are not proven to be optimal by the solver, hence, Table 3 provide the optimality gaps for the solutions of Instance 3. This is one of the primary limitations of the formulation, as due to its complexity, even small instances can not be easily solved to optimality (or proven to be optimal by the solver).

The results show in Instances 2 and 3 that, as the number of trains to schedule increases, the POT-flex formulation leads to equal or lower objectives than the POT formulation by not scheduling certain trains. As the demand for transit stations increases, the importance of a timetable that provides a lower perceived travel time for transit stations at the expense of a lower frequency for another line becomes an apparent trade-off for the model and can lead to significant improvements. In Instance 2, this can lead to up to a 26.3% improvement by not scheduling 4 trains. While the large scale of this improvement is likely due to the small size of Instance 2, we can observe similar improvements of smaller magnitude for Instance 3 by reducing the frequency of ℓ_2, leading to a 2% improvement in objective. Furthermore, these improvements are made despite the larger optimality gap of the POT-flex solution, resulting from larger feasibility region of the POT-flex problem. The fact that such improvements are possible, even in small instances, shows the price that one may pay by assuming fixed frequencies.

6 Conclusion

In this paper, we introduce and study the Passenger Oriented Timetabling Problem with flexible frequencies (POT-flex). We develop a MILP formulation and provide insights on the advantages of providing more freedom to the timetabling model through experiments on three instances. The POT-flex formulation allowed to find solutions for instances where the maximum frequencies could originally not be simultaneously realised, and showed up to 2% improvements in total perceived passenger time for the largest tested instance. These improvements all came from the ability of the model to select the optimal line frequencies with respect to the demand. These improvements represent the cost that fixed frequency can have on timetabling.

References

1 Eva Barrena, David Canca, Leandro C Coelho, and Gilbert Laporte. Single-line rail rapid transit timetabling under dynamic passenger demand. *Transportation Research Part B: Methodological*, 70:134–150, 2014.

2 Michael R Bussieck, Thomas Winter, and Uwe T Zimmermann. Discrete optimization in public rail transport. *Mathematical programming*, 79(1):415–444, 1997.

3 Florian Fuchs, Alessio Trivella, and Francesco Corman. Enhancing the interaction of railway timetabling and line planning with infrastructure awareness. *Transportation Research Part C: Emerging Technologies*, 142:103805, 2022.

4 Johann Hartleb and Marie Schmidt. Railway timetabling with integrated passenger distribution. *European Journal of Operational Research*, 298(3):953–966, 2022.

5 Johann Hartleb, Marie Schmidt, Dennis Huisman, and Markus Friedrich. Modeling and solving line planning with integrated mode choice. *Available at SSRN 3849985*, 2021.

6 Dennis Huisman, Leo G Kroon, Ramon M Lentink, and Michiel JCM Vromans. Operations research in passenger railway transportation. *Statistica Neerlandica*, 59(4):467–497, 2005.

7 Mor Kaspi and Tal Raviv. Service-oriented line planning and timetabling for passenger trains. *Transportation Science*, 47(3):295–311, 2013.

8 Christian Liebchen and Rolf H Möhring. The modeling power of the periodic event scheduling problem: railway timetables—and beyond. In *Algorithmic methods for railway optimization*, pages 3–40. Springer, 2007.

9 Berenike Masing, Niels Lindner, and Christian Liebchen. Periodic timetabling with integrated track choice for railway construction sites. Technical report, Zuse Institute Berlin, 2022.

10 Mathias Michaelis and Anita Schöbel. Integrating line planning, timetabling, and vehicle scheduling: a customer-oriented heuristic. *Public Transport*, 1(3):211–232, 2009.

11 Gert-Jaap Polinder, Valentina Cacchiani, Marie Schmidt, and Dennis Huisman. An iterative heuristic for passenger-centric train timetabling with integrated adaption times. *Computers & Operations Research*, page 105740, 2022.

12 Gert-Jaap Polinder, Leo Kroon, Karen Aardal, Marie Schmidt, and Marco Molinaro. Resolving infeasibilities in railway timetabling instances. *Available at SSRN 3106739*, 2018.

13 Gert-Jaap Polinder, Marie Schmidt, and Dennis Huisman. Timetabling for strategic passenger railway planning. *Transportation Research Part B: Methodological*, 146:111–135, 2021.

14 Philine Schiewe and Anita Schöbel. Periodic timetabling with integrated routing: Toward applicable approaches. *Transportation Science*, 54(6):1714–1731, 2020.

15 Anita Schöbel. Line planning in public transportation: models and methods. *OR spectrum*, 34(3):491–510, 2012.

16 Anita Schöbel. An eigenmodel for iterative line planning, timetabling and vehicle scheduling in public transportation. *Transportation Research Part C: Emerging Technologies*, 74:348–365, 2017.

17 Paolo Serafini and Walter Ukovich. A mathematical model for periodic scheduling problems. *SIAM Journal on Discrete Mathematics*, 2(4):550–581, 1989.

18 Michael Siebert and Marc Goerigk. An experimental comparison of periodic timetabling models. *Computers & Operations Research*, 40(10):2251–2259, 2013.

19 Raimond Wüst, Stephan Bütikofer, Severin Ess, Claudio Gomez, Albert Steiner, Marco Laumanns, and Jacint Szabo. Periodic timetabling with 'track choice'-pesp based on given line concepts and mesoscopic infrastructure. In *Operations Research Proceedings 2018: Selected Papers of the Annual International Conference of the German Operations Research Society (GOR), Brussels, Belgium, September 12-14, 2018*, pages 571–578. Springer, 2019.

20 Yuting Zhu, Baohua Mao, Yun Bai, and Shaokuan Chen. A bi-level model for single-line rail timetable design with consideration of demand and capacity. *Transportation Research Part C: Emerging Technologies*, 85:211–233, 2017.

A Notation

Table 4 Notation of the sets, variables, and constants used throughout the paper.

Sets		
Notation	Description	
$\mathcal{OD}$	Set of all Origin-Destination pairs	
$\mathcal{L}$	Set of lines in the network	
$\mathcal{R}^k$	Set of routes serving an $\mathcal{OD}$-pair k	
V	Set of events	
V^k	Set of starting event of routes serving an $\mathcal{OD}$-pair k	
$V[\ell, tr]$	Set of events related to train tr in line ℓ	
A	Set of Activities	
$A[\ell, tr]$	Set of activities related to train tr in line ℓ	
Constants		
Notation	Description	
T	Cycle Period	
d_k	Number of passengers per cycle period T for an $\mathcal{OD}$-pair k	
γ_t	Penalty value for a transfer activity in a route	
γ_w	Penalty factor for the waiting time until the next route chosen	
M_k^τ	Big-M penalty value for a route containing a train that is not scheduled	
Variables		
Notation	Description	Domain
π_i	time at which event $i \in V$ happens	$\{0, \ldots, T-1\}$
y_{ij}	duration of activity $(i, j) \in A$	$\mathbb{Z}_{\geq 0}$
p_{ij}	modulo parameter used for the shift from one cycle period to another, for activity $(i, j) \in A$	$\mathbb{Z}_{\geq 0}$
$\tau_{\ell,tr}$	binary variable indicating whether a train (ℓ, tr) is scheduled	{0,1}
Y_r	perceived travel time by a passenger from an $\mathcal{OD}$-pair k for a route $r \in \mathcal{R}$	$\mathbb{Z}_{\geq 0}$
$\Delta_{v,v'}$	time difference between event v and v'	$[0, T]$
L_v^k	number of minutes before event v, in which no other departure event for $\mathcal{OD}$-pair k takes place	$[0, T]$
Y_v^k	perceived travel time for passengers of $\mathcal{OD}$-pair k, from the timing of event v onwards	$\mathbb{Z}_{\geq 0}$
$\alpha_{v,v'}$	binary variable ensuring the correct determination of the time difference between event v and v'	{0,1}
W_v^k	expected waiting time for passenger for $\mathcal{OD}$-pair k, for who event v is the next departure event	$[0, T/2]$

B Lower Bound for Non-Scheduled Train Penalty in Route Length Computation

For computational stability, it is important to chose a value of M_k^τ that is as low as possible. Consequently, we chose M_k^τ as the maximum travel time over all routes that the corresponding $\mathcal{OD}$-pair could take, plus the waiting time for a full period, that is

$$M_k^\tau := \max_{r' \in \mathcal{R}^k} \{\bar{Y}_{r'}\} + \gamma_w T = \max_{r' \in \mathcal{R}^k} \left\{ \sum_{(i,j) \in r'} u_{ij} + \gamma_t \cdot \mathbb{1}_t(i,j) \right\} + \gamma_w T. \tag{13}$$

This ensures that there always will be a route in $\mathcal{R}^k$ that has a shorter travel time than M_k^τ. Hence, no route containing an activity related to a train that is not scheduled will be chosen.

C Linearisation of the Mixed Integer Linear Program

C.1 Linearisation of the Minimum Time Difference Between Two Routes

Since L_v^k appears both in constraint (12i) and in the objective function, our first step is to find a way to linearise this variable. Let us introduce a variable A_v^k that denotes for an $\mathcal{OD}$-pair k the time difference between the starting event v and its predecessor starting event. That is, $A_v^k := \Delta_{\hat{v},v}$ for $\hat{v}$ being the departure event in V^k that precedes v, or

$$A_v^k := \begin{cases} \min_{\hat{v}\in V^k\setminus v}\{(\pi_v - \pi_{\hat{v}}) \mod T\} & \text{if } |V^k| > 1 \\ T & \text{otherwise} \end{cases}$$

Then, for each $\mathcal{OD}$-pair k, The variable A_v^k is defined using the following set of constraints:

$$0 \le \Delta_{v,v'} = \pi_{v'} - \pi_v + T\alpha_{v,v'} \qquad \forall v \in V^k, \forall v' \in V^k \setminus \{v\} \tag{14a}$$

$$\alpha_{v,v'} + \alpha_{v',v} = 1 \qquad \forall v \in V^k, \forall v' \in V^k \setminus \{v\} \tag{14b}$$

$$0 \le A_v^k \le \Delta_{v',v} \qquad \forall v \in V^k, \forall v' \in V^k \setminus \{v\} \tag{14c}$$

$$\sum_{v\in V^k} A_v^k = T \tag{14d}$$

Constraints (14a) computes the time difference between two events v and v', and constraints (14b) allows us to determine which event happens first within the period. If $\alpha_{v,v'} = 0$, then event v is scheduled before event v' in the period (and therefore $\pi_{v'} > \pi_v$). Constraint (14c) restricts the maximum value of A_v^k such that A_v^k can be at most the minimum time difference between v and any other event v'. Together with constraints (14d), which ensures that the sum of all times between events is be equal to T, this set of constraints ensures that A_v^k is the minimum length of time between v and the next event v'.

This property is kept even when v and/or v' belong to non-scheduled trains as, because $Y_v^k = \min_{v'\in V^k}\left\{Y_r + \Delta_{v,v'} \cdot \gamma_w | r \in \mathcal{R}^k, \sigma(r) = v'\right\}$, even if a train is not scheduled, the next scheduled train will be selected due to the large penalty for a non-scheduled train.

C.2 Linearisation of the Objective Function

Due to the relationship between W_v^k and L_v^k defined in Constraints (12l), the objective function can be rewritten as

$$\sum_{k\in\mathcal{OD}} \frac{d_k}{T} \sum_{v\in V^k} L_v^k \cdot (\gamma_w \cdot W_v^k + Y_v^k) = \sum_{k\in\mathcal{OD}} \frac{d_k}{T} \sum_{v\in V^k} \frac{\gamma_w}{2}(L_v^k)^2 + L_v^k \cdot Y_v^k \tag{15}$$

As the objective is quadratic with respect to L_v^k, we must linearise it. Using the previously defined variable A_v^k, we define a new variable $x_{v,d}^k$ such that

$$x_{v,d}^k = \begin{cases} 1 & \text{if } A_v^k \ge d \\ 0 & \text{otherwise} \end{cases} \qquad \forall k \in \mathcal{OD}, v \in V^k, d \in \{1,\dots,T\}$$

$$A_v^k = \sum_{d=1}^{T} x_{v,d}^k$$

This allows us to rewrite $(A_v^k)^2$ as follows:

$$(A_v^k)^2 = \sum_{d=1}^{T} (2d-1) \cdot x_{v,d}^k$$

Furthermore, we introduce the variable $R^k_{v,d} = x^k_{v,d} \cdot Y^k_v$ such that $R^k_{v,d}$ takes the value Y^k_v (length of shortest route starting from v for $\mathcal{OD}$-pair k) if the interval A^k_v corresponding to v is greater than or equal to d, and the value 0 otherwise. To set R^d_v to the required values we impose that

$$Y^k_v - u^k_v \times (1 - x^k_{v,d}) \leq R^k_{v,d} \leq u^k_v \times x^k_{v,d}. \tag{16a}$$

where u^k_v is a parameter defining an upper bound on the length of a shortest route over all timetables. Since the upper-bound is most likely defined based on the maximum penalty a cancelled train will have, then

$$u^k_v = \max_{r \in \mathcal{R}^k} \left\{ \sum_{(\ell,tr)\in r} M^\tau_k \right\}$$

Which represents the maximum amount of times that the penalty M^τ_k can be applied for an $\mathcal{OD}$-pair k. Given the set $\mathcal{R}^k$ of possible routes, this can easily be computed beforehand. Using those two new variables, we can rewrite the objective as:

$$\begin{aligned}
\sum_{k\in\mathcal{OD}} \frac{d_k}{T} \sum_{v\in V^k} L^k_v \cdot (\gamma_w \cdot W^k_v + Y^k_v) &= \sum_{k\in\mathcal{OD}} \frac{d_k}{T} \sum_{v\in V^k} \frac{\gamma_w}{2}(A^k_v)^2 + A^k_v \cdot Y^k_v \\
&= \sum_{k\in\mathcal{OD}} \frac{d_k}{T} \sum_{v\in V^k} \sum_{d=1}^{T} \left[\frac{\gamma_w}{2}(2d-1)\cdot x^k_{v,d} + x^k_{v,d}\cdot Y^k_v\right] \\
&= \sum_{k\in\mathcal{OD}} \frac{d_k}{T} \sum_{v\in V^k} \sum_{d=1}^{T} \left[\frac{\gamma_w}{2}(2d-1)\cdot x^k_{v,d} + R^k_{v,d}\right]
\end{aligned}$$

C.3 Linearisation of the Minimum Perceived Travel Time

The variable Y^k_v represents, for an $\mathcal{OD}$-pair k, the minimum perceived travel time of a passenger who arrived at the station between the starting event $v \in V^k$ and the starting event preceding v. Constraints (12k) model Y^k_v using a minimum that we must linearise. To that end, we define the binary variable $z^k_{v,v',r}$ such that

$$z^k_{v,v',r} = \begin{cases} 1 & \text{if passengers wait from event } v \text{ to } v' \text{ to use the route } r, \\ 0 & \text{otherwise,} \end{cases}$$
$$\forall k \in \mathcal{OD}, \forall v, v' \in V^k, \forall r \in \mathcal{R}^k : \sigma(r) = v'. \tag{17}$$

Note that v and v' can be the same event, and r refers to all possible routes starting with event v'. Given $z^k_{v,v',r}$, Constraints (12k) can now be rewritten for every $k \in \mathcal{OD}$ and for every $v \in V^k$ as the set of constraints

$$Y^k_v \leq Y_r + \gamma_w \Delta_{v,v'} \qquad \forall v' \in V^k, \forall r \in \mathcal{R}^k : \sigma(r) = v', \tag{18a}$$
$$Y^k_v \geq Y_r + \gamma_w \Delta_{v,v'} - M^k_v \times (1 - z^k_{v,v',r}) \qquad \forall v' \in V^k, \forall r \in \mathcal{R}^k : \sigma(r) = v', \tag{18b}$$
$$\sum_{v'\in V^k} \sum_{r\in\mathcal{R}^k:\sigma(r)=v'} z^k_{v,v',r} = 1. \tag{18c}$$

Constraints (18a) and (18b) provide respectively an upper- and lower-bound for Y^k_v. The big-M value M^k_v is a value large enough to ensure that the lower-bound of Y^k_v is always the minimum perceived passenger travel time at event v. Finally, Constraints (18c) ensure that only one route is selected for passengers arriving between the starting event v and its predecessor.

Again, for computational stability, M_v^k has to be a small as possible, but large enough to make Constraints (18b) redundant if $z^k_{v,v',r} = 0$. We can take

$$M_v^k = \gamma_w T + \max_{r \in \mathcal{R}^k} \{\overline{Y}_r\} - \max_{r \in \mathcal{R}^k} \{\underline{Y}_r\} \tag{19}$$

where $\overline{Y}_r$ and $\underline{Y}_r$ denote respectively the highest and lowest possible value for variable Y_r.

D Empirical Experiment Parameters

D.1 Event Activity Network Parameters

Table 5 Activity Constraint Bounds of Instance 1.

Activity constraint bounds						
	Headway:	[8,52]	Transfer:	[5,64]	Dwell:	[2,3]
Drive	S1-S3:	[16,18]	S2-S3:	[11,13]	S3-S4:	[21,24]
	S4-S5:	[10,11]	S4-S6:	[15,17]		

Table 6 Activity Constraint Bounds of Instance 2. **If a line goes through a transit station but does not stop at the station, the dwell activity constraint bounds are [0,0].*

Activity constraint bounds						
	Headway:	[5,55]	Transfer:	[5,64]	Dwell*:	[2,3]
Drive	S1-S2(ℓ_1):	[21,24]	S2-S3(ℓ_1):	[21,24]		
	S1-S2(ℓ_2):	[16,18]	S2-S3(ℓ_2):	[16,18]		

Table 7 Activity Constraint Bounds of Instance 3. **If a line goes through a transit station but does not stop at the station, the dwell activity constraint bounds are [0,0].*

Activity constraint bounds						
	Headway:	[5,55]	Transfer:	[5,64]	Dwell:	[2,3]
Drive	S1-S2:	[10,11]	S2-S3:	[11,13]	S3-S4:	[2,3]
	S4-S5:	[5,6]	S3-S6:	[20,22]	S3-S7	[31,35]

D.2 Origin-Destination Matrices

	S1	S2	S3
S1	0.0	85.7	8.6
S2	85.7	0.0	85.7
S3	8.6	85.7	0.0

(a) $\mathcal{OD}(0.1)$ for Instance 2.

	S1	S2	S3
S1	0.0	60	60
S2	60	0.0	60
S3	60	60	0.0

(b) $\mathcal{OD}(1)$ for Instance 2.

	S1	S2	S3
S1	0.0	15.0	150.0
S2	15.0	0.0	15.0
S3	150.0	15.0	0.0

(c) $\mathcal{OD}(10)$ for instance 2.

	S1	S2	S3	S4	S5	S6
S1	0.0	60	60	60	60	60
S2	60	0.0	60	60	60	60
S3	60	60	0.0	60	60	60
S4	60	60	60	0.0	60	60
S5	60	60	60	60	0.0	60
S6	60	60	60	60	60	0.0

(d) $\mathcal{OD}(1)$ for Instance 1.

	S1	S2	S3	S4	S5	S6	S7
S1	0.0	60	60	60	60	60	60
S2	60	0.0	60	60	60	60	60
S3	60	60	0.0	60	60	60	60
S4	60	60	60	0.0	60	60	60
S5	60	60	60	60	0.0	60	60
S6	60	60	60	60	60	0.0	60
S7	60	60	60	60	60	60	0.0

(e) $\mathcal{OD}(1)$ for Instance 3.

	S1	S2	S3	S4	S5	S6	S7
S1	0.0	104.6	10.9	104.6	10.9	10.9	10.9
S2	104.6	0.0	104.6	105.0	104.6	104.6	104.6
S3	10.9	104.6	0.0	104.6	10.9	10.9	10.9
S4	104.6	105.0	104.6	0.0	104.6	104.6	104.6
S5	10.9	104.6	10.9	104.6	0.0	10.9	10.9
S6	10.9	104.6	10.9	104.6	10.9	0.0	10.9
S7	10.9	104.6	10.9	104.6	10.9	10.9	0.0

(f) $\mathcal{OD}(0.1)$ for Instance 3.

	S1	S2	S3	S4	S5	S6	S7
S1	0.0	11.37	113.5	11.36	113.5	113.5	113.5
S2	11.37	0.0	11.37	11.36	11.36	11.36	11.36
S3	113.5	11.37	0.0	11.37	113.5	113.5	113.5
S4	11.36	11.36	11.37	0.0	11.37	11.36	11.36
S5	113.5	11.36	113.5	11.37	0.0	113.5	113.5
S6	113.5	11.36	113.5	11.36	113.5	0.0	113.5
S7	113.5	11.36	113.5	11.36	113.5	113.5	0.0

(g) $\mathcal{OD}(10)$ for Instance 3.

Figure 5 $\mathcal{OD}$-Matrices used for experiments.