An Integrated Uncertainty and Sensitivity Analysis for Spatial Multicriteria Models

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Abstract

This paper introduces an integrated Uncertainty and Sensitivity Analysis (US-A) approach for Spatial Multicriteria Models (SMM). The US-A approach evaluates uncertainty and sensitivity by considering both criteria values and weights, providing spatially distributed measures. A geodiversity assessment case study demonstrates the application of US-A, identifying influential inputs driving uncertainty in specific areas. The results highlight the importance of considering both criteria values and weights in analyzing model uncertainty. The paper contributes to the literature on spatially-explicit uncertainty and sensitivity analysis by providing a method for analyzing both categories of SMM inputs: evaluation criteria values and weights, and by presenting a novel form of visualizing their sensitivity measures with bivariate maps.

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1 Introduction

Uncertainty analysis (UA) and sensitivity analysis (SA) are two complementary methods of evaluating uncertainty present in model inputs and, by extension, in model results [12]. UA quantifies outcome variability given model input uncertainties, and is, therefore, forward-looking as it focuses on evaluating how the uncertainty of inputs propagates through the model and affects its output values. However, UA does not inform about the magnitude of individual inputs' influence on model output variability. This information can be obtained from SA that relates the output variability to model inputs and evaluates how much each source of uncertainty contributes to the overall variability of the output. In this sense, SA is a backward-looking approach that complements UA.

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Spatial Multicriteria Models (SMM) implemented in the context of GIS-based multicriteria analysis employ either value function-based methods or outranking relation-based methods to arrive at a rank-order/classification of spatially-explicit choice alternatives [8]. In SMM that employ value function methods, the rank order is determined by a synthetic score expressing the overall strength of each choice alternative vis-à-vis other alternatives under consideration. The score is calculated by integrating criteria values with weights using a combination rule. Due to potential errors in criteria values and the subjectivity of weights, both types of inputs can become potential sources of uncertainty affecting the SMM output. The overall impact of uncertainty can be represented by a measure of output variability (e.g., variance), which is also a proxy of output uncertainty. In order to isolate influential inputs driving the model's output uncertainty, one can employ SA. Ultimately, the purpose of UA combined with SA is to improve the model's reliability and its value for policy and decision-making.

2 Related work

Two approaches to UA-SA – local and global, have been proposed for SMM. In the local approach, the values of model inputs are varied one at a time (OAT) while keeping other inputs unchanged. This approach has been popular among modelers due to its simplicity, tractability, and low computational cost [15]. Yet, in SMM based on compensatory decision rules (i.e., Weighted Linear Combination, Analytical Hierarchy Process), model inputs do interact, and the OAT approach does not address these interaction effects. In contrast, the global approach accounts for model input interactions by more or less systematically sampling the entire input value space [7]. The downside of the global approach is its computational cost. Different solutions to accelerating global SA for spatial models have been proposed, including parallelization [1], [5] and surrogate models [11].

In an early example of global approach for SMM, [3] used variance decomposition-based SA to investigate model's solution stability in light of uncertainties affecting criteria values and weights. In their study, SA was performed on aggregated criteria values and weights, producing one measure of sensitivity for each input for the entire study area. This approach to UA-SA takes spatially explicit inputs, identifies among them the influential ones that drive the model's output variability, and returns non-spatial estimates of sensitivity without providing a crucial piece of information – namely, where in the study area this influence plays out. Others, including [6], [2], and [10] proposed a spatially explicit and integrated approach to UA-SA of SMM, henceforth referred to as US-A, based on global variance decomposition, in which the output of SMM results in spatially distributed measures of uncertainty and sensitivity. Their work, however, addressed only one category of uncertain inputs: criteria weights. The work presented here extends it by providing a method for analyzing both categories of SMM input: criteria values, and weights. Additionally, it presents a novel form of visualizing their sensitivity measures with bivariate maps.

3 Methods

The US-A of variable criteria and weights is presented in Figure 1. In this approach, weights Wn are represented as probability distributions, whereas criteria Cn are represented as sets of k multiple alternative layers. Since both types of inputs are stochastic, a given SMM *f(W,C)* has to be calculated multiple times, each time with a different vector of input values. Each calculation uses n scalars for W and n maps for C, where the scalars are derived from weights' respective probability distributions and the maps from their respective sets of realizations.

The sampling used to generate the vectors is called Sobol's quasi-random with radials and is described in [14]. As a result, we obtain a distribution of SMM spatial outputs, for which we can calculate different aggregation statistics' maps like mean or standard deviation. Both statistics can then be used jointly (Figure 2) as an uncertainty map.

The next step involves spatially-explicit variance decomposition, independently applied to every spatial unit *(su)* in the study area (e.g., raster cell, vector polygon). Variance decomposition involves subdividing the total variance of *su* creating partial variances for each input [14], [13]. The procedure produces two sensitivity indices per input – First Order Effects Index and Total Effects Index. The former is the input's fractional contribution to the total variance when the given input is treated independently from all other inputs. The latter is the input's fractional contribution to the total variance due to its independent influence and interactions with other inputs. Consequently, the difference between the Total Effects Index and the First Order Effects Index is the input's interactions (Figure 3, legend). The final results comprise 2N sensitivity maps (i.e., one map per each W and one per each C) depicting regions of input's combined (i.e., bivariate) "first order and interactions" influence on SMM outputs.

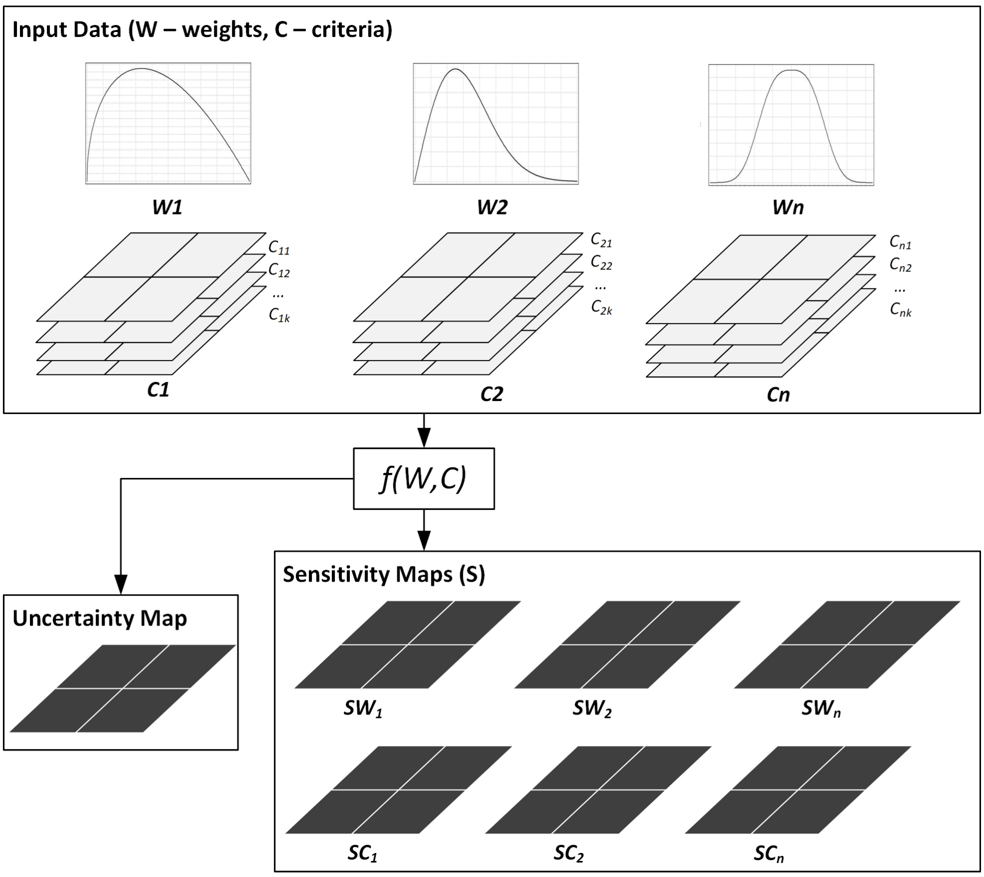

Figure 1 A framework for an extended US-A incorporating the analysis of criteria values and weights.

4 Case study

US-A was employed to assess the uncertainty and sensitivity of multicriteria geodiversity assessment [16], for the Karkonosze National Park (KNP) in southwestern Poland. The park is known for its unique relief and the richness of landforms, including mountain-top planation surfaces, glacial kettles, granite tors with fanciful shapes, waterfalls, and peat bogs. A multicriteria model developed for the purpose of assessment included seven criteria (lithological features, relief energy, landforms, land cover and land use, soils, solar radiation and the topographical wetness index), their relative importance weights, and it was based on a weighted linear combination function for aggregating criteria values with weights. The criteria values and weights were collected from 57 experts in geodiversity and/or Earth sciences using a geo-questionnaire [4]. The study area, the model, and the data collection approach are described in detail in [9].

5 Results

Uncertainty analysis is the first step of US-A (Fig. 1). Figure 2 shows its results, including 1) standardized, average geodiversity score (0.0 – 1.0 scale) calculated for each of 212 first order watersheds (assessment units) based on 2000 model runs, and 2) standard deviation representing the measure of uncertainty. Each model run used a sample of input values drawn from discrete uniform probability distributions of criteria maps discrete non-uniform distributions of weights. The sampling scheme was based on Sobol's quasi-random sampling sequence that improves the uniformity of samples in the parameter space [13]. Many watersheds in Fig. 2 exhibit high average values of geodiversity (0.79 – 0.7) and medium-low standard deviation (0.08 – 0.06). We focus our analysis on three watersheds rendered in black in Fig. 2, representing high average geodiversity (0.79 – 0.7) and relative high uncertainty (0.09 – 0.08). These watersheds, which are highlighted in red circles (Fig. 2), represent areas characterized by the richness of geomorphological forms. Two of them (lower right red circle), located in the eastern part of the park, include the headwaters of Sowia Valley in the eastern part of Black Range. The third watershed, located in the western part of the park (upper right), covers Snow Kettles – the second deepest complex of glacial kettles in the park.

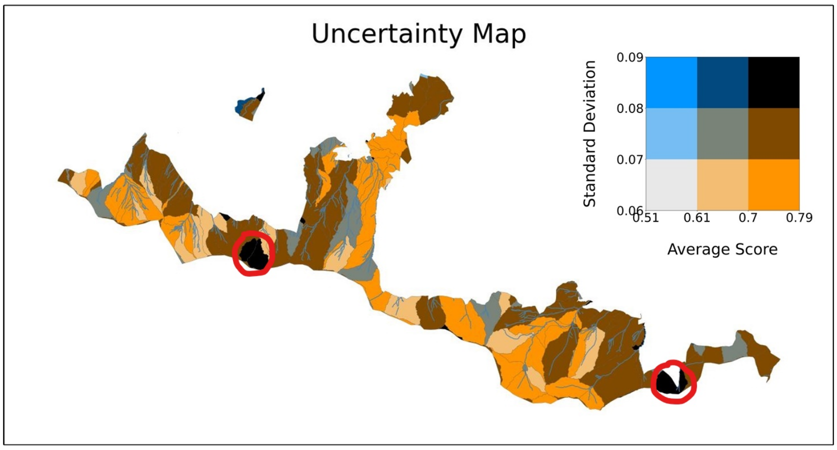

Figure 2 Spatial distribution of average geodiversity and standard deviation in KNP.

In order to identify inputs driving the uncertainty of the selected watersheds, we used the combined “first order and interactions” effects for each of the model’s 14 inputs (seven criteria + seven weights). As described in section 3, variance decomposition produces two sensitivity indices for each criterion and each weight. A challenge in mapping first and total effects sensitivity indexes in the presence of many inputs is cognitive difficulty in interpreting 2N sensitivity maps. The values of indexes are typically rendered on coincident maps (side-by-side) requiring a lot of visual back and forth. The overcome this challenge, we used a bivariate map for each input, which allowed us to present the distribution of both index values on one map per input (Fig. 3). The examination of the sensitivity maps in Figure 3 reveals that both landforms and lithology criteria contribute to a relatively high uncertainty (high standard deviation) of geodiversity values in the three watersheds. Specifically, the landforms criterion affects geodiversity of the watersheds covering Sowia Valley and the eastern part of Black Range (lower rights) and the lithology criterion impacts geodiversity of the watershed covering Snow Kettles (upper right). This could be addressed, for example, by obtaining higher quality input data for the criteria, which in turn might reduce the uncertainty of assessment. The other input contributing to high uncertainty is the relief energy criterion, but only for the watershed that covers Snow Kettles (upper left).

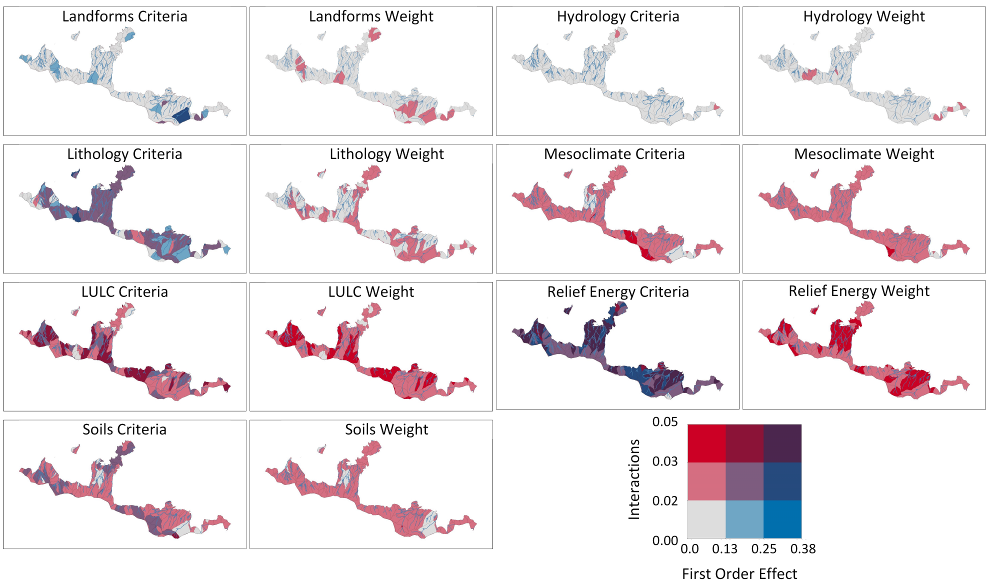

Figure 3 Spatial distribution of First Order and Interactions (Total Order) effects across 14 input factors.

6 Conclusion

The work presented here shows that considering only criteria weights in US-A may give us an incomplete understanding of important factors driving multicriteria model output uncertainty. Notably, the framework presented in Figure 1 lends itself to incorporating in US-A potential sources of the model’s output uncertainty other than criteria values and weights. Other considerations, not accounted for in this study, are the model’s decision rule represented by aggregation function(s) and the selection of criteria used in the model. They can be addressed in future research.

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