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Aims and Scope
The periodical *Dagstuhl Reports* documents the program and the results of Dagstuhl Seminars and Dagstuhl Perspectives Workshops.
In principal, for each Dagstuhl Seminar or Dagstuhl Perspectives Workshop a report is published that contains the following:

- an executive summary of the seminar program and the fundamental results,
- an overview of the talks given during the seminar (summarized as talk abstracts), and
- summaries from working groups (if applicable).

This basic framework can be extended by suitable contributions that are related to the program of the seminar, e. g. summaries from panel discussions or open problem sessions.

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Report from Dagstuhl Seminar 15241

Computational Social Choice: Theory and Applications

Edited by

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Abstract

This report documents the program and the outcomes of Dagstuhl Seminar 15241 "Computational Social Choice: Theory and Applications". The seminar featured a mixture of classic scientific talks (including three overview talks), open problem presentations, working group sessions, and five-minute contributions ("rump session"). While there were other seminars on related topics in the past, a special emphasis was put on practical applications in this edition.

Seminar June 7–12, 2015 – http://www.dagstuhl.de/15241
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1 Executive Summary

Britta Dorn
Nicolas Maudet
Vincent Merlin

Computational social choice is an interdisciplinary research area dealing with the aggregation of preferences of groups of agents in order to reach a consensus decision that realizes some social objective. Economists typically view markets as an optimal mean for coordinating the activities and allocation of resources across a group of heterogeneous agents based on their utilities or preferences. By contrast, the methods of social choice, broadly defined, focus on coordination mechanisms that do not rely on prices, monetary/resource transfer or market structures, while still defining social objectives that account for individual preferences. Some classic (but certainly not exhaustive) topics of study in social choice topics include:

- voting procedures, where a single alternative must be taken given the preferences of individuals group members;
- fair division, which deals with the distribution of goods among a group reflecting individual preferences and fairness criteria;
- matching problems, in which agents/items are matched in a way that respects both preferences and other constraints.

The theoretical treatment of these problems is concerned with the existence of solutions which could be defended on normative grounds. In classical social choice, desirable solution concepts satisfy certain properties, such as: efficiency, non-discriminatory treatment of agents;

envy-freeness; stability (or equilibrium) with respect to incentives; non-manipulability; and a variety of others. Over the past 15 years, the computational properties of these solution concepts have emerged as critically important to their theoretical viability and practical impact. Computer scientists in both AI and theoretical computer science have developed efficient algorithms for realizing certain social choice functions, proven the computational intractability of others, studied the theoretical and practical communication requirements of these procedures, and developed computational tools to sharpen our understanding of incentives, manipulation, and other important phenomena. Applying a computational lens to these theoretical investigations has led to breakthroughs that have supported a variety of real-world applications like web-page ranking, fair buy-sell/exchange protocols, and the development of much more socially efficient exchanges for organ transplantation.

At the same time, the era of networked communication and "big data" has made it easier than ever to infer people's preferences and have them engage with ever larger groups. This has opened up tremendous opportunities for the application of social choice to a wider range of "lower stakes, higher frequency" group decisions. Hence, it introduces new challenges for social choice – many mechanisms for the problems above have been designed using assumptions that – while suitable for "high stakes" domains like political voting, or matching in labor markets and organ donations – are entirely untenable in other domains.

The objective of the seminar was to continue the series of meetings on theoretical computational social choice previously held in Dagstuhl, but the emphasis was on problems which have practical relevance. We have addressed in particular three lines of works concerning issues in social choice: voting, matching, and fair division. The seminar brought together 41 researchers from 18 countries and various fields such as computer science, mathematics, social choice theory, economics, political sciences, and industry. The meeting gathered both participants focusing on the theoretical foundations of computational social choice, and those seeking to apply social choice mechanisms to real-world problems of both the high-stakes/low-frequency and the low-stakes/high-frequency variety.

The technical program of the seminar included overview talks, regular seminar talks, a rump session and slots for communication and work on open problems. The three overview talks presented open questions and challenges in multiwinner voting (complemented by a panel discussion), economics and computation, and in matching in the context of assignments of teachers to schools. The 26 regular seminar talks covered the three lines of work concerning voting, matching, and fair division/resource allocation. Current trends in these fields as reflected by the contributions include allocation of indivisible items under ordinal preferences, the study of well-behaved preference structures (e.g. single-peaked, single-crossing), multiwinner elections, mixed voting systems, and several highly challenging special cases of matching problems. Challenges from real-world applications included online fair division for the distribution of food donations to a food bank, assignments of referees to papers for scientific reviewing, peer grading in massive online open courses, online voting and online participation, sharing cars, junior doctor allocation, and house swapping. Furthermore, several online platforms dedicated to social choice were presented and discussed during the seminar. Precious feedback was collected by the teams of developers.

The program offered the possibility to present open problems and provided slots for working groups on these topics as well as a final session for presentation of outcomes. Several working groups were formed some of which obtained first results during the seminar week. The research projects initiated in these groups are still ongoing. Many participants also used these slots for collaboration with their co-authors that were present at the seminar.

The rump session consisted of 17 five minute contributions, ranging from announcements of events related to the community, over presentation of tools for preference aggregation and online voting, preference libraries for datasets, applications like sharing cars, to short research talks and presentations of open problems.

To conclude, the seminar acknowledged that more and more contributions in computational social choice are driven by real world issues, with many potential applications for industry and policy making. It confirmed that theoretical considerations enable, justify and guarantee the quality of practical applications. Conversely, the specific features and constraints of applications provide novel theoretical challenges and new directions for foundational research.

The participants greatly appreciated the time devoted to working group sessions and benefited from the seminar in various ways: by learning about new problems, many of them being directly inspired from real world issues; by being introduced to several existing tools; by having the possibility to interact and to develop new collaborations. A next event will be the COMSOC workshop in Toulouse in June 2016. It will be co-located with the meeting of the COST Action IC1205 "Computational Social Choice", and one day will be devoted to applications and interactions with industry, in line with the 15241 Dagstuhl seminar.

We would like to thank all participants for their contributions, discussions, ideas and collaborations, making this seminar a very productive and enjoyable one. In particular, we sincerely thank the team of Schloss Dagstuhl for the great support and excellent organization.

2 Table of Contents

3 Overview of Talks

3.1 Two Desirable Fairness Concepts for Allocation of Indivisible Objects under Ordinal Preferences

Haris Aziz (NICTA – Sydney, AU)

Joint work of Aziz, Haris; Gaspers, Serge; Mackenzie, Simon; Walsh, Toby
Main reference H. Aziz, S. Gaspers, S. Mackenzie, T. Walsh, "Two Desirable Fairness Concepts for Allocation of Indivisible Objects under Ordinal Preferences," Manuscript, 2014.
URL https://dl.dropboxusercontent.com/u/30901094/fair_overview.pdf

Fair allocation of indivisible objects under ordinal preferences is an important problem. Envy-freeness is not only incompatible with Pareto optimality but it also NP-complete to achieve. In view of this predicament, we first frame allocation of indivisible objects as randomized assignment with integrality requirements. We then use the stochastic dominance relation to devise two natural notions of proportionality. Since an assignment may not exist even for the weaker notion of proportionality, we propose relaxations of the concepts – optimal weak proportionality and optimal proportionality. For both concepts, we propose algorithms to compute fair assignments under ordinal preferences. Both new fairness concepts appear to be desirable in view of the following: they are compatible with Pareto optimality, admit efficient algorithms to compute them, are based on proportionality, and are guaranteed to exist.

3.2 Challenges in Online Participation

Dorothea Baumeister (Heinrich-Heine-Universität Düsseldorf, DE)

URL http://www.fortschrittskolleg.de

In the interdisciplinary graduate school "Online Participation" researchers from computer science, business economics, communication science, political science, law, and sociology work together with several industry partners to analyze the potential of online participation. Up to now most online participation projects are organized as a simple forum, which is an inadequate representation for a discussion. The concepts of argumentation frameworks give the possibility to formalize a discussion, they are however not suitable for non-experts in this field. One central topic in this project is to develop an online tool that provides a convenient solution to this problem, where on the one hand the discussion is more structured than in a simple forum, but on the other hand is also applicable for ordinary users. In this talk I will present some first ideas of how this may be achieved, and point out some challenges both from a practical and a theoretical point of view.

3.3 Integer programming methods for special college admissions problems

Péter Biró (Hungarian Academy of Sciences – Budapest, HU)

Joint work of Biró, Péter; McBride, Iain
Main reference P. Biró, I. McBride, "Integer Programming Methods for Special College Admissions Problems," in Proc. of the 8th Int'l Conf. on Combinatorial Optimization and Applications (COCOA'14), LNCS, Vol. 8881, pp. 429–443, Springer, 2014.
URL http://dx.doi.org/10.1007/978-3-319-12691-3_32

We develop Integer Programming (IP) solutions for some special college admission problems arising from the Hungarian higher education admission scheme. We focus on four special features, namely the solution concept of stable score-limits, the presence of lower and common quotas, and paired applications. We note that each of the latter three special feature makes the college admissions problem NP-hard to solve. Currently, a heuristic based on the Gale-Shapley algorithm is being used in the application. The IP methods that we propose are not only interesting theoretically, but may also serve as an alternative solution concept for this practical application, and other similar applications.

3.4 Taming the Whale. Lessons learned about online voting and real people

Sylvain Bouveret (LIG – Grenoble, FR)

Joint work of Bouveret, Sylvain; Blanch, Renaud; Karanikolas, Nikos; Cos, Corentin
URL http://whale3.noiraudes.net/

Whale is an online voting system where people can vote on different alternatives, using rankings, scores, or approval ballots, and see the results given by different voting rules. The aim of this talk is to give some lessons (or questions) learned from observing laypeople using the system. From these observations we derive three challenges for online voting (or poll) systems: the first one concerns voting and data visualization, the second concerns the need for taking into account incomplete preferences and the third one is about voting and combinatorial domains.

3.5 How to Divide Things Fairly

Steven J. Brams (New York University, US)

Joint work of Brams, Steven J.; Kilgour, D. Marc; Klamler, Christian
Main reference S. . Brams, D. M. Kilgour, C. Klamler, "How to Divide Things Fairly," Manuscript, 2014.
URL http://www.politics.as.nyu.edu/docs/IO/2578/SA9.pdf

We analyze a simple sequential algorithm (SA) for allocating indivisible items that are strictly ranked by n ≥ 2 players. It yields at least one Pareto-optimal allocation which, when $n = 2$, is envy-free unless no envy-free allocation exists. However, an SA allocation may not be maximin or Borda maximin – maximize the minimum rank, or the Borda score, of the items received by a player. Although SA is potentially vulnerable to manipulation, it would be difficult to manipulate in the absence of one player's having complete information about the other players' preferences. We discuss the applicability of SA, such as in assigning people to committees or allocating marital property in a divorce.

3.6 The Paradox of Voting Systems

Steven J. Brams (New York University, US)

Joint work of Brams, Steven J.; Potthoff, Richard F.
Main reference S. J. Brams, R. F. Potthoff, "The Paradox of Grading Systems," Manuscript, 2015.
URL http://www.politics.as.nyu.edu/docs/IO/2578/GradingParadox.pdf

We distinguish between (i) voting systems in which voters can rank candidates and (ii) those in which they can grade candidates, such as approval voting, in which voters can give two grades – approve (1) or not approve (0) – to candidates. While two grades rule out a discrepancy between the average-grade winners, who receive the highest average grade, and the superior-grade winners, who receive more superior grades in pairwise comparisons (akin to Condorcet winners), more than two grades allow it. We call this discrepancy between the two kinds of winners the paradox of grading systems, which we illustrate with several examples and whose probability we estimate for sincere and strategic voters through a Monte Carlo simulation. We discuss the trade-off between (i) allowing more than two grades, but risking the paradox, and (ii) precluding the paradox, but restricting voters to two grades.

3.7 Fishburn's Maximal Lotteries: A randomized rule that is immune to splitting electorates, cloning alternatives, abstention, and crude manipulation.

Felix Brandt (TU München, DE)

Joint work of Aziz, Haris; Brandt, Felix; Brandl, Florian; Brill, Markus; Hofbauer, Johannes; Seedig, Hans Georg

This talk summarizes a number of recent papers on maximal lotteries, a randomized voting rule that was proposed by Fishburn in the 1980s. Maximal lotteries satisfy various consistency conditions, economic efficiency, computational efficiency, participation, and a weak variant of strategyproofness.

3.8 Control and Bribery for Approval Voting through Two Edge Cover Generalizations

Robert Bredereck (TU Berlin, DE)

Joint work of Bredereck, Robert; Talmon, Nimrod
Main reference R. Bredereck, N. Talmon, "NP-hardness of two edge cover generalizations with applications to control and bribery for approval voting," Information Processing Letters, 116(2):147–152, 2015.
URL http://dx.doi.org/10.1016/j.ipl.2015.09.008

We close three gaps with respect to the computational complexity of voting problems by showing NP-hardness of two generalizations of the Edge Cover problem, which were conjec-

tured to be polynomial-time solvable. More precisely, our results imply that 2-Approval Constructive Control by Adding Weighted Voters, 2-Veto Constructive Control by Deleting Weighted Voters, and k-Veto-\$Bribery for k from $\{2, 3\}$ are (strongly) NP-complete.
Given an undirected graph and an integer q, the Edge Cover problem asks for a subgraph with at most q edges such that each vertex has degree at least one. Both generalizations introduce weights on the edges and an individual demand $b(v)$ for each vertex v. The first generalization, named Simple b-Edge Weighted Cover, requires the edge set to have a total weight of at most q while each vertex v is to be adjacent to at least $b(v)$ edges. The second generalization, named Simple Weighted b-Edge Cover, requires the edge set to contain at most q edges while each vertex v is to be adjacent to edges of total weight at least $b(v)$.

3.9 Computing the Optimal Game

Markus Brill (Duke University – Durham, US)

Joint work of Brill, Markus; Freeman, Rupert; Conitzer, Vincent
Main reference M. Brill, R. Freeman, V. Conitzer, "Computing the Optimal Game," in Proc. of the 2nd Workshop on Exploring Beyond the Worst Case in Computational Social Choice (EXPLORE'15), held as part of the 14th Int'l Conf. on Autonomous Agents and Multiagent Systems (AAMAS'15), to appear; pre-print available from workshop webpage.
URL http://www.explore-2015.preflib.org/wp-content/uploads/2015/04/paper_8.pdf

In many multiagent environments, a designer has some, but limited control over the game being played. In this paper, we formalize this by considering incompletely specified games, in which some entries of the payoff matrices can be chosen from a specified set. We show that it is NP-hard for the designer to decide whether she can make her choices so that no action in a given set gets played in equilibrium. Hardness holds even in zero-sum games and even in weak tournament games (which are symmetric zero-sum games whose entries are all -1, 0, or 1). The latter case is closely related to the necessary winner problem for a social-choice-theoretic solution concept. We then give a mixed-integer linear programming formulation for weak tournament games and evaluate it experimentally.

3.10 Aggregating partial rankings with applications to peer grading in MOOCs

Ioannis Caragiannis (CTI & University of Patras, GR)

Joint work of Caragiannis, Ioannis; Krimpas, George A.; Voudouris, Alexandros A.

We investigate the potential of using ordinal peer grading for the evaluation of students in massive online open courses (MOOCs). According to such grading schemes, each student receives a few assignments (by other students) which she has to rank. Then, a global ranking (possibly translated into numerical scores) is produced by combining the individual ones. This is a novel application area for social choice concepts and methods where the important problem to be solved is as follows: how should the assignments be distributed so that the collected individual rankings can be easily merged into a global one that is as close as possible to the ranking that represents the relative performance of the students in the assignment? Our main theoretical result suggests that using very simple ways to distribute the assignments so that each student has to rank only k of them, a Borda-like aggregation method can recover a $1 - O(1/k)$ fraction of the true ranking when each student correctly ranks the assignments she receives. Experimental results strengthen our analysis further and also demonstrate that the same method is extremely robust even when students have imperfect capabilities as graders. Our results provide strong evidence that ordinal peer grading can be a highly effective and scalable solution for evaluation in MOOCs.

3.11 House swapping with engaged and divorcing pairs

Katarína Cechlárová (Pavol Jozef Šafárik University – Košice, SK)

Joint work of Cechlárová, Katarína; Fleiner, Tamás; Jankó, Zsuzsanna

We study a modification of a housing market that allows engaged pairs and divorcing couples. An engaged pair owns two houses and wants to move together into one house, while a divorcing couple owns one house and needs two different houses. We show that the problem to maximize the number of moving agents is inapproximable but fixed parameter tractable.

3.12 Assignment of teachers to schools – a new variation on an old theme

Katarína Cechlárová (Pavol Jozef Šafárik University – Košice, SK)

Several countries more or less successfully use centralized matching schemes for assigning teachers to vacant positions at schools. We explore combinatorial and computational aspects of a possible similar scheme motivated by the situation characteristic for Slovak and Czech education system where each teacher specializes in two subjects. We present a model that takes into consideration that schools may have different capacities for each subject and show that its combinatorial structure leads to intractable problems even under several strong restrictions concerning the total number of subjects, partial capacities of schools and the number of acceptable schools each teacher is allowed to list. We propose several approximation algorithms. Finally, we present integer programming models and their application to real data.

3.13 Parliamentary Voting Procedures: Agenda Control, Manipulation, and Uncertainty

Jiehua Chen (TU Berlin, DE)

Joint work of Bredereck, Robert; Chen, Jiehua; Niedermeier, Rolf; Walsh, Toby
Main reference R. Bredereck, J. Chen, R. Niedermeier, T. Walsh, "Parliamentary Voting Procedures: Agenda Control, Manipulation, and Uncertainty," in Proc. of the 24th International Joint Conference on Artificial Intelligence (IJCAI'15), pp. 164–170, AAAI Press, 2015.
URL http://ijcai.org/papers15/Abstracts/IJCAI15-030.html

We study computational problems for two popular parliamentary voting procedures: the amendment procedure and the successive procedure. While finding successful manipulations or agenda controls is tractable for both procedures, our real-world experimental results indicate that most elections cannot be manipulated by a few voters and agenda control is typically impossible. If the voter preferences are incomplete, then finding possible winners is NP-hard for both procedures. Whereas finding necessary winners is coNP-hard for the amendment procedure, it is polynomial-time solvable for the successive one.

3.14 Structure in Dichotomous Preferences

Edith Elkind (University of Oxford, GB)

Many hard computational social choice problems are known to become tractable when voters' preferences belong to a restricted domain, such as those of single-peaked or single-crossing preferences. However, to date, all algorithmic results of this type have been obtained for the setting where each voter's preference list is a total order of candidates. The goal of this paper is to extend this line of research to the setting where voters' preferences are dichotomous, i.e., each voter approves a subset of candidates and disapproves the remaining candidates. We propose several analogues of the notions of single-peaked and single-crossing preferences for dichotomous profiles and investigate the relationships among them. We then demonstrate that for some of these notions the respective restricted domains admit efficient algorithms for computationally hard approval-based multi-winner rules.

3.15 Multiwinner Voting: New Perspectives and New Challenges

Piotr Faliszewski (AGH University of Science & Technology – Krakow, PL)

Joint work of Faliszewski, Piotr; Elkind, Edith; Skowron, Piotr; Slinko, Arkadii; Lang, Jérôme
Main reference E. Elkind, P. Faliszewski, P. Skowron, A. Slinko, "Properties of Multiwinner Voting Rules," in Proc. of the 13th Int'l Conf. on Autonomous Agents and Multi-Agent Systems (AAMAS'14), pp. 53–60, IFAAMAS/ACM, 2014.
URL http://dl.acm.org/citation.cfm?id=2615743

In a multiwinner election we are given a set of candidates, a set of voters with their preferences regarding the candidates, and a positive integer K. A multiwinner voting rule picks a group of some K candidates. However, as opposed to the case of single-winner elections where it is clear that the voting rule should pick a candidate that is in some sense "the best one", the goal of a multiwinner voting rule depends on the particular application. Some natural applications include:

(a) Shortlisting: We want to pick a group of K candidates that we will inspect further and pick the best one from them (shortlisting happens, for example, when we want to decide who to hire and we focus on some group of promising applicants).

(b) Picking a representative committee: Parliamentary elections are the most natural example here. We are picking a group of K members of a parliament. The elected candidates should represent the society well.

(c) Picking items for users to share: A natural example includes picking what movies to put in the airplane's entertainment system to maximize the satisfaction level of the passengers.

Each of these applications requires a voting rule with different properties. For example, if we have two similar candidates then a rule used for shortlisting should either pick them both or reject them both, whereas a rule for movie selection should perhaps pick one of them, but certainly not both.

In this tutorial we will present several prominent multiwinner voting rules, discuss some recent results (with the focus on axiomatic properties and the complexity of winner determination), and present some avenues for future research. In particular, it is quite clear that the search for interesting multiwinner rules has only started!

3.16 Participation and Strategyproofness: Insights via SAT Solving

Christian Geist (TU München, DE)

Joint work of Brandl, Florian; Brandt, Felix; Geist, Christian; Hofbauer, Johannes
Main reference F. Brandl, F. Brandt, C. Geist, J. Hofbauer, "Strategic Abstention Based on Preference Extensions: Positive Results and Computer-Generated Impossibilities," in Proc. of the 24th International Joint Conference on Artificial Intelligence (IJCAI'15), pp. 18–24, AAAI Press, 2015.
URL http://ijcai.org/papers15/Abstracts/IJCAI15-010.html

Similar to the well-known and much studied problem of strategic misrepresentation, a common flaw of some voting rules, known as the no-show paradox, is that agents may obtain a more preferred outcome by abstaining from an election. We study both these types of strategic manipulation (misrepresentation and abstention) for set-valued, majoritarian voting rules based on Kelly's and Fishburn's preference extensions. We survey a range of recent results, both positive and negative. For example, we show that, whenever there are at least five alternatives, every Pareto-optimal voting rule suffers from both types of manipulation with respect to Fishburn's extension. This is achieved by reducing the statement to finite – yet very large – problems, which are encoded as formulae in propositional logic and then shown to be unsatisfiable by a SAT solver. We also provide human-readable proofs which we extracted from minimal unsatisfiable cores of the formulae. On the positive side, we prove that every voting rule that satisfies a natural condition cannot be manipulated with respect to Kelly's extension and discover discriminating examples of such rules.

3.17 Open Problems: Identifying k-majority Digraphs

Christian Geist (TU München, DE)

Joint work of Brandt, Felix; Geist, Christian; Seedig, Hans Georg
Main reference F. Brandt, C. Geist, H. G. Seedig, "Identifying k-Majority Digraphs via SAT Solving," in Proc. of the 1st Workshop on Exploring Beyond the Worst Case in Computational Social Choice (EXPLORE'14), held as part of the 13th Int'l Conf. on Autonomous Agents and Multiagent Systems (AAMAS'14), to appear; pre-print available from workshop webpage.
URL http://www.explore14.preflib.org/wp-content/uploads/2014/03/paper_8.pdf

Many hardness results of computational social choice make use of the fact that every directed graph may be induced as the pairwise majority relation of some preference profile. However, this fact requires a number of voters that is almost linear in the number of alternatives. For the general question of how many voters it takes to induce a given majority graph many answers have been given (e.g, by McGarvey, 1953; Stearns, 1959; Erdos and Moser, 1964; Fiol, 1992), but quite a few questions remain open.

3.18 Pnyx: A Powerful and User-friendly Tool for Preference Aggregation

Christian Geist (TU München, DE)

Joint work of Brandt, Felix; Chabin, Guillaume; Geist, Christian
Main reference F. Brandt, G. Chabin, C. Geist, "Pnyx: A powerful and user-friendly tool for preference aggregation," in Proc. of the 14th Int'l Conf. on Autonomous Agents and Multi-Agent Systems (AAMAS'15), pp. 1915–1916, IFAAMAS/ACM, 2015.
URL http://dl.acm.org/citation.cfm?id=2773502
URL http://pnyx.dss.in.tum.de

Pnyx is an easy-to-use and entirely web-based tool for preference aggregation that does not require any prior knowledge about social choice theory. The tool is named after a hill in Athens called Pnyx, which was the official meeting place of the Athenian democratic assembly and is therefore known as one of the earliest sites in the creation of democracy. Pnyx is available at pnyx.dss.in.tum.de.

3.19 Role Based Hedonic Games

Judy Goldsmith (University of Kentucky – Lexington, US)

Joint work of Spradling, Matthew; Goldsmith, Judy
Main reference SM. Spradling, J. Goldsmith, "Stability in Role Based Hedonic Games," in Proc. of the 28th Int'l Florida Artificial Intelligence Research Society Conf. (FLAIRS'15), pp. 85–90, AAAI Press, 2015.
URL http://www.aaai.org/ocs/index.php/FLAIRS/FLAIRS15/paper/view/10383

In the hedonic coalition formation game model called Roles Based Hedonic Games (RBHG), agents view teams as compositions of available roles. An agent's utility for a partition is based upon which role she fulfills within the coalition and which additional roles are being fulfilled within the coalition. We consider optimization and stability problems for settings with variable power on the part of the central authority and on the part of the agents. We prove most of these problems to be NP-complete or coNP-complete. We introduce heuristic methods for approximating solutions for a variety of these hard problems. We validate heuristics on real-world data scraped from League of Legends games.

3.20 Gibbard-Satterthwaite Games

Umberto Grandi (Toulouse 1 Capitole University, FR)

Joint work of Elkind, Edith; Grandi, Umberto; Rossi, Francesca; Slinko, Arkadii
Main reference E. Elkind, U. Grandi, F. Rossi, A. Slinko, "Gibbard-Satterthwaite Games," in Proc. of the 24th Int'l Joint Conf. on Artificial Intelligence (IJCAI'15), pp. 533–539, AAAI Press, 2015; pre-print available from author's webpage.
URL http://ijcai.org/papers15/Abstracts/IJCAI15-081.html
URL http://www.irit.fr/~Umberto.Grandi/publications/ElkindEtAlIJCAI2015.pdf

The Gibbard-Satterthwaite theorem implies the ubiquity of manipulators – voters who could change the election outcome in their favor by unilaterally modifying their vote. In this paper, we ask what happens if a given profile admits several such voters. We model strategic interactions among Gibbard–Satterthwaite manipulators as a normal-form game. We classify the 2-by-2 games that can arise in this setting for two simple voting rules, namely Plurality and Borda, and study the complexity of determining whether a given manipulative vote weakly dominates truth-telling, as well as existence of Nash equilibria.

3.21 The McKelvey Uncovered Set and Pareto Optimality

Paul Harrenstein (University of Oxford, GB)

Joint work of Brandt, Felix; Geist, Christian; Harrenstein, Paul

We consider the notion of Pareto optimality under the assumption that only the pairwise majority relation is known and show that the set of necessarily Pareto optimal alternatives coincides with the McKelvey uncovered set. In fact, every majority relation admits a single consistent profile in which precisely all covered alternatives are Pareto dominated. As a consequence, the McKelvey uncovered set constitutes the coarsest Pareto optimal majoritarian social choice function.

3.22 Maximin Envy-Free Division of Indivisible Items

Christian Klamler (Universität Graz, AT)

Joint work of Brams, Steven J.; Kilgour, D. Marc; Klamler, Christian

Assume that two players have strict rankings over an even number of indivisible items. We propose algorithms to find allocations of these items that are maximin – maximize the minimum rank of the items that the players receive – and are envy-free and Pareto-optimal if such allocations exist. We show that neither maximin nor envy-free allocations may satisfy other criteria of fairness, such as Borda maximinality. Although not strategy-proof, the algorithms would be difficult to manipulate unless a player has complete information about its opponent's ranking. We assess the applicability of the algorithms to real-world problems, such as allocating marital property in a divorce or assigning people to committees or projects.

3.23 Consistent House Allocation with Existing Tenants

Bettina Klaus (University of Lausanne, CH)

Joint work of Karakaya, Mehmet; Klaus, Bettina; Schlegel, Jan Christoph

Abdulkadiroglu and Sonmez (1999) introduced the house allocation with exiting tenants model and the class of YRMH-IGYT (you request my house – I get your turn) rules. Sonmez and Unver (2010) showed that a rule for house allocation problems with existing tenants is Pareto-optimal, individually-rational, strategy-proof, weakly neutral, and consistent if and only if it is a YRMH-IGYT rule. The class of YRMH-IGYT rules is a subclass of the class of TTC (top trading cycles) rules (based on priority structures). We characterize the subclass of all TTC rules that are consistent by a new acyclicity requirement for the underlying priority structure and thereby extend previous results by Ergin (2000) and Kesten (2006) for house allocation (without existing tenants). Finally, we analyze what happens when we drop weak neutrality and consider rules satisfying Pareto-optimality, individual-rationality, strategy-proofness, and consistency.

3.24 School Choice: Nash Implementation of Stable Matchings through Rank-Priority Mechanisms

Flip Klijn (CSIC – Barcelona, ES)

Joint work of Jaramillo, Paula; Kayi, Çagatay; Klijn, Flip

We study the school choice problem (Abdulkadiroglu and Sonmez, 2003) where students are to be matched to schools through a clearinghouse. We focus on the class of rank-priority mechanisms, to which the Boston (or immediate acceptance) mechanism belongs. We provide a necessary and sufficient condition for the Nash implementation of the set of stable matchings.

3.25 To approve or not to approve: This is not the only question

Annick Laruelle (Ikerbasque – Bilbao, ES)

Joint work of Alcantud, José Carlos R.; Laruelle, Annick
Main reference J. C. R. Alcantud, A. Laruelle, "Dis&approval voting: a characterization," Social Choice and Welfare, 43(1):1–10, 2014.
URL http://dx.doi.org/10.1007/s00355-013-0766-7

In polls many citizens express some dissatisfaction with politicians. Usual ways to voice this dissatisfaction in elections are absenteeism, spoiled or blank votes, or voting for a fringe candidate. Why can citizens only vote in favor of a single candidate and not against a candidate? Why can't voters express their opinion on every candidate? Alcantud and Laruelle (2014) study a method that permits to express dissatisfaction and express an opinion on each candidate. Under the disapproval rule the voter is asked to cast a positive, null or negative vote on each candidate. The candidate who obtains the highest difference between the number of positive votes and the number of negative votes is elected. In this presentation we discuss the potential advantage of this method, and further lines of research.

3.26 The IAC Probability of a Divided Verdict in a Simple U.S. Presidential Type Election

Michel Le Breton (University of Toulouse I, FR)

Joint work of de Mouzon, Olivier; Laurent, Thibault; Le Breton, Michel; Lepelley, Dominique

During the talk, we presented an asymptotic estimate of the probability $\phi(n)$ of a divided verdict in a U.S. presidential type election in the simplest possible unbiased setting when the phenomenon can occur: three equipopulated districts with n voters per district. The novelty of this paper, in contrast to all the existing literature (IC and IAC^*), is to assume that votes are drawn result from an IAC (Impartial Anonymous Culture) probability model. In the IC and IAC^* settings, $\phi(n)$ converge to a finite limit which is $3\frac{\arccos(\frac{\sqrt{3}}{3})}{\pi} - \frac{3}{4} \simeq 16.226\%$ for IC and $\frac{1}{8} = 12.5\%$ for IAC^*. Through the use of numerical methods, it is conjectured that $\sqrt{n}\,\phi(n)$ converges to a limit when n (the size of the electorate in one district) tends to infinity. It implies that $\phi(n)$ converge to 0. It is also demonstrated that: $\phi_m(n) \leq \phi(n)$ where $\phi_m(n) = O\left(\frac{1}{\sqrt{n}}\right)$ and an upper bound $\phi_M(n)$ is also introduced whose convergence properties are examined.

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3.27 Matching in Practice: Junior Doctor Allocation in Scotland

David Manlove (University of Glasgow, GB)

Joint work of Irving, Rob; Kwanashie, Augustine; Manlove, David; McBride, Iain

Matching problems typically involve assigning agents to commodities, possibly on the basis of ordinal preferences or other metrics. These problems have large-scale applications to centralized matching schemes in many countries and contexts. In this short talk I will describe the matching problems featuring in the allocation of junior doctors to Scottish hospitals, as part of the Scottish Foundation Allocation Scheme (SFAS). I will outline the computational methods for their solution and give an overview of results arising from real data connected with SFAS. Algorithms arising from this research were deployed by the National Health Service Education for Scotland as part of SFAS between 1999-2012.

3.28 OWAs for Voting and Matching

Nicholas Mattei (UNSW – Sydney, AU)

Motivated by the common problem of assigning referees to papers for conference organizing and reviewing we survey the existing work and complexity of assignment problems when one or both sides of the market have maximum and minimum capacity constraints. This setting has a number of practical real world applications beyond conference reviewing including shift work assignment and other one (and two) sided assignment settings where one (or both) sides have capacity constraints. Drawing inspiration from various rules used in voting, we use order weighted averages (OWAs) to implement a novel and flexible assignment mechanism which allows one to balance the trade-off between egalitarian and utilitarian objectives. We show that this problem can be solved in polynomial time and report on a set of experiments using real world data from conferences.

3.29 Approximation Algorithms for Power Allocation Problems in AC Electric Systems

Trung Thanh Nguyen (Masdar Institute – Abu Dhabi, AE)

Joint work of Elbassioni, Khaled; Nguyen, Trung Thanh
Main reference K. Elbassioni, T. T. Nguyen, "Approximation Schemes for Binary Quadratic Programming Problems with Low cp-Rank Decompositions," arXiv:1411.5050v1 [cs.DS] , 2014.
URL http://arxiv.org/abs/1411.5050v1

Yu and Chau [AAMAS2013] introduced the Complex Demand Knapsack Problem (CKP), a variant of KP with (one) quadratic constraint motivated by the allocation of power in

Alternating Current (AC) electric systems. Formally, there is a set of users with power demands, each demand is characterized by two components, active power and reactive power. A typical way for expressing the power demand is to use complex numbers of which the real and imaginary parts correspond to the active and reactive power, whereas the magnitudes express as the so-called apparent power. Each user i will give a certain utility if her demand is fulfilled. The goal is to maximize the sum of utilities of the chosen users such that the magnitude of the sum of satisfied demands should not exceed the capacity. Yu and Chau [AAMAS2013] gave a $\frac{1}{2}$-approximation algorithm for CKP and this was improved recently by Chau et al. [AAMAS2014] to a polynomial-time approximation scheme (PTAS); this is the best result we can hope for given that a fully PTAS for the problem does not exist unless P=NP (Woeginger [SODA99]). In this work, we investigate a scenario with more than one number of quadratic constraints as it captures the model of allocating powers in multiple periods. In addition, we consider more general objective functions such as submodular functions and sum-of-ratio functions (i.e., sum of ratios of linear functions). Our contributions are as follows: for the linear objective function and a fixed number of quadratic constraints, we propose a PTAS based on the convex-programming method, which relies on the polynomial solvability of the corresponding convex relaxation problem. Also, we obtain a $(1/e - \epsilon)$-approximation algorithm for the submodular case, for any constant $\epsilon > 0$, by using the geometric approach; the key idea is to make use of the geometry of the problem to reduce it to a multi-dimensional knapsack problem, which can be solved using enumeration and dynamic programming for the linear objective case, or LP-rounding for the submodular case. Finally, we develop a PTAS for the sum-of-ratios objective function under the condition that the number of ratios is constant, by following the multi-objective approach.

3.30 Economics and Computation: Five Challenges in Algorithmic Game Theory, Computational Social Choice, and Fair Division

Joerg Rothe (Heinrich-Heine-Universität Düsseldorf, DE)

Joint work of Rothe, Joerg; Baumeister, Dorothea; Elkind, Edith; Erdelyi, Gabor; Faliszewski, Piotr; Hemaspaandra, Edith; Hemaspaandra, Lane A.; Lang, Jérôme; Lindner; Claudia; Rothe; Irene
Main reference J. Rothe (Ed.), 'Economics and Computation: An Introduction to Algorithmic Game Theory, Computational Social Choice, and Fair Division," ISBN 978-3-662-47904-9, Springer Texts in Business and Economics, Springer, 2016.
URL http://www.springer.com/de/book/9783662479032

In this survey talk, I present five challenges from various areas at the interface of economics and computer science. These challenges are also described in the new book: "Economics and Computation: An Introduction to Algorithmic Game Theory, Computational Social Choice, and Fair Division" J. Rothe (editor and co-author). Springer Texts in Business and Economics, Springer-Verlag, in press, to appear in 2015.

3.31 Refining the complexity of the sports elimination problem

Ildikó Schlotter (Budapest University of Technology & Economics, HU)

Joint work of Potpinková, Eva; Cechlárová, Katarína; Schlotter, Ildikó
Main reference K. Cechlárová, E. Potpinková, I. Schlotter, "Refining the complexity of the sports elimination problem," Discrete Applied Mathematics, Vol.. 199, pp. 172–186, 2016; pre-print available from author's webpage.
URL http://dx.doi.org/10.1016/j.dam.2015.01.021
URL http://www.cs.bme.hu/~ildi/pub/sport.pdf

The sports elimination problem asks whether a team participating in a competition still has a chance to win, given the current standings and the remaining matches to be played among the teams. This problem can be viewed as a graph labeling problem, where arcs receive labels that contribute to the score of both endpoints of the arc, and the aim is to label the arcs in a way that each vertex obtains a score not exceeding its capacity. We investigate the complexity of this problem in detail, using a multivariate approach to examine how various parameters of the input graph (such as the maximum degree, the feedback vertex/edge number, and different width parameters) influence the computational tractability. We obtain several efficient algorithms, as well as certain hardness results.

3.32 Voting Manipulation Games from Epistemic Game Theory Perspective

Arkadii Slinko (University of Auckland, NZ)

So far the main tool for analyzing voting games have been the concept of Nash equilibrium. The concept of Nash equilibrium – as prediction of rational behavior – assumes that individuals have unlimited computing capabilities and can resolve the infinite loops arising from logical reasoning: I think that he thinks that I think that etc. In reality, players' reasoning goes only to a finite depth. Level-k models were introduced by Stahl and Wilson (1994, 1995) to describe experimental data better. Selten (1998) wrote: "Boundedly rational strategic reasoning seems to avoid circular concepts. It directly results in a procedure by which a problem solution is found."

I will show how level-k models can be used to analyze voting manipulation games. In particular, I will show that L_2 voters have to play the game that was previously called Gibbard-Satterthwaite game by Elkind-Grandi-Rossi-Slinko (2015). In particular, for such a voter for some voting rules it will be coNP-hard to decide whether a given manipulation strategy weakly dominates her sincere strategy.

3.33 Online Fair Division: Modelling a Food Bank problem

Toby Walsh (UNSW – Sydney, AU)

I discuss online models of fair division designed to capture different features of a real world charity problem. Food is donated to a food bank and must be allocated to different charities. This allocation problem has many traditional features. We want to allocate food fairly between the different charities that feed different sectors of the community. In addition, the allocation does not use money as these are all charities. However, the problem also has other features not traditionally found in the academic literature on fair division. One of the main novelties is that it is online. Food is donated throughout the day and we must start allocating and distributing it almost immediately, before we know what else will be donated.

3.34 Private and Efficient Repeated Allocation

Jia Yuan Yu (IBM Research – Dublin, IE)

Joint work of Corless, Martin; Häusler, Florian; Griggs, Wynita; Shorten, Robert; Stüdli, Sonja; Wirth, Fabian; Yu, Jia Yuan
Main reference W. Griggs, J. Y. Yu, F. Wirth, F. Haeusler, R. Shorten, "On the Design of Campus Parking Systems with QoS guarantees," arXiv:1506.02818v1 [math.OC], 2015.
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URL http://arxiv.org/abs/1404.5064v3

An unknown number of agents consumes a common good resource in a repeated fashion. We show that if the agents control their consumptions according to a certain policy, then these consumptions converge to a socially efficient allocation. This is achieved without communicating any of the utility functions of the agents, and with very limited broadcast signals from a central authority. We present one such policy for divisible goods and one for indivisible goods.

3.35 Aggregating binary relations: universal scoring rules via inner product

William S. Zwicker (Union College – Schenectady, US)

Are plurality voting, the Kemeny rule, approval voting, the Borda count, and the mean rule all versions of the same aggregation rule? Yes, in a sense. The *median procedure* is a well known method (see e.g. [1], [11]) for aggregating binary relations (on some finite set A of alternatives) that belong to a class $\mathcal{C}_I$ of *feasible inputs* into a binary relation (on the same set A) belonging to a possibly different class $\mathcal{C}_O$ of *feasible outputs*. By relaxing the assumption that $\mathcal{C}_I \subseteq \mathcal{C}_O$ and exploiting the classes $\mathcal{D}$ of *dichotomous weak orders* (weak orders with exactly two indifference classes) and $\mathcal{U}$ of *univalent orders* (dichotomous weak orders whose "top" I-class is a singleton), we show that each of the aforementioned rules[1] can be obtained as a restriction of the median procedure to appropriate classes $\mathcal{C}_I$ and $\mathcal{C}_O$.

Our approach throughout is to view the median procedure itself as a generalized *relational* scoring rule in which each "ballot" R_i in $\mathcal{C}_I$ assigns a real number scoring weight $F(R_i, R)$ to each binary relation R on A; the outcome of the amalgamation is then the $R \in \mathcal{C}_O$ accumulating the greatest total weight, as summed over all ballots. This generalizes the more traditional definition of *scoring rule*, in which ballots award scoring weights to individual alternatives in A, rather than to binary elations on A; the idea is thus similar to the generalizations considered in [5], [6], [15], [17], and [19]. The scoring weights are given by a Euclidean inner product $F(R_i, R) = \mathcal{J}(R_i) \cdot \mathcal{J}(R)$, where $\mathcal{J}$ embeds binary relations on A as vertices of a hypercube in some suitable Euclidean space $\Re^k$. This inner product formulation yields a Euclidean form of *distance rationalization* that is universal, in that the same metric is used for each restriction of the median rule, as well as a *mean proximity* representation that is similarly universal.[2] The median procedure itself is generated by the "hypercube" embedding $\mathcal{J}$ in particular. Alternative embeddings give rise to different sets of rules obtained via the same restriction process. One such alternative uses a permutahedron in place of a hypercube, and yields the Borda count as a *social welfare function* (in which the outcome is a ranking of alternatives) as one of its restrictions; the hypercube seems to be capable of capturing only the *social choice function* form of Borda (in which the outcome is a winning alternative). In general, however, the possibility of interesting alternatives to the hypercube embedding has not been well explored.

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[1] Although most of these are voting rules, the Mean rule (see [7]) is not, nor is the form of cluster analysis considered in [1]; more generally, the applicability of the median procedure to amalgamation goes well beyond its application to voting rules.

[2] See [1], [2], [4], [8], [9], [10], [12], [13], and [16] for distance rationalization, and [19] for mean proximity representations.

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4 Working Groups

4.1 Working group: Course Allocation

KatarínaCechlárová, Bettina Klaus, David Manlove, Jiehua Chen, Péter Biró, Nicholas Mattei, and Haris Aziz

The problem we worked on was concerned with matching applicants to courses in the presence of quotas (on both sides of the market), ordinal preferences of applicants over courses, and course pre-requisites.

Formally, there is a set A of applicants and a set C of courses. Each applicant and course has a quota, indicating the maximum number of entities of the other type that it can be matched to. Each applicant finds acceptable a subset of the courses and ranks this subset in strict order of preference. Thus applicants have ordinal preferences over individual courses. These preferences extend lexicographically to subsets of courses. So for example if applicant a_1 prefers c_1 to c_2 to c_3 to c_4 then she prefers $\{c_1, c_4\}$ to $\{c_2, c_3\}$ according to the lexicographic rule.

There is a directed acyclic graph $D = (C, A)$ whose vertices correspond to the courses, and an arc (c_i, c_j) means that c_i is a pre-requisite for c_j. Thus an applicant cannot be assigned c_j unless she is also assigned c_i. In the basic model we assume that the pre-requisites are the same for all applicants, but in generalizations the directed graph may differ from one applicant to another. In special cases of the basic model we may restrict the structure of D.

The problem is to find a Pareto optimal matching. We looked at three generalizations of the serial dictatorship mechanism and found that two always constructed Pareto optimal matchings, whilst one did not in some cases. Katka also showed that the problem of finding a maximum cardinality Pareto optimal matching is NP-hard. Among the further questions to be investigated are the following: (i) is there a strategy-proof mechanism for generating all Pareto optimal matchings; (ii) is it possible to determine in polynomial time whether a given matching is Pareto optimal?

4.2 Working group: (Control and) Bribery in k-Approval Voting – Open Problems

Robert Bredereck, Judy Goldsmith, and Gerhard Woeginger

Our project was to identify and resolve open problems with respect to computational complexity for Control and Bribery in k-Approval Voting. To this end, we took [1] as our starting point.

We first discovered that all open questions concerning election control have already been solved in [2], but we did not find results for the bribery questions. The main open problem left was the computational complexity of (priced, uniform) bribery for 2-Approval. Given is an election (C, V) with C being the set of candidates and V being the set of voters, a preferred candidate $p \in C$, a price function $w : V \to \mathbb{N}$ and some budget B. The question is whether one can make p become a winner of the 2-approval election by bribing voters with total cost of at most B. Whereas this problem is strongly NP-hard for 3-Approval and trivially solvable for 1-Approval (Plurality), the 2-Approval case was still open.

Within the seminar, we translated the core of the problem into a graph problem and showed this to be polynomial-time solvable. To identify the border between tractability and intractability more precisely, we also looked at a slightly more general variant of the problem where the price of bribing a voter also depends on the bribed vote. We showed that this non-uniform bribery model is already NP-hard for 2-Approval whereas it remains polynomial-time solvable for 1-Approval.

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4.3 Working Group: Mixed Voting Systems

Vincent Merlin and Michel Le Breton

For the elections of parliaments, political science traditionally distinguishes between proportional systems, and systems that use plurality or majority. However, Blais and Massicote (1999) pointed out that one fifth of countries uses mixed voting systems to elect their first (or unique) chamber. Most of these rules, on the top of electing part of the members of the parliament in districts (using a plurality or majority formula), also elects supplementary members at large in broader jurisdictions (such as regions, states, landers, or the country itself) using another formula. However, social choice theory, as well as computer science, has paid little attention to the study of the systems that try to combine different electoral principles. The objective of this group was to present several of these mixed voting rules, and to promote their study in the computational social choice community. Le Breton presented in detail one of the rare theoretical papers on the subject, due to Paul Edelman (2005): in this paper he evaluated the power (or influence) of the voters when they have to choose between two parties in their districts. However, on the top of electing D members of the parliament locally, L extra seats are attributed at large to the party that obtains a majority of the ballot at the national level. Generalization and extension of this result were also outlined by Michel Le Breton. Nevertheless, the issues on this topic remain numerous: Which voting model shall we use to describe the behavior of the voters when they may cast more than one ballot? How to measure the power of the voters when there are several ways (local, national) to influence the outcome? What are the properties of these systems? Which property do they fail to fulfill? Can we even propose axiomatic characterizations of such systems? What are the manipulation and control strategies for these systems, and what are their complexity? How can we translate the models of bribery and lobbying to this context? Given that many projects of electoral reforms (e.g. France, Italy) propose to move towards mixed voting systems, the researchers present at the session agreed that the subject was of crucial importance in the field and needed further investigations.

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5 Rump Session

The rump session featured the announcement by Umberto Grandi of the sixth workshop in the series of COMSOC workshops. Pnyx, a tool for preference aggregation, was presented by Christian Geist, and Nicolas Mattei reported on PrefLib, a reference library of preference data and links. Several research talks were presented:

- Vincent Merlin took the example of the Eurovision European Song Contest to illustrate the problem of the determination of potential and necessary winners for scoring rules.
- Edith Elkind reported on joint work with her PhD student Dominik Peters on a hardness result for single-peaked preferences on trees.

- Ioannis Caragiannis talked about the optimal parliament problem, whose composition maximizes the number of issues for which the majority of the parliament and the public opinion agree on.
- Jérôme Lang defined the "Dagstuhl boat trip problem", a specific hedonic game with a Stackelberg flavor, in the style of the problem [1] which was born at Dagstuhl seminar 12101 on computational social choice, and motivated participants for collaboration on this problem.
- Umberto Grandi presented several issues that remain open in the literature on opinion diffusion on networks, such as existence of convergence, or search for the most influential nodes.
- Steven Brams presented the "catch-up game", where two players alternate choosing subsets of numbers, without replacement, from $\{1, 2, \ldots, n\}$ such that each player's running sum is equal to or greater than – but not more than by the smallest new number he chooses – his opponent's running sum on the last turn.
- Piotr Faliszewski referred to the domain restriction VI for approval votes by E. Elkind and M. Lackner. He showed that dichotomous preference profiles that are not VI can be obtained from single-crossing ones and asked for an efficient algorithm to recognize these.
- Péter Biró raised the question of the search for an optimal matching when preferences are uncertain.
- Jia Yuan Yu proposed two open problems: How should one assign parking spots to cars in a fair fashion so that each driver is inconvenienced as equally as possible, and how could ride-sharing (matching travelers) be used to avoid bursty arrivals at congested roads.
- Judy Goldsmith talked on generating CP-nets in an independent and identically distributed way.
- Flip Klijn presented results he recently obtained with P. Jaramillo and C. Kayi about the possibility to induce truth telling for the stable matchings generated by rank-priority mechanisms.
- Nicolas Maudet talked about the length of paths that items can take, when traded from agent to agent in distributed resource allocation settings.
- Michel le Breton reported on results he obtained with T. Laurent, D. Lepelley and O. De Mouzon on the likelihood of election inversions in two-tier voting systems.
- Annick Laruelle presented the "disapproval rule" which enables voters to express their dissatisfaction with candidates, in contrast to absenteeism, spoiled or blank votes, or voting for a fringe candidate.

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Report from Dagstuhl Seminar 15242

Complexity of Symbolic and Numerical Problems

Edited by

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Abstract

This report documents the program and the outcomes of Dagstuhl Seminar 15242 "Complexity of Symbolic and Numerical Problems".

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1 Executive Summary

Peter Bürgisser
Felipe Cucker
Marek Karpinski
Nicolai Vorobjov

The seminar was dedicated to Prof. Dima Grigoriev on the occasion of his 60th birthday. Its aim was to discuss modern trends in computational real algebraic geometry, in particular, areas related to solving real algebraic and analytic equations and inequalities. Very recent new developments in the analysis of these questions from the point of view of *tropical mathematics* were also presented.

Historically there were two strands in the computational approach to polynomial systems' solving. One is the tradition of numerical analysis, a classical achievement of which is the *Newton's method*. Various other approximation algorithms were developed since then, some based on the idea of a *homotopy*. Numerical analysis did not bother to introduce formal models of computations (and hence computational complexity considerations) but developed refined methods of estimations of convergency rates. Another tradition emerged from algebra, particularly in classical works of Cayley, Sylvester and Macaulay. Algebraic results concerning *real* solutions go further back to the Descartes' rule and Sturm sequences. An important contribution to the subject from logic was Tarski's constructive quantifier elimination procedures for algebraically closed and real closed fields. The computations considered in this tradition are exact, under modern terminology – "symbolic". They naturally fit into standard models of computation (Turing Machines, straight-line programs, computation trees) thus lending themselves to complexity analysis.

Until 1990s these two strands developed largely independently. One of the important unifying ideas became the concept of a *real numbers* (or *BSS*) machine suggested by Blum, Shub and Smale which can be considered as a model of computation for the numerical analysis. This idea led to Smale's 9th and 17th problems, which became an inspiration for many researchers in the field.

The seminar considered a wide set of questions related to the current state of the symbolic and numeric approaches to algorithmic problems of real algebraic and analytic geometry, also from the novel perspective of tropical and max/plus mathematics.

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3 Overview of Talks

In this section we give a very coarse general outline of the results presented at the seminar. The reader should consult the *Abstracts* section for a more precise description of the talks.

One of the conceptually most simple and computationally efficient methods for deciding consistency of systems of polynomial equations over algebraically closed fields is based on *Effective Hilbert's Nullstellensatz* – the theorem known since late 1980s. Over real closed fields, where polynomial inequalities make sense, the analogy of the classical Hilbert's Nillstellensatz is *Positivstellensatz* (in different versions). This seminar was one of the first conferences where the new breakthrough result was announced: an elementary recursive bound for Effective Positivstellensatz and Hilbert 17-th problem, proved by H. Lombardi, D. Perrucci, and M.-F. Roy.

In tropical mathematics, the research of algorithmic and complexity issues had started only very recently, and mostly in linear algebra. One of the first complexity results in polynomial algebra, *Tropical Effective Nullstellensatz*, was reported at the seminar by V. Podolskii (joint work with D. Grigoriev). This remarkable achievement starts a new chapter in symbolic computer algebra. In the talk of A. Weber, complexity aspects of tropical algebra were applied to bio-chemical reaction networks.

In complexity theory, the Graph Isomorphism problem is one of the most exciting since it is not known to be NP-complete or belong to P. On the other hand, the problem is polynomially equivalent to finding the automorphism group of a colored graph. In the talk of I. Ponomarenko, for the important particular case of the latter problem, the Cartan association scheme, an algorithm with polynomial complexity was presented.

Talks of F. Cucker, D. Amelunxen, and P. Bürgisser were dedicated to the latest advances in complexity theory of numerical algorithms, foundations of which were developed in the recent monograph "Condition" by Bürgisser and Cucker. Cucker presented a theory of complexity for numerical computations that takes into account the condition of the input data and allows for roundoff in the computations. Amelunxen proposed a new model for probabilistic analysis of condition numbers which, when applied to the convex feasibility problem, yields a dramatic improvement in complexity (joint work with M. Lotz). Bürgisser presented a proof of the condition number theorem, characterizing the *condition* of computing a point in the intersection of a fixed complex projective variety with an input linear subspace of the complement dimension.

A number of talks described recent breakthroughs concerning problems in classical complexity theory. D. Grigoriev applied a technique of cluster algebras to close a long-standing problem on the comparative complexity power between all possible subsets of operations $+$, $-$, $\times$, $/$ in arithmetic circuits. This is done via computing Schur functions (joint work with S. Fomin and G. Koshevoy). É. Shost gave in his talk an alternative algorithm for computing Schur functions. N. Vorobjov presented complexity lower bounds for testing membership in semi-algebraic sets on algebraic computation trees and arithmetic networks (joint work with A. Gabrielov). Using recent advances in o-minimal topology the classical lower bounds of Yao and Montaña-Morais-Pardo were expanded to singular homology on arbitrary semi-algebraic sets. Within the theme of classical complexity, K. Meer suggested a new, algebraic, proof of the real number PCP (probabilistically checkable proof) theorem (joint work with M. Baartse). This result is an exact match with the main motive of the seminar: interplay between symbolic and numerical approaches to computation.

A group of seminar talks discussed aspects of computer algebra. J. Davenport reported on practical improvements in Cylindrical Algebraic Decomposition algorithm (a subroutine extensively used in, e.g., Maple), in the presence of equational constraints. S. Basu presented a theory of symmetric groups acting on symmetric real algebraic varieties, semi-algebraic sets, and symmetric complex varieties in affine and projective spaces, defined by polynomials of fixed degrees. He gave polynomial bounds on the number of irreducible representations of such groups, as well as their multiplicities (joint work with C. Riener). P. Koiran explained a version of a fewnomial theorem. It turns out that in the case of two polynomial equations in two variables, when one has a degree $d \geq 1$ and another t monomials, the number of real solutions is polynomial in d and t when this number is finite. This result is in sharp contrast with the famous Khovanskii's general bound, which is exponential in t. M. Safey El Din presented a new algorithm for computing roadmaps in smooth algebraic sets, having the lowest complexity achieved so far (joint work with É. Schost). A classical topic in computer algebra was re-run by E. Kaltofen, who reproduced his 1987 talk on fast multiplication of polynomials over arbitrary rings with various modern witty comments.

The seminar featured a number of talks on a broad subject of convex geometry which are related to both symbolic and numerical computing. D. Pasechnik considered the problem of reconstructing a measure in $\mathbb{R}^d$ from a truncated multi-sequence of its moments, in an important particular case of a measure with piecewise-polynomial density supported on a compact polyhedron. He showed that this problem can be solved exactly (joint work with N. Gravin, and B. and M. Shapiro). M. Lotz discussed various applications of spherical integral geometry, in particular the complexity theory of conic optimization and convex optimization approaches to solving underdetermined systems of equations. T. Theobald revisited a classical problem of the complexity of deciding containment of one polyhedron in another, where polyhedra can be defined either by linear inequalities or as convex hulls, in any combination. The novel approach uses sums of squares technique (joint work with K. Kellner).

Modern cryptography was represented by the talk of V. Spilrain. He explained a revolutionary approach to building public key cryptosystems, based on laws of classical physics, and not using any trapdoor functions (joint work with D. Grigoriev).

A. O. Slissenko presented a novel view on the work of an algorithm as a process of the decreasing uncertainty (entropy) about the output. The complexity aspect of this approach requires understanding of what is the speed of this decreasing. A technique is suggested which allows to develop an adequate definition.

The talk by E. Hirsch was devoted to a major conjecture in proof complexity: the existence of an algorithm, called acceptor, that is optimal on all propositional tautologies. It was claimed that in the presence of errors such optimal algorithm exists (joint with D. Itsykson, I. Monakhov, A. Smal).

4 Abstracts of talks

4.1 An elementary recursive bound for effective Positivstellensatz and Hilbert 17-th problem

Marie-Françoise Roy (University of Rennes, FR)

Joint work of Marie-Françoise Roy, Henri Lombardi, Daniel Perrucci

We prove the first elementary recursive bounds in the degrees for Positivstellensatz and Hilbert 17-th problem, which is the expression of a nonnegative polynomial as a sum of squares of rational functions. We obtain a tower of five exponentials. A precise bound in terms of the number and degree of the polynomials and their number of variables is provided. (See http://arxiv.org/abs/1404.2338v2.)

4.2 Tropical Effective Nullstellensatz

Vladimir Podolskii (Steklov Institute – Moscow, RU)

Joint work of Vladimir Podolskii, Dima Grigoriev

A tropical (or min-plus) semiring is a set real numbers, possibly with infinity, endowed with two operations: tropical addition, which is just usual minimum operation, and tropical multiplication, which is usual addition. Tropical polynomials can be defined analogously to classical polynomials. In tropical algebra, a tuple $\mathbf{x}$ is a solution to a multivariate polynomial $\min(g_1(\mathbf{x}), g_2(\mathbf{x}), \ldots, g_k(\mathbf{x}))$, where $g_i(\mathbf{x})$'s are tropical monomials, if the minimum is attained at least twice. If we consider a convex piece-wise linear function given by a tropical polynomials, then roots correspond to non-smoothness points of the function.

In this talk we present a tropical effective analog of Hilbert's Nullstellensatz.

4.3 Recognizing the Cartan association schemes in polynomial time

Ilia Ponomarenko (Steklov Institute – St. Petersburg, RU)

It is well known that the Graph Isomorphism Problem is polynomially equivalent to finding the automorphism group of a colored graph. In the present talk, we deal with a special case of the latter problem. Namely, the Cartan association scheme can be thought of as the complete colored graph the color classes of which are the orbits of a finite group with BN-pair that acts on the cosets of the Cartan subgroup $B \cap N$. We show that the following problem can be solved in a polynomial time (in the number of vertices): given a colored complete graph check whether it is a Cartan scheme associated with a simple group and (if so) find the automorphism group of the graph.

4.4 Empiric investigations of some seemingly slowly growing complexity parameters of bio-chemical reaction networks

Andreas Weber (Universität Bonn, DE)

Joint work of Andreas Weber, Dima Grigoriev, Ovidiu Radulescu, Satya Swarup Samal

Bio-chemical reaction networks are bi-partitite graphs involving the reacting species and the reactions as vertices. These can range from a few for so called *network motifs* up to several thousands (e.g., for networks reconstructing yeast metabolism or human metabolism). Assuming well-mixing and mass action kinetics, the dynamics of the networks is given by a system of ordinary differential equations with polynomial vector fields. Whereas a priori only little special structure is known to reduce the complexity of several computational tasks on the given systems, it has recently been shown that the tree width of the networks is growing slowly (and even being smaller than 6 for most networks). In this talk we will focus on computing tropical equilibrations, which are important for several purposes, e.g. for model reduction, but is an NP-complete task in general. Performing computations on the BIOMODELS database we found that the number of maximal solution polytopes is much smaller than had to be expected.

4.5 A Theory of Complexity, Condition, and Roundoff

Felipe Cucker (City University – Hong Kong, HK)

We develop a theory of complexity for numerical computations that takes into account the condition of the input data and allows for roundoff in the computations. We follow the lines of the theory developed by Blum, Shub, and Smale for computations over $\mathbb{R}$ (which in turn followed those of the classical, discrete, complexity theory as laid down by Cook, Karp, and Levin among others). In particular, we focus on complexity classes of decision problems and paramount among them, on appropriate versions of the classes P, NP and EXP of polynomial, nondeterministic polynomial, and exponential time, respectively. We prove some basic relationships between these complexity classes and exhibit natural NP-complete problems.

4.6 On a blindspot in probabilistic analysis of condition numbers

Dennis Amelunxen (City University – Hong Kong, HK)

Joint work of Dennis Amelunxen, Martin Lotz

The common practice in the probabilistic analysis of condition numbers suffers from a strange imbalance in the model, which seems to have gone unnoticed so far. On the one hand it is emphasized that the use of condition numbers takes into account real-world limitations of numerical computations such as the effects of floating-point arithmetics, while on the other hand an overly strict mathematical model is used, which prevents the inclusion of important effects in high-dimensional geometry that are loosely summarized by the term “concentration of measure”.

We propose a small but decisive change in the model used in the probabilistic analysis, which broadly consists of replacing the nullset of ill-posed inputs by an exponentially small set. While this change is easily accepted from an applications point of view, the resulting change in the probabilistic behavior of the condition number can be dramatic. Indeed, in the case of the convex feasibility problem the expectation of Renegar’s condition number (not its logarithm!) has constant “weak average-case complexity”, as opposed to infinity, which is the answer in the classical form and which is in stark contrast to what is being observed in practice. The argument for this result is surprisingly simple.

4.7 Subtraction-free computations and cluster algebras

Dima Grigoriev (Lille I University, FR)

Joint work of Dima Grigoriev, S. Fomin, G. Koshevoy

Using cluster transformations we design subtraction-free algorithms for computing Schur polynomials and for generating spanning trees and arborescences polynomials. The latter provides an exponential complexity gap between circuits admitting arithmetic operations $+, \times, /$ versus $+, \times$. In addition, we establish an exponential complexity gap between circuits admitting $+, -, \times, /$ versus $+, \times, /$. Together with V. Strassen’s result on “Vermeidung von Divisionen” this closes a long-standing problem on comparative complexity power between all possible subsets of operations $+, -, \times, /$.

4.8 On Entropic Convergence of Algorithms

Anatol Slissenko (Université Paris-Est Créteil, FR)

This talk presents an attempt to find an information based view on the work of an algorithm. Unfortunately, there is no mathematical notion of information that adequately reflects our intuition. The only related notion, as far as I know, is that of entropy – that is a measure of uncertainty.

It is intuitively clear that an algorithm, while computing a function, diminishes the uncertainty of its knowledge about the result. The question is how to estimate quantitatively the speed of this decreasing of the uncertainty. In order to define entropy we introduce a probabilistic measure on the domain of deterministic algorithm on the basis of principle of Maximal Uncertainty: the uncertainty about the result is maximal if all the results are equiprobable. The measure is over inputs of a fixed length (for better intuition one may think that this set is of exponential size). It depends only on the graph of the computed function, not on the algorithm. Denote by f the function computed by an algorithm $\mathfrak{A}$, and by $\boldsymbol{dm}(f)$ and $\boldsymbol{rn}(f)$ the set of its inputs of a fixed size and respectively its range $f(\boldsymbol{dm}(f))$. The measure is $\boldsymbol{P}(f^{-1}(v)) = \frac{1}{|\boldsymbol{rn}(f)|}$, and it is uniform on $f^{-1}(v)$, $v \in \boldsymbol{rn}(f)$.

After that, and this is the next conceptual difficulty, we introduce a mapping of events of the algorithm under study, into partitions of subsets of $\boldsymbol{dm}(f)$. The work of $\mathfrak{A}$ is treated as the set of its traces. A trace $\boldsymbol{tr}(X)$ of $\mathfrak{A}$ for an element $X \in \boldsymbol{dm}(f)$ is a sequence of commands executed by $\mathfrak{A}$ for input X (plus the initial state that we do not make explicit), each such execution being an update (assignment) or a guard (evaluated as true); these are *events*, and $\boldsymbol{tr}(X, t)$ is the event at an instant t.

The mapping of events into partitions is based on some notion of similarity $\sim$ of events (which is assumed to be an equivalence). First, an event $E = \boldsymbol{tr}(X, t)$ is associated with a set $\widehat{E}$ of inputs X' such that $\boldsymbol{tr}(X, t) \sim \boldsymbol{tr}(X', t')$ for some t'. This set defines its ordered partition $\pi(E)$ into intersection of $\widehat{E}$ with $f^{-1}(v)$ for a fixed order of $\boldsymbol{rn}(f)$. For such a partition its entropy conditioned by $\widehat{E}$ is denoted $\mathcal{D}(E)$.

A metric function can be defined on $\pi(E)$. Not all events are informative, e.g., the loop counter gives nothing. Modulo this remark, sequences $\big(\mathcal{D}(\boldsymbol{tr}(X,t))\big)_t$, $X \in \boldsymbol{dm}(f)$, and the space $\mathfrak{R}(t) = \{\pi(\boldsymbol{tr}(X, \tau)) : X \in \boldsymbol{dm}(f) \wedge \tau \geq t\}$ give descriptions of entropic convergence of $\mathfrak{A}$.

4.9 Nature-based information security

Vladimir Shpilrain (City University of New York, US)

Joint work of Vladimir Shpilrain, Dima Grigoriev

We use various laws of classical physics to offer several solutions of Yao's millionaires' problem without using any one-way functions. We also describe informationally secure public key encryption protocols, i.e., protocols secure against passive computationally unbounded adversary. This introduces a new paradigm of decoy-based cryptography, as opposed to "traditional" complexity-based cryptography.

4.10 Equational Constraints and Cylindrical Algebraic Decomposition

James H. Davenport (University of Bath, GB)

Quantifier Elimination by Cylindrical Algebraic Decomposition is easier if there is a global equational constraint $f = 0 \wedge \cdots$. We have recently extended this to local equational constraints (R.J. Bradford, J.H. Davenport, M. England, S. McCallum, and D.J. Wilson, Proceedings ISSAC 2013, 125–132). We present this, and work in press (`arXiv1401.0645; 1501.04466`) on multiple equational constraints, and where the local equational constraints do not give a global constraint. This last is particularly useful for motion planning and branch cut applications. See http://staff.bath.ac.uk/masjhd/Slides/JHDatDagstuhlJune2015.pdf.

4.11 Condition of intersecting a fixed projective variety with a given linear subspace

Peter Bürgisser (TU Berlin, DE)

Let $Z \subseteq \mathbb{P}^n$ be a fixed complex projective variety of dimension m and put $s := n - m$. We define the *condition* $\kappa(L, z)$ of computing a point of intersection $z \in Z \cap L$ for an input $L \in \text{Grass}(\mathbb{P}^n, s)$ in the Grassmann manifold of s-dimensional linear subspaces as the operator norm of the (locally defined) solution map $L \mapsto z$ (if the intersection is transversal and z is smooth). We characterize $\kappa(L, z)$ in terms of the minimal principal angle between the tangent spaces $T_z Z$ and $T_z L$, and use this to prove a *condition number theorem* that characterizes $\kappa(L, z)$ as the inverse distance of L to a local Schubert variety of ill-posedness. Hence a probabilistic analysis of the maximum condition number $\kappa(L) := \max\{\kappa(L, z) \mid z \in Z \cap L\}$ is reduced to bounding the volume of the ε-tube around the hypersurface $\Sigma \subseteq \text{Grass}(\mathbb{P}^n, s)$ of ill-posed subspaces touching Z. As a first step towards this goal, we prove that $\text{vol}(\Sigma)/\text{vol}(\text{Grass}(\mathbb{P}^n, s)) = \deg(\Sigma)(s + 1)(n - s)/\pi$.

4.12 An algebraic proof of the real number PCP theorem

Klaus Meer (BTU Cottbus, DE)

Joint work of Klaus Meer, M. Baartse

The PCP theorem is a major achievement in theoretical computer science in the last two decades. There exist two intrinsically different proofs of it. The original one by Arora et al. being algebraic in nature, and a more recent one by Dinur based on graph theoretic techniques.

We are interested in PCP theorems for the real number model of computation introduced by Blum, Shub, and Smale. In earlier work we could prove the real number PCP theorem to hold along the lines of Dinur's proof. In this talk we report on an algebraic proof of the theorem. It is close in structure to the original one by Arora et al., but needs additional efforts to deal with several problems arising on the way.

4.13 On the isotypic decomposition of cohomology modules of symmetric semi-algebraic sets: polynomial bounds on multiplicities

Saugata Basu (Purdue University – West Lafayette, US)

Joint work of Saugata Basu, Cordian Riener

We consider symmetric (as well as multi-symmetric) real algebraic varieties and semi-algebraic sets, as well as symmetric complex varieties in affine and projective spaces, defined by polynomials of fixed degrees. We give polynomial (in the dimension of the ambient space) bounds on the number of irreducible representations of the symmetric group which acts on these sets, as well as their multiplicities, appearing in the isotypic decomposition of their cohomology modules with coefficients in a field of characteristic 0. We also give some applications of our methods in proving lower bounds on the degrees of defining polynomials of certain symmetric semi-algebraic sets, as well as improved bounds on the Betti numbers of the images under projections of (not necessarily symmetric) bounded real algebraic sets.

Finally, we conjecture that the multiplicities of the irreducible representations of the symmetric group in the cohomology modules of symmetric semi-algebraic sets defined by polynomials of fixed degrees are computable with polynomial complexity, which would imply that the Betti numbers of such sets are also computable with polynomial complexity. This is in contrast with general semi-algebraic sets, for which this problem is provably hard (#**P**-hard).

4.14 Rational moment generating functions and polyhedra

Dmitrii V. Pasechnik (University of Oxford, GB)

Joint work of Dmitrii V. Pasechnik, Nick Gravin, Boris Shapiro, Michael Shapiro

The problem of reconstructing a measure in $\mathbb{R}^d$ from a (truncated) multi-sequence of its moments has important applications, and is in general very hard to solve. We concentrate on a natural case of a measure m with piecewise-polynomial density supported on a compact polyhedron P, and show that such problems can be solved exactly, due to existence of a natural integral transform of the measure (known as Fantappie transformation), which is a rational function $F_m(u)$.

The denominator of $F_m(u)$ is the product of linear functions of the form $1 - \langle u, v\rangle$, with v belonging to certain finite multiset $V(P)$. $F_m(u)$ is closely related to a more well-known Laplace transform $L_m(u)$ of a related "conified" measure arising in the theory of hyperplane arrangements. It is an interesting problem to reconstruct F_m (or L_m) from the noisy data; this would entail approximate Pade approximation and/or factoring into linear terms.

There are interesting applications of L_m to compact (not necessarily convex) polyhedra P. Let $I(P)$ be the indicator function of P. Then $I(P)$ can be decomposed (up to a measure 0 subset) as a sum, with $+1$ or -1 coefficients, of $I(D)$, where D runs through simplices with vertices in $V(P)$. This can be viewed as a non-convex generalisation of triangulations of convex polytopes.

4.15 Optimal proving algorithms

Edward A. Hirsch (Steklov Institute – St. Petersburg, RU)

Joint work of Edward A. Hirsch, D. Itsykson, I. Monakhov, A. Smal

The existence of a (p)-optimal propositional proof system is a major open question in (proof) complexity; Krajíček and Pudlák (1989) show that this question is equivalent to the existence of an algorithm (acceptor) that is optimal on all propositional tautologies.

We show that in the presence of errors such optimal algorithms *do* exist. The concept is motivated by the notion of heuristic algorithms. Namely, we allow the algorithm to claim a small number of false "theorems" (according to any polynomial-time samplable distribution on non-tautologies) and err with bounded probability on other inputs.

This construction can be viewed as "physical" proof as opposed to "mathematical" proof: the validity of a candidate algorithm is established using an "experiment" (drawing non-theorems at random and feeding them to the algorithm).

4.16 On the intersection of a sparse curve and a low-degree curve: A polynomial version of the lost theorem

Pascal Koiran (ENS – Lyon, FR)

Consider a system of two polynomial equations in two variables:

$$F(X,Y) = G(X,Y) = 0,$$

where $F \in \mathbb{R}[X,Y]$, has degree $d \geq 1$, $G \in \mathbb{R}[X,Y]$ and has t monomials. We show that the system has only $O(d^3t + d^2t^3)$ real solutions when it has a finite number of real solutions. This is the first polynomial bound for this problem. In particular, the bounds coming from the theory of fewnomials are exponential in t, and count only nondegenerate solutions. More generally, we show that if the set of solutions is infinite, it still has at most $O(d^3t + d^2t^3)$ connected components. By contrast, the following question seems to be open: if F and G have at most t monomials, is the number of (nondegenerate) solutions polynomial in t?

The authors' interest for these problems was sparked by connections between lower bounds in algebraic complexity theory and upper bounds on the number of real roots of "sparse like" polynomials.

4.17 Another subtraction-free algorithm for computing Schur functions

Éric Schost (University of Western Ontario – London, CA)

In recent work, Fomin, Grigoriev and Koshevoy give subtraction-free algorithms for the computation of Schur functions (and some of their generalizations), following earlier works by Koev and Demmel. In this talk, we present another algorithm, which is hinted at as a remark in Fomin et al.'s paper.

4.18 On nearly optimal algorithms for computing roadmaps of real algebraic sets

Mohab Safey El Din (UPMC – Paris, FR)

Joint work of Mohab Safey El Din, Éric Schost

Canny introduced roadmaps of semi-algebraic sets as a key tool for reducing connectivity queries in arbitrary dimension to connectivity queries on 1-dimensional semi-algebraic sets. Indeed, roadmaps are (semi-)algebraic curves which have a connected intersection with all connected components of the semi-algebraic set under consideration (and contain the query-points).

In 2010, we introduced a new technique for computing roadmaps improving the long-standing complexity bounds derived from Canny-like procedures for computing roadmaps. This has led to several developments by Basu et al.

We obtained recently the first nearly optimal algorithm, i.e. running in time $D^{O(n\log(n))}$ when the input is an n-variate reduced regular sequence of degree $\leq D$ defining a smooth and bounded real algebraic set.

In this talk, we show how to remove the boundedness assumption and report on first practical results, showing that roadmaps can now be computed in practice for non-trivial examples.

4.19 Fast Multiplication of Polynomials over Arbitrary Rings

Erich Kaltofen (North Carolina State University – Raleigh, US)

As a nostalgic reprise of the time when Dima Grigoriev and I were young researchers, and in the memory of my co-author David G. Cantor (1935–2012), I will repeat my 1987 talk at Zürich, with overhead transparencies and such, on the algebraic complexity of polynomial multiplication.

I will also mention Mark Giesbrecht's 1997 application of one of our ideas to computing integral solutions to sparse systems of linear equations, and recent results based on Martin Führer's fast integer multiplication algorithm.

Unfortunately, unlike the matrix multiplication exponent, the

$$O(n\log(n)\log\log(n))$$

complexity still seems to remain the best after those 28 years.

4.20 Conic integral geometry and applications

Martin Lotz (Manchester University, GB)

Integral geometry and geometric probability, going back to the work of Blaschke and Santaló, deal with measures on spaces of geometric objects, and can answer questions about the probability that random geometric objects intersect. We discuss various applications of (spherical) integral geometry: from the complexity theory of conic optimization to the analysis of convex optimization approaches to solving underdetermined systems of equations. In particular, it is shown how integral geometry naturally gives rise to a complete explanation of phase transition phenomena for the applicability of convex regularization to data recovery problems. We also introduce combinatorial methods, based on the theory of hyperplane arrangements, to compute the conic intrinsic volumes of various cones of interest.

4.21 Sum of squares certificates for containment of H-polytopes in V-polytopes

Thorsten Theobald (Goethe-Universität Frankfurt am Main, DE)

Joint work of Thorsten Theobald, Kai Kellner

Given an H-polytope P and a V-polytope Q, the decision problem whether P is contained in Q is co-NP-complete. This hardness remains if P is restricted to be a standard cube and Q is restricted to be the affine image of a cross polytope. While this hardness classification by Freund and Orlin dates back to 1985, there seems to be only limited progress on that problem so far.

Based on a formulation of the problem in terms of a bilinear feasibility problem, we study sum of squares certificates to decide the containment problem. These certificates can be computed by a semidefinite hierarchy. As a main result, we show that under mild and explicitly known preconditions the semidefinite hierarchy converges in finitely many steps. In particular, if P is contained in a large V-polytope Q (in a well-defined sense), then containment is certified by the first step of the hierarchy.

4.22 Topological lower bounds for computation trees and arithmetic networks

Nicolai Vorobjov (University of Bath, GB)

Joint work of Nicolai Vorobjov, Andrei Gabrielov

We prove that the height of any algebraic computation tree for deciding membership in a semialgebraic set $\Sigma \subset \mathbb{R}^n$ is bounded from below by

$$\frac{c_1 \log(\mathrm{b}_m(\Sigma))}{m+1} - c_2 n,$$

where $\mathrm{b}_m(\Sigma)$ is the m-th Betti number of Σ with respect to "ordinary" (singular) homology, and c_1, c_2 are some (absolute) positive constants. This result complements the well known lower bound by Yao for *locally closed* semialgebraic sets in terms of the total *Borel-Moore* Betti number.

We also prove that if $\rho : \mathbb{R}^n \to \mathbb{R}^{n-r}$ is the projection map, then the height of any tree deciding membership in Σ is bounded from below by

$$\frac{c_1 \log(\mathrm{b}_m(\rho(\Sigma)))}{(m+1)^2} - \frac{c_2 n}{m+1}$$

for some positive constants $c_1,\ c_2$.

We illustrate these general results by examples of lower complexity bounds for some specific computational problems.

An analogous theory is developed for *arithmetic networks*, a computational model aimed to capture the idea of a parallel computation in its simplest form. Here we generalize lower bounds of Montaña, Morais and Pardo (who considered locally closed semialgebraic sets relative to Borel-Moore homology) to arbitrary semialgebraic sets relative to singular homology.

5 Open Problems

The open problems session was held on Tuesday, the 9th of June.

5.1 Determinantal Witnesses and Matchings

Marek Karpinski (Universität Bonn, DE)

Given a bipartite graph $G = (V, E)$ with n vertices and the adjacency matrix $A = [a_{ij}]$. Existence of a perfect matching in G is equivalent to checking the identity to zero of a symbolic determinant $S = \mathrm{Det}([a_{ij}x_{ij}])$. Evaluate S at the points $((p_{11})^i, \ldots, (p_{nn})^i)$ for i between 1 and $n!$ and p_{jk} being "consecutive" prime numbers. Denote the values of a symbolic determinant S at those points by a_i.

It is known that existence of a matching in G is equivalent to the existence of an index i, $1 \le i \le n!$, such that the number a_i is nonzero (Grigoriev, Karpinski 1987). Define a *determinantal witness* wt (G) of G to be a minimal index i with that property, and the n-dimensional witness wt_n to be a maximum of the determinatal witnesses of all graphs with n vertices.

Give an explicit construction of a bipartite graph such that $\mathrm{wt}(G) \ge 4$. Can the upper bound $n!$ on wt_n be reduced to a subexponential (or even polynomial) bound? Shedding some light on those issues will constitute a significant progress in the area.

5.2 Complexity of solving tropical or min-plus linear systems

Dima Grigoriev (Lille I University, FR)

A min-plus linear system has a form

$$\min_{1\le j\le n}\{a_{ij} + x_j\} = \min_{1\le j\le n}\{b_{ij} + x_j\},\ 1 \le i \le m$$

where the integer coefficients a_{ij}, b_{ij} fulfil bounds $|a_{ij}|, |b_{ij}| < M$. Solvability is asked in integers. This class of systems is polynomially equivalent to tropical linear systems and to mean pay-off games.

The long-standing question is in existence of a polynomial complexity (so, polynomial in $n, m, \log M$) algorithm for solving min-plus (or tropical) linear system. The known algorithms have complexity polynomial either in n, m, M or in $n^m, \log M$.

5.3 Construction of explicit disperser $f : \{0,1\}^n \to \{0,1\}^{o(n)}$

E. A. Hirsch (Steklov Institute – St. Petersburg, RU)

Construct an explicit disperser $f : \{0,1\}^n \to \{0,1\}^{o(n)}$ of the following kind: f should be non-constant on every possible set of solutions of size at least $2^{n/100}$ of a set of $O(n)$ quadratic equations over $\mathbb{F}_2$.

5.4 Conjecture: Log-concavity of conic intrinsic volumes

Dennis Amelunxen (City University – Hong Kong, HK)

If $C \subseteq \mathbb{R}^d$ is a (convex) polyhedral cone then its kth intrinsic volume can be defined as the probability that the projection onto the cone of a uniformly random point on the unit sphere falls into the relative interior of a k-dimensional face of C:

$$v_k(C) = \text{Prob}\{\Pi_C(\mathbf{p}) \in \text{relint}(F) |\ F\ k\text{-dimensional face of } C\},$$

where $\Pi_C(\mathbf{z}) = \mathbf{x}$ with $\|\mathbf{z} - \mathbf{x}\| = \min\{\|\mathbf{z} - \mathbf{y}\| \mid \mathbf{y} \in C\}$ and $\mathbf{p} \sim \text{Uniform}(S^{d-1})$.

(For more information about conic intrinsic volumes see `arXiv:1412.1569` and the references given therein.)

Conjecture: The intrinsic volumes of a cone form a log-concave sequence.

In technical terms, if $C \subseteq \mathbb{R}^d$ closed convex cone, then for all $1 \le k \le d-1$,

$$v_k(C)^2 \ge v_{k-1}(C)\, v_{k+1}(C).$$

It is known that these inequalities hold in dimension $d \le 4$ (which in connection with the stability of log-concavity under convolution yields an infinite set of positive examples in any dimension), and recent investigations about the behavior of the intrinsic volumes in high dimensions support the plausibility of these inequalities. A proof of this conjecture could be seen as a conic analog of the famous Alexandrov-Fenchel inequalities.

5.5 Lower bounds for sums of powers of degree 1 univariate polynomials

Pascal Koiran (ENS – Lyon, FR)

We consider representations of polynomials $f \in K[X]$ under the form

$$f(X) = \sum_{i=1}^{k} \alpha_i (x + a_i)^{e_i}.$$

The problem is to find explicit polynomials f of degree d which require at least $k = \Omega(d)$ terms in any representation of this form. Such polynomials are known for the field $K = \mathbb{R}$, but the problem seems to be open for $K = \mathbb{C}$. Some background can be found in the paper "lower bounds by Birkhoff interpolation" (in preparation).

5.6 Sign-representation

Vladimir Podolskii (Steklov Institute – Moscow, RU)

Consider a sequence of Boolean functions $\{f_n\}_{n\in\mathbb{N}}$, where $f_n : \{1, 2\}^n \to \{-1, 1\}$. We say that polynomials $p_n \in \mathbb{Z}[x_1, \ldots, x_n]$ sign-represent this sequence of functions if for all n and for all $\mathbf{x} \in \{1, 2\}^n$ we have $f_n(\mathbf{x}) = \operatorname{sign} p_n(\mathbf{x})$.

Suppose we know that the sequence $\{f_n\}_{n\in\mathbb{N}}$ can be sign-represented by a sequence of polynomials $\{p_n\}_{n\in\mathbb{N}}$ with the number of monomials growing polynomially in n. Can we say that the same sequence of functions can be sign-represented by a sequence of polynomials with polynomial number of monomials and with any bound on the degree?

The background on the problem can be found in the paper http://eccc.hpi-web.de/report/2013/021/.

5.7 Complexity of testing membership to Kronecker polytopes

Peter Bürgisser (TU Berlin, DE)

Kronecker coefficients play a crucial role in geometric complexity theory for proving lower bounds for tensor rank and determinantal complexity.

Let λ, μ, ν be three partitions of k into at most n parts. The Kronecker coefficient $g(\lambda, \mu, \nu)$ is the multiplicity of the irreducible $\mathrm{GL}_n(\mathbb{C})^3$-representation of type (λ, μ, ν) in the the space of forms of degree k on $\mathbb{C}^n \otimes \mathbb{C}^n \otimes \mathbb{C}^n$. Let $\Delta(n)$ denote the closure of the set of $\frac{1}{k}(\lambda, \mu, \nu)$ such that $g(\lambda, \mu, \nu) > 0$. It is known that $\Delta(n)$ is a convex polytope. We study the problem $\mathrm{KRON} - \mathrm{POLYTOPE}$ of testing membership to $\Delta(n)$ for given partitions λ, μ, ν, each given as a list of n integers encoded in binary (n is varying and part of the input). Recently, it was shown by Bürgisser, Christandl, Mulmuley, and Walter that KRON − POLYTOPE $\in$ NP $\cap$ coNP.

Is there a polynomial time algorithm for testing membership to Kronecker polytopes?

5.8 Cylindrical decomposition with topologically regular cells

Nicolai Vorobjov (University of Bath, GB)

Let $X \subset \mathbb{R}^n$ be a bounded set definable in an o-minimal structure over the reals, e.g., a semialgebraic or a subanalytic set. It is well known that $\mathbb{R}^n$ admits a cylindrical cell decomposition compatible with X. It is obvious from the definition that each cell of the decomposition is a *topological cell*, i.e., a homeomorphic image of a standard open ball. However, there are examples (see Section 4 in S. Basu, A. Gabrielov, and N. Vorobjov, J. European Math. Soc., 15, 2, 2013, 635-657) when a definable cylindrical cell is not *topologically regular*[1].

Conjecture: For any definable bounded $X \subset \mathbb{R}^n$ there is a cylindrical cell decomposition of $\mathbb{R}^n$, compatible with X, such that each cell, contained in X, is topologically regular.

This conjecture is proved in two cases: $\dim X \leq 2$ and $\dim X = 3,\ n = 3$ (S. Basu, A. Gabrielov, and N. Vorobjov, `arXiv:1402.0460`).

[1] A set Y is called *topologically regular cell* if the pair $(\overline{Y}, Y)$ is homeomorphic to the pair $(\overline{B}, B)$, where B is the standard open ball, and the bar denotes the closure operation.

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Report from Dagstuhl Seminar 15251

Sparse Modelling and Multi-exponential Analysis

Edited by

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Abstract

The research fields of harmonic analysis, approximation theory and computer algebra are seemingly different domains and are studied by seemingly separated research communities. However, all of these are connected to each other in many ways.

The connection between harmonic analysis and approximation theory is not accidental: several constructions among which wavelets and Fourier series, provide major insights into central problems in approximation theory. And the intimate connection between approximation theory and computer algebra exists even longer: polynomial interpolation is a long-studied and important problem in both symbolic and numeric computing, in the former to counter expression swell and in the latter to construct a simple data model.

A common underlying problem statement in many applications is that of determining the number of components, and for each component the value of the frequency, damping factor, amplitude and phase in a multi-exponential model. It occurs, for instance, in magnetic resonance and infrared spectroscopy, vibration analysis, seismic data analysis, electronic odour recognition, keystroke recognition, nuclear science, music signal processing, transient detection, motor fault diagnosis, electrophysiology, drug clearance monitoring and glucose tolerance testing, to name just a few.

The general technique of multi-exponential modeling is closely related to what is commonly known as the Pad/'e-Laplace method in approximation theory, and the technique of sparse interpolation in the field of computer algebra. The problem statement is also solved using a stochastic perturbation method in harmonic analysis. The problem of multi-exponential modeling is an inverse problem and therefore may be severely ill-posed, depending on the relative location of the frequencies and phases. Besides the reliability of the estimated parameters, the sparsity of the multi-exponential representation has become important. A representation is called sparse if it is a combination of only a few elements instead of all available generating elements. In sparse interpolation, the aim is to determine all the parameters from only a small amount of data samples, and with a complexity proportional to the number of terms in the representation.

Despite the close connections between these fields, there is a clear lack of communication in the scientific literature. The aim of this seminar is to bring researchers together from the three mentioned fields, with scientists from the varied application domains.

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1 Executive Summary

Annie Cuyt
George Labahn
Avram Sidi
Wen-shin Lee

The seminar brought together a number of researchers from polynomial interpolation, rational approximation and exponential analysis. The five day seminar centered around talks on Exponential Analysis (Day 2), Rational Approximation (Day 3) and Sparse Interpolation (Day 4). Applications were grouped on Day 1 in order to challenge the participants to discuss them further while related topics, mainly from Numerical Linear Algebra, were scheduled on Day 5.

The seminar itself started with a talk by Cuyt and Lee pointing out the considerable intersection of the three main themes, particularly as they all strongly overlap. In order to reach out to industry and connect the scientific research to the industrial needs, several participants working at industrial or real-life applications were invited for a presentation on the first day of the seminar. Then interaction about these topics would occur naturally throughout the week. We mention talks on Mobile sampling and sensor networks (Karlheinz Gröchenig), High-speed fluorescence lifetime imaging (David Li), The estimation of variable star periods (Daniel Lichtblau) and Imaging of structured arrays (Adhemar Bultheel).

In the past the three communities have mostly been following distinct paths of research and methods for computation. One of the highlights of the seminar was the realization of significant commonalities between the communities, something nicely pointed out in the talk of Roche. Prony's method takes center stage in this case, with its origin in 1795 being used to solve problems in exponential analysis. Prony's method appeared much later in the case of sparse polynomial interpolation with it's use by Blahout, Ben-or/Tiwari, and Giesbrecht/Labahn/Lee. Prony's method takes samples at multiples of a common point to determine the support and then makes use of separate Hankel methods for determining the individual coefficients or weights of the expression.

Numerical conditioning was a significant issue in many talks at the seminar. Beckermann and later Matos looked at numerical conditioning of Padé and rational approximation problems. In the former case Beckermann used the close relationship of Padé approximation to Prony's method to point out that the latter is, for the most part, a provably ill-conditioned problem. Still there were a number of approaches in both areas which attempted to address this conditioning issue. In the case of numerical computation of sparse polynomial interpolants, use is made of randomization to produce a better conditioning of the problem, primarily by separating the roots appearing in Prony's problem. A similar idea also appears in exponential analysis making use of the notion of stride length. In both cases the object is to spread out the roots which arise in Prony's method.

Rather than spreading out the roots one can instead spread out the coefficients of a sparse polynomial/exponential expression for improving numerical performance. Sparse interpolation does this by making use of the concept of diversification where the coefficients are spread out multiplying evaluation points using a random multiplier. A corresponding concept in exponential analysis is the use of shifted samples which is useful to address the problem of anti-aliasing.

Sparse interpolation also makes use of the concept of small primes sparse interpolation where exponents are reduced modulo a small prime. This recovers the exponents modulo the small prime. Doing this for a number of small primes (which can be done in parallel) allows one to reconstruct the true exponents. Of course one encounters the problem of collisions and inadvertant combinations of exponents. It was noticed at the seminar that exponential analysis has a corresponding technique which made use of sub-sampling. Collisions in this case correspond to aliasing. Again the different communities reported on their methods for overcoming such collisions/aliasing problems.

Researchers at the seminar also showed interest in multivariate Prony methods. In the case of sparse interpolation one encounters Zippel's method while in exponential analysis there are projection methods. In these cases one attempts recursive methods for estimating the support of the underlying multivariable expression. In the case of multivariate polynomial interpolation a second approach is to convert the multivariate problem into a univariate problem by making use of randomized Kronecker substitution. Exponential sums takes a similar approach using random lattice projection.

While there were strong commonalities between the main research areas, there were also some strong differences between the topics noted at the seminar. The most telling of these differences was the analysis of exponential sums which have polynomial, rather than constant coefficients. Such expressions appear naturally when modeling solutions of linear differential equations where the associated polynomial has repeated roots. Of course such problems have considerable numerical issues when the roots of the associated polynomial are close but not numerically equal. Sidi and Batenkov both pointed out the importance and difficulties when dealing with such problems.

The seminar was also important for illustrating the applications of the three research areas. In many cases the applications involved the need to only work with sums having a small sparse support rather than with the complete set of possible nonzero elements. Methods from the multivariate Prony problem were exploited by Collowald and Hubert to determine new cubature formulas invariant to some specific finite groups action. Markovsky showed the similarities to the exponential sum problems with the notion of low rank approximation of structured matrices. Software was also discussed. Numerical analysis of errors on experimental runs also brought up the issue of the type of random distributions used when simulating errors for the experiments.

2 Table of Contents

3 Overview of Talks

3.1 Multidimensional approximation of functions sampled at unequally spaced points by sums of exponentials

Fredrik Andersson (Lund University, SE)

Joint work of Andersson, Fredrik; Carlsson, Marcus; Wendt, Herwig

Let f be sampled at unequally spaced points $x_m \in \mathbb{R}^d$. We consider the problem of finding

$$g(x) = \sum_{k=1}^{K} c_k e^{2\pi i x \cdot \xi_k}, \tag{1}$$

so that $f(x_m) \approx g(x_m)$.

Let Ξ and Υ be subsets of equally spaced grids in $\mathbb{R}^d$ and let $\Omega = \Xi + \Upsilon = \{x + y : x \in \Xi, y \in \Upsilon\}$. Given a function g on Ω, consider the generalized multidimensional Hankel operator

$$\Gamma_g h(x) = \sum_{y \in \Upsilon} g(x+y)h(y), \quad x \in \Xi \tag{2}$$

By Kronecker;s theorem Γ_g has rank K if g is of the form (1). It also turns out that range of Γ_g is the space of all linear combinations of the functions $e^{2\pi i x \cdot \xi_k}$ on Ξ (See Lemma 4.2 of [1]). Let us represent the operator with the matrix $\mathbf{\Gamma}_g$.

Let $\mathbf{J}$ be an interpolation matrix that interpolates the values at the equally spaced point in Ω to unequally spaced points $\Psi = \{x_m\}_{m=1}^M$ in $\mathbb{R}^d$. In order to approximate the function f sampled at Ψ using K exponentials, we consider the optimization problem

$$\begin{aligned} &\underset{g}{\text{minimize}} \quad \sum_{m=1}^{M} |(\mathbf{J}g)_m - f(x_m)|^2 \\ &\text{subject to} \quad \text{rank } \mathbf{\Gamma}_g = K \end{aligned} \tag{3}$$

where $(\mathbf{J}g)_m$ is the interpolated value of g at x_m.

We follow the setup in [4], and formulate (3) using the alternating direction method of multipliers [5]. The problem formulations is not convex, and there is hence no guarantee that the procedure will converge. However, it will typically give a matrix values of g such that the singular values σ_k of $\mathbf{\Gamma}_g$ are small if $k > K$. To estimate the (multidimensional) frequencies ξ_k associated with g we can then follow the approach gives in [2, 3] by solving systems of polynomial equations with coefficients taken from the singular vectors of $\mathbf{\Gamma}_g$ for $k > K$.

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3.2 Fourier-Sparsity Testing of Boolean Functions

Andrew Arnold (University of Waterloo, CA)

Joint work of Arnold, Andrew; Blais, Eric

We consider the problem of testing whether a function f has at most s nonzero Fourier coefficients, in which case we say f is s-sparse, given black-box access to f. We restrict our attention to perhaps the simplest case when f is a Boolean function acting on n bits. The analogous problem of learning the Fourier transform of an s-sparse Boolean function f was studied in previous work by Kushilevitz and Mansour [SIAM J. Computing, vol 22. (1992)], and Levin [J. Symb. Logic, vol 58. (1993)], the latter resulting in an $O(ns)$ Monte Carlo Sparse Fourier Transform (SFT) algorithm. Their work was the foundation for subsequent Sparse Fourier Transform algorithms in more general settings.

We say an algorithm is an ϵ-tester for sparse Boolean functions if it accepts if f is s-sparse and rejects if f is ϵ-far from s-sparse in terms of ℓ_2 norm, each with probability at least 2/3. Gopolan et al. [SIAM J. Computing, vol 40. (2011)], gave the first such tester with query-complexity polynomial in s and ϵ^{-1}.

We improve upon this result, present a sparsity tester with query-complexity $O(s \log s\epsilon^{-2} + \epsilon^{-4})$. Our tester relies on dimensionality-reduction techniques developed in the aforementioned previous work. Using these techniques, we reduce sparsity testing to the problem of homomorphism testing, which in turn may be solved via the Blum- Luby-Rubinfeld (BLR) linearity test [J. Comput. Syst. Sci. Int., vol 47. (1993)].

3.3 Numerical stability of the parameter estimation problem in sparse generalized exponential sums

Dmitry Batenkov (Technion – Haifa, IL)

Joint work of Batenkov, Dmitry; Yomdin, Yosef

We consider the parameter estimation problem in sparse generalized exponential sums of the form $m(k) = \sum_{j=1}^{s} e^{\imath x_j k} \sum_{\ell=0}^{d_j-1} a_{\ell,j} k^{\ell}$, when $m(k)$ are known only approximately.

We provide estimates on the component-wise condition numbers of the parameters x_j and $a_{\ell,j}$ above, and show that they can be accurately recovered by sampling at arithmetic progressions and polynomial homotopy methods.

We also discuss the application of these ideas to the problem of recovering a piecewise-smooth function (including the positions of the discontinuities) from its Fourier coefficients.

3.4 On the conditioning of the Padé map and related questions

Bernhard Beckermann (University of Lille, FR)

Joint work of Beckermann, Bernhard; Matos, Ana C.

Padé approximants play an important role in signal processing, sparse interpolation and exponential analysis. In this talk we will report about recent results from [1] concerning the forward and backward conditioning of the (real) Padé map, which sends a vector of Taylor coefficients onto the normalized vector of coefficients of the Padé numerator and denominator. In particular, we show that this map is not necessarily well conditioned for robust Padé approximants in the sense of Trefethen et al. [2].

We will also discuss the condition number of related non-linear maps.

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3.5 Sub-Nyquist spectral analysis

Matteo Briani (University of Antwerp, BE)

Joint work of Briani, Matteo; Cuyt, Annie; Lee, Wen-shin

In the field of sparse interpolation, parametric methods aim to retrieve the values of parameters of a linear combination of exponential functions from samples in a uniform time grid. These samples are collected following the Shannon-Nyquist theorem that dictates the minimum sampling rate that prevents the aliasing effect. In this paper we explain how it is possible, by means of undersampling, to use a coarser time grid and still be able to solve the aliasing effect. This reflects into a better conditioning of the problem and this behavior is explained by means of the ill-disposedness and a link to Padé approximation theory. Avoiding the aliasing effect, and using a coarser time grid, it is possible to perform several smaller independent analysis from the original set of samples. Joining these analysis together we obtain a method that brings higher accuracy to the existing parametric methods and introduces an extra parameter that can be use as validator. This is especially useful when the parametric method has to deal with signals consisting of close frequencies in a broad spectrum.

3.6 Order parameter for images of structured arrays

Adhemar Bultheel (KU Leuven, BE)

Joint work of Bultheel, Adhemar; Kaatz, Forrest

In nature (e.g. a bee honeycomb, muscle structure, crystals) or in engineering (e.g. micro lens arrays, nano pore/pillar arrays, solar cells) two-dimensional highly regular arrays are produced. Hexagonal, square or triangular grids are most common. Perfect symmetry of the grids does not exist in practical situations. Given the image of some array, one may analyse the properties of each of the individual nodes of the grid and compute parameters like their size, the location of their centers, perhaps their orientation, etc. These parameters could be combined to define some number indicating the deviation from the ideal grid.

We have tried to compute some order parameter from the Fourier transform of the image. For example a perfect hexagonal array has a Fourier spectrum that consists of a central peak, surrounded by six smaller peaks and their harmonics. This is a sparse exponential representation. The more the nodes in the image are dislocated from the perfect grid, the more noise will show up in the spectrum. Thus the amount of noise in the Fourier domain can be used as a measure for the disorder of the original grid.

Unfortunately, images may depend on many parameters (number of nodes, size of the nodes, shape of the nodes, orientation of the image,. . .) so that the Fourier technique only works in a rather restrictive number of situations and it is probably not useful in practical situations.

3.7 A moment matrix approach to symmetric cubatures

Mathieu Collowald (INRIA Sophia Antipolis – Méditerranée, FR)

Joint work of Collowald, Mathieu; Hubert, Evelyne

Quadrature and sparse interpolation are closely linked. The common key issue is the construction of a linear form

$$\Lambda : \mathbb{R}[x] \to \mathbb{R}, p \mapsto \sum_{j=1}^{r} a_j p(\xi_j)$$

from the knowledge of its restriction to $\mathbb{R}[x]_{\leq d}$. The unknowns are the weights a_j and the nodes ξ_j.

Cubature is a generalization of quadrature in higher dimension. An approach based on moment matrices was proposed in [2, 4]. We give a basis-free version in terms of the Hankel operator $\mathcal{H}$ associated to Λ. The existence of a cubature of degree d with r nodes boils down to conditions of ranks and positive semidefiniteness on $\mathcal{H}$. The nodes are then the solutions of a generalized eigenvalue problem.

Standard domains of integration are symmetric under the action of a finite group. It is natural to look for cubatures that respect this symmetry [1, 3]. Introducing adapted bases obtained from representation theory, the symmetry constraint allows to block diagonalize the Hankel operator $\mathcal{H}$. The size of the blocks is explicitly related to the orbit types of the nodes. From the computational point of view, we then deal with smaller-sized matrices both for securing the existence of the cubature and computing the nodes.

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3.8 Exponential analysis, Sparse interpolation and Padé approximation

Annie Cuyt (University of Antwerp, BE)

Joint work of Cuyt, Annie; Lee, Wen-shin

A common underlying problem statement in many applications is that of determining the number of components, and for each component the value of the frequency, damping factor, amplitude and phase in a multi-exponential model. It occurs, for instance, in magnetic resonance and infrared spectroscopy, vibration analysis, seismic data analysis, electronic odour recognition, keystroke recognition, nuclear science, music signal processing, transient detection, motor fault diagnosis, electrophysiology, drug clearance monitoring and glucose tolerance testing, to name just a few.

The general technique of multi-exponential modeling is closely related to what is commonly known as the Padé-Laplace method in approximation theory, and the technique of sparse interpolation in the field of computer algebra. The problem of multi-exponential modeling is an inverse problem and therefore may be severely ill-posed, depending on the relative location of the frequencies and phases. Besides the reliability of the estimated parameters, the sparsity of the multi-exponential representation has become important. A representation is called sparse if it is a combination of only a few elements instead of all available generating elements.

Despite the close connections between these fields, there is a clear lack of communication in the scientific literature. The aim of this seminar is to bring researchers together from the three mentioned fields, with scientists from the varied application domains.

3.9 Inverse Problems regularised by Sparsity

Pier Luigi Dragotti (Imperial College London, GB)

Modelling signals as sparse in a proper domain has proved useful in many signal processing tasks and, here, we show how sparsity can be used to solve inverse problems. We first recall that many inverse problems involve the reconstruction of continuous-time or continuous-space signals from discrete measurements and show how to relate the discrete measurements to some properties of the original signal (e.g., its Fourier transform at specific frequencies). Given this partial knowledge of the original signal, we then solve the inverse problem using sparsity. We focus on two specific problems which have important practical implications: localisation of diffusion sources from sensor measurements and reconstruction of planar domains from samples. First, we show how to reconstruct specific planar domains whose contours are determined using implicit functions, then we localise diffusion sources using a variation of the 'reciprocity gap' method which involves analytic test functions.

In both cases, the problem is solved by building a Prony's type system and by building structured matrices which, in the ideal settings, are simultaneously Toeplitz and rank deficient.

3.10 Mobile Sampling

Karlheinz Groechenig (Universität Wien, AT)

Joint work of Groechenig, Karlheinz; Romero, Jose Luis; Unnikrishnan, Jayakrishnan; Vetterli, Martin

We study the design of sampling trajectories for stable sampling and the reconstruction of bandlimited spatial fields using mobile sensors. The spectrum is assumed to be a symmetric convex set. As a performance metric we use the path density of the set of sampling trajectories that is defined as the total distance traveled by the moving sensors per unit spatial volume of the spatial region being monitored. Focussing first on parallel lines, we identify the set of parallel lines with minimal path density that contains a set of stable sampling for fields bandlimited to a known set. We then show that the problem becomes ill-posed when the optimization is performed over all trajectories by demonstrating a feasible trajectory set with arbitrarily low path density. However, the problem becomes well-posed if we explicitly specify the stability margins. We demonstrate this by obtaining a non-trivial lower bound on the path density of an arbitrary set of trajectories that contain a sampling set with explicitly specified stability bounds.

This is joint work with Jose Luis Romero, Univ. of Vienna, Jayakrishnan Unnikrishnan and Martin Vetterli from Ecole Polytechnique Fédérale de Lausanne (EPFL), Switzerland.

3.11 Error-Correcting Sparse Interpolation in Chebyshev Basis

Erich Kaltofen (North Carolina State University – Raleigh, US)

Joint work of Arnold, Andrew; Kaltofen, Erich L.

We present an error-correcting interpolation algorithm for a univariate black-box polynomial that has a sparse representation using Chebyshev polynomials as a term basis. Our algorithm assumes that an upper bound on the number of erroneous evaluations is given as input, and is a generalization of the algorithm by Lakshman and Saunders [SIAM J. Comput., vol. 24 (1995)] for interpolating sparse Chebyshev polynomials and the techniques in error-correcting sparse interpolation in the usual basis of consecutive powers of the variable due to Comer, Kaltofen, and Pernet [Proc. ISSAC 2012 and 2014]. We prove the correctness of our list-decoder-based algorithm with a Descartes-rule-of-signs-like property for sparse polynomials in Chebyshev basis. We also give a new algorithm that reduces the sparse interpolation in Chebyshev basis to that in power basis, thus making the many techniques for the sparse interpolation in power basis, for instance, supersparse (lacunary) interpolation over large finite fields, available to interpolation in Chebyshev basis. Furthermore, we can customize the randomized early termination algorithms from Kaltofen and Lee [J. Symb. Comput., vol. 36 (2003)] to our new approach.

3.12 A multivariate generalization of Prony's method

Stefan Kunis (Universität Osnabrück, DE), Ulrich von der Ohe (Universität Osnabrück, DE)

Joint work of Kunis, Stefan; Peter, Thomas; Römer, Tim; von der Ohe, Ulrich

A classical solution to the problem of parameter reconstruction for an exponential sum from a finite number of samples is given by Prony's method and the parameters are recovered as roots of a single univariate polynomial. We present a generalization of this method for exponential sums in an arbitrary finite number of variables and realize the parameters as common roots of several multivariate polynomials. Finally, the coefficients of the exponential sum arise as solutions to a linear system of equations.

In the first part of the talk we explain this approach and its algebraic properties. Provided we sample the exponential sum on an equidistant grid with a number of grid points in each coordinate direction bounded from below by the number parameters, unique reconstruction is guaranteed and this bound is shown to be sharp. In its simplest form, the reconstruction method consists of setting up a certain multilevel Toeplitz matrix of the samples, compute a basis of its kernel, and compute by some method of choice the set of common roots of the multivariate polynomials whose coefficients are given in the second step.

The second part of the talk is dedicated to numerical properties of our approach. Provided the number of grid points in each coordinate direction is bounded from below by some small constant divided by the separation distance of the parameters, the kernel of the above Toeplitz matrix can be stably computed. Moreover, we relate our approach to a recent semidefinite optimization formulation and show a couple of numerical experiments.

3.13 Behavior preserving extension of univariate and bivariate functions

David Levin (Tel Aviv University, IL)

Given function values on a domain D_0, possibly with noise, we examine the possibility of extending the function to a larger domain D, $D_0 \subset D$. In addition to smoothness at the boundary of D_0, the extension on $D \setminus D_0$ should also inherit behavioral trends of the function on D_0, such as growth and decay or even oscillations. The approach chosen here is based upon the framework of linear models, univariate or bivariate, with constant or varying coefficients.

3.14 Estimating Variable Star Periods from Unevenly Sampled Light Curve Data

Daniel Lichtblau (Wolfram Research – Champaign, US)

Joint work of Lichtblau, Daniel; Bryant, Jeffrey

A problem of interest in astronomy is determining the period of variable stars. Data collection is of necessity irregular (can only sample on clear nights) and noisy (from light pollution, atmospheric differences, etc.) We describe several ways in which period estimation can be performed on such data. Some are by now classical (from 60's–80's). One newer method will use Diophantine approximation.

3.15 High-speed fluorescence lifetime imaging (FLIM) instruments with fast hardware-friendly exponential analysis

David Li (The University of Strathclyde – Glasgow, GB)

Fast fluorescence lifetime imaging (FLIM) techniques are powerful tools for visualising protein interaction networks in living cells. FLIM has been used for cancer diagnosis, assessing drug efficacy in cancer therapy, understanding brain functions, etc. It can also sense physiological parameters such as Ca^{2+}, pH, O_2, temperature, viscosity, etc [1, 2]. For real- time applications, such as visualising neuronal activities or fast biophysical phenomena, it is desirable to apply innovative solid-state single-photon sensors [3, 4] and fast hardware embedded exponential analysis processors that can boost FLIM imaging [5, 6, 7]. But is it easy to have a hardware-friendly and high-efficient exponential analysis method for such applications?

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3.16 Structured low-rank approximation: Theory, algorithms, and applications

Ivan Markovsky (Free University of Brussels, BE)

URL http://slra.github.io/

Mathematical engineering continuously addresses new applications and solves new problems. The expansion of existing methods and applications makes it difficult to maintain a common theoretical framework. This talk shows the potential of the structured low-rank approximation setting to unify problems of data modeling from diverse application areas. An example treated in more details in the presentation is identification of a linear time-invariant system from observed trajectories of the system. We present an optimization method based on the variable projection principle. The method can deal with data with exact and missing (unknown) values. Problems with exact and missing values occur in data driven simulation and control – a new trend of model-free methods for system dynamical analysis and control.

3.17 Well conditioned rational functions approximants versus numerically co-prime polynomials

Ana C. Matos (Lille I University, FR)

Joint work of Matos, Ana C.; Beckermann, Bernd; Labahn, George

Rational functions like for instance Padé approximants play an important role in signal processing, sparse interpolation and exponential analysis. However, for a successful modeling with help of rational functions we want to make sure that there is no "similar" rational function being degenerate, i.e., having strictly smaller degree of both degrees of numerator and denominator. In particular, we prefer having rational functions without Froissart doublets (i.e., roots close to a pole), and without spurious poles (i.e., simple poles having small residuals).

In a recent paper [1] we showed that, provided that the Sylvester matrix built with the coefficients of the numerator and denominator is well-conditioned, the corresponding rational function has neither Froissart doublets nor spurious poles, and this is also true to sufficiently "close" rational functions. Here closeness is measured with two different metrics, in terms of the chordal distance of the values on the unit disk, or in terms of the distance of normalized coefficient vectors. The paper [1] also contained a comparison of these two metrics.

In [2] the authors introduced a measure for numerical coprimeness representing the minimal distance in the coefficient vector metric to a couple of degenerate polynomials (with a joint root allowing for canceling the fraction). They also showed that if the underlying Sylvester matrix is well-conditioned then a couple of polynomials is numerically coprime, the reciprocal being wrong.

The aim of this talk is to provide precise inequalities implying that also the larger class of rational functions with numerator and denominator being numerically coprime do not have neither Froissart doublets nor spurious poles.

This is a joint work with Bernd Beckermann and George Labahn.

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3.18 Using noise to detect faint signals: tricks with Padé approximants to Z-transforms

Luca Perotti (Texas Southern Unversity – Houston, US)

Joint work of Perotti, Luca; Bessis, Daniel; Regimbau, Tania

Adding small amounts of noise is a recognized method to stabilize Padé approximants; due to the nonlinearity of the Padé approximant, larger amounts of noise can be added to generate different time series and thus increase the statistics when detection probabilities are low. Recently we proposed a new technique based on the observation that the presence of even a weak signal significantly perturbs the universal properties of noise poles and zeros of the Padé approximants to the Z-transform of a data series. For data from two channels, combined in a single complex sequence, the different behavior of poles corresponding to complex noise and poles corresponding to coherent signal can also be used as a signature of the presence of a signal in heavy noise.

3.19 The generalized Prony method and its application I and II

Thomas Peter (Universität Osnabrück, DE), Gerlind Plonka (Universität Göttingen, DE)

Joint work of Peter, Thomas; Plonka, Gerlind

In this paper, we want to present a new very general approach for the reconstruction of sparse expansions of eigenfunctions of suitable linear operators. This approach provides us with a tool to unify all Prony-like methods on the one hand and to essentially generalize the Prony approach on the other hand. Thus it will establish a much broader field of applications of the method. In particular, we will show that all well-known Prony-like reconstruction methods for exponentials and polynomials known so far, can be seen as special cases of this approach. For example, the new insight into Prony-like methods enables us to derive new reconstruction algorithms for orthogonal polynomial expansions including Jacobi, Laguerre, and Hermite polynomials. The approach also applies to finite dimensional vector spaces, and we derive a deterministic reconstruction method for M-sparse vectors from only $2M$ measurements.

The talk will be split into two parts given by the two authors. In the **first part** we concentrate on deriving the new general approach to apply Prony's method to sparse expansions of eigenfunctions of linear operators and present the close connection to the well-known Prony-method.

The **second part** of the talk is especially dedicated to the advantages that the new more general insights give us for applications, as e.g. the use of different linear operators, the influence of the choice of functionals in case of noisy data and further numerical issues.

3.20 High dimensional approximation with trigonometric polynomials

Daniel Potts (TU Chemnitz, DE)

Joint work of Lutz Kämmerer, Lutz; Potts, Daniel; Volkmer, Toni

In this talk, we present algorithms for the approximation of multivariate functions by trigonometric polynomials. The approximation is based on sampling of multivariate functions on rank-1 lattices. To this end, we study the approximation of functions in periodic Sobolev spaces of dominating mixed smoothness. The proposed algorithm based mainly on a one-dimensional fast Fourier transform, and the arithmetic complexity of the algorithm depends only on the cardinality of the support of the trigonometric polynomial in the frequency domain. Therefore, we investigate trigonometric polynomials with frequencies supported on hyperbolic crosses and energy based hyperbolic crosses in more detail. Furthermore, we present algorithms where the support of the trigonometric polynomial is unknown.

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3.21 Using univariate algorithms to solve multivariate problems

Daniel Roche (U.S. Naval Academy – Annapolis, US)

A key feature of sparse interpolation algorithms is that their complexity should scale nicely (often linearly) with the number of variables in the unknown function. In fact, such algorithms can usually be decomposed into two parts: a "base case" univariate interpolation algorithm, and a method to reduce a given multivariate problem to one or more instances of a univariate one.

We will look at both historical and very recent approaches to the second part, the multivariate-to-univariate reduction. As has been frequently observed, many of these reductions are essentially orthogonal to the choice of underlying univariate algorithm, allowing for a wide range of hybrid approaches – not all of which are equally effective. We will examine the strengths and weaknesses of the various variable reduction strategies, and aim to give some insights into how they may be most effectively chosen and applied to new problems.

3.22 New approaches to Vector-Valued Rational Interpolation

Avraham Sidi (Technion – Haifa, IL)

We discuss some recent vector-valued rational interpolation procedures for vector-valued functions $F(z)$, $F : \mathbb{C} \to \mathbb{C}^N$. The interpolants produced by these procedures are all of the simple form

$$R_{p,k}(z) = \frac{U_{p,k}(z)}{V_{p,k}(z)} = \frac{\sum_{j=0}^{k} c_j\, \psi_{1,j}(z)\, G_{j+1,p}(z)}{\sum_{j=0}^{k} c_j\, \psi_{1,j}(z)}.$$

Here

$$\psi_{m,n}(z) = \prod_{r=m}^{n} (z - \xi_r), \quad n \geq m \geq 1; \quad \psi_{m,m-1}(z) = 1, \quad m \geq 1,$$

and $G_{m,n}(z)$ is the vector-valued polynomial of interpolation to $F(z)$ at the points ξ_i, $m \leq i \leq n$. The c_j are scalars, and they are determined in different ways by the different methods. As such, $R_{p,k}(z)$ interpolates $F(z)$ at ξ_i, $1 \leq i \leq p$, in the generalized Hermite sense.

We first discuss the algebraic properties of these interpolants, namely, their uniqueness, symmetry, and reproducing properties. We next discuss their use in approximating vector-valued meromorphic functions $F(z)$ in the complex plane.

Next, choosing the interpolation points appropriately, for $p \to \infty$ and k fixed, we derive de Montessus type convergence results for the interpolants and Koenig type convergence results for their poles and residues, which show that these interpolants, despite their simple appearance, are effective approximation tools. Especially interesting Koenig type results are obtained when the residues of $F(z)$ form a mutually orthogonal set. (Note that, for any type of rational interpolation problem, whether scalar or vector, the crucial test for deciding whether these are useful approximation tools is the existence of de Montessus and Koenig type theories.)

Finally, we consider the fully confluent case in which all interpolation points ξ_i coincide, and show the connection of the resulting interpolants with Krylov subspace methods.

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3.23 Efficient spectral estimations by MUSIC and related algorithms

Manfred Tasche (Universität Rostock, DE)

Joint work of Tasche, Manfred; Potts, Daniel; Volkmer, Toni

In the spectral estimation, one has to determine all parameters of a univariate resp. multivariate exponential sum h, if only finitely many (noisy) sampled data of h are given. A frequently used method for spectral estimation is the known MUSIC algorithm. Another popular methods are ESPRIT and the approximate Prony method (APM). We show that both MUSIC and APM are based on an orthogonal projection onto a so-called noise space, whereas ESPRIT uses an orthogonal projection onto the orthogonal complement of the noise space, the so-called signal space. These orthogonal projections can be constructed by (partial) singular value decomposition or QR decomposition of a rectangular Hankel matrix formed by the given sampled data of h.

In this talk, we describe that MUSIC and the related algorithms can be efficiently realized by sampling of h on special grids and using sparse fast Fourier transforms. Numerical experiments illustrate the procedure.

3.24 Towards simplified construction of subresultant matrix of multiple univariate polynomials

Akira Terui (University of Tsukuba, JP)

For three or more inputs of univariate polynomials with the real coefficients, we discuss a new construction of subresultant-like matrix which enable us to estimate the degree of the greatest common divisor (GCD) of the input polynomials from its rank. Such matrix is used in approximate GCD algorithms using optimization techniques with its degree is given in advance, especially for constructing constraints. Therefore, in these algorithms, it is important to construct the matrix in a more simplified form to make the overall algorithm more efficient. In this talk, for those purposes, we discuss towards a proposal of a new simplified construction of the matrix.

3.25 Hankel and Quasi-Hankel low-rank matrix completion: a convex relaxation

Konstantin Usevich (GRIPSA Lab – Saint Martin d'Hères, FR)

Joint work of Comon, Pierre; Usevich, Konstantin

The completion of matrices with missing values under the rank constraint is a non-convex optimization problem. A popular convex relaxation is based on minimization of the nuclear norm (sum of singular values) of the matrix. For this relaxation, an important question is when the two optimization problems lead to the same solution. This question was addressed in the literature mostly in the case of random positions of missing elements and random known elements. In this contribution, we analyze the case of structured matrices with fixed pattern of missing values, in particular, the case of Hankel and quasi-Hankel matrix completion, which appears as a subproblem in the computation of symmetric tensor canonical polyadic decomposition. Similar matrix completion problems appear in other applications, where a function can be approximated as a sum of complex exponentials (time series analysis, medical imaging). We extend existing results on completion of rank-one real Hankel matrices to completion of rank-r complex Hankel and quasi-Hankel matrices.

3.26 A deterministic sparse FFT algorithm for vectors with short support

Katrin Wannenwetsch (Universität Göttingen, DE)

Joint work of Wannenwetsch, Katrin; Plonka, Gerlind

It is well known that usual FFT algorithms for the discrete Fourier transform of a vector of length N require $\mathcal{O}(N \log N)$ arithmetical operations. Within the last years, there has been a great interest in sublinear time Fourier algorithms for sparse vectors.

In this talk we consider the special case where a signal $\mathbf{x} \in \mathbb{C}^N$ is known to vanish outside a support interval of length $m < N$. If the support length m of $\mathbf{x}$ or a good bound of it is a-priori known we derive a sublinear algorithm to compute $\mathbf{x}$ from its discrete Fourier transform $\widehat{\mathbf{x}} \in \mathbb{C}^N$. The proposed algorithm is deterministic and numerically stable.

In case of exact Fourier measurements we require only $\mathcal{O}(m \log m)$ arithmetical operations. For noisy measurements, we propose a stable $\mathcal{O}(m \log N)$ algorithm.

This is joint work with Gerlind Plonka.

3.27 Sparsity with Symbolic Polynomials

Stephen M. Watt (University of Western Ontario – London, CA)

We are interested in algorithms for “symbolic polynomials”, that is multivariate polynomials generalized so the exponents are themeselves integer-valued multivariate polynomials, for example $x^{n^2/2-n/2} - y^m$. These objects may be used to model parameterized families of Laurent polynomials, with integer evaluations of the exponent variables giving specific Laurent polynomials. We have shown elsewhere that when polynomials with coefficents in a particular ring form a unique factorization domain, then so do the corresponding symbolic polynomials. We have given algorithms to compute their GCDs and factorizations in this case. Some of these algorithms rely on reduction to algorithms on sparse polynomials with many more variables, as will be explored in this talk. We additionally describe some new directions on Groebner bases for symbolic polynomials.

3.28 Reconstruction of Structured Functions from Sparse Fourier Data

Marius Wischerhoff (Universität Göttingen, DE)

In several scientific areas, such as radio astronomy, computed tomography, and magnetic resonance imaging, the reconstruction of structured functions from the knowledge of samples of their Fourier transform is a common problem. For the analysis of the examined object, it is important to reconstruct the underlying original signal as exactly as possible. We aim to uniquely recover structured functions from only a small number of Fourier samples. For this purpose, the Prony method, which is a deterministic method for the recovery of sparse trigonometric functions, is used as key instrument to derive algorithms for unique recovery by means of a smallest possible set of Fourier data.

We will give an overview of reconstruction results for different function classes, and we will consider two classes in detail.

First, we will examine linear combinations of N non-uniform shifts of a given bivariate function. Here, the unknown shift parameters and corresponding coefficients in the linear combination are recovered from sparse Fourier data. Unique recovery of the parameters is possible by using only $3N + 1$ Fourier samples on three lines through the origin. For this purpose, two predetermined lines are considered, while the third sampling line is chosen dependently on the results obtained by employing the samples from the first two lines. The presented approach can be generalized to the case of d-variate functions with $d > 2$.

Secondly, we turn to the reconstruction of polygonal shapes in the real plane. Here, a convex or non-convex polygonal domain D with N vertices is considered. It is shown that the vertices and their order can be reconstructed by taking $3N$ samples of the Fourier transform of the characteristic function of the polygonal domain D. Again, two predetermined sampling lines and an appropriately chosen third line are considered.

3.29 Accuracy of Spike-Train Fourier Reconstruction for Near-Colliding Nodes

Yosef Yomdin (Weizmann Institute – Rehovot, IL)

Joint work of Akinshin, Andrey; Batenkov, Dmitry; Yomdin, Yosef

We study reconstruction of “spike-train” signals F of the form

$$F(x) = \sum_{j=1}^{d} a_j \delta(x - x_j),$$

from their Fourier transform $\hat{F}(s)$, known for $s \in [-N, N]$, with an absolute error not exceeding $\epsilon > 0$. We concentrate on “near-collision” situations where the nodes x_j are known to form an l elements cluster of a size $h \ll 1$.

We show that in such situations the geometry of error amplification in the reconstruction is governed by the “Prony foliations” S_q whose leaves are defined by the Prony equations $\sum_{j=1}^{d} a_j x_j^k = \gamma_k$, with $k = 0, \ldots, q \leq l$, and with the arbitrary right-hand sides γ_k. On this base we give an “absolute” (i.e. valid with any reconstruction method) lower bound for the “worst case” reconstruction error of F from $\hat{F}$. We show that for the measurement error

$\epsilon > C_1(hN)^{2l-1}$, the inside configuration of the cluster nodes (in the worst case scenario) cannot be reconstructed at all.

Combining a proper rescaling with the "Decimation method" we show that for $\epsilon < C_2(hN)^{2l-1},\ C_2 \ll C_1$, an accurate (up to an error $\alpha h,\ \alpha \ll 1$) reconstruction of the cluster nodes is possible. The same algorithm reconstructs the non-cluster nodes and amplitudes with the full accuracy (of order $\frac{\epsilon}{N}$).

3.30 Semidefinite Representations of Noncompact Convex Sets

Lihong Zhi (MMRC – Beijing, CN)

Joint work of Guo, Feng; Wang, Chu; Zhi, Lihong

We consider the problem of the semidefinite representation of a class of non-compact basic semialgebraic sets. We introduce the conditions of pointedness and closedness at infinity of a semialgebraic set and show that under these conditions our modified hierarchies of nested theta bodies and Lasserre's relaxations converge to the closure of the convex hull of S. Moreover, if the PP-BDR property is satisfied, our theta body and Lasserre's relaxation are exact when the order is large enough; if the PP-BDR property does not hold, our hierarchies converge uniformly to the closure of the convex hull of S restricted to every fixed ball centered at the origin. We illustrate through a set of examples that the conditions of pointedness and closedness are essential to ensure the convergence. Finally, we provide some strategies to deal with cases where the conditions of pointedness and closedness are violated.

3.31 Trivariate polynomial approximation on Lissajous curves

Stefano de Marchi (University of Padova, IT)

Joint work of Bos Len; Vianello Marco; de March, Stefano
Main reference L. Bos, S. De Marchi, M. Vianello, "Trivariate polynomial approximation on Lissajous curves," arXiv:1502.04114v1 [math.NA], 2015.
URL http://arxiv.org/abs/1502.04114v1

We study Lissajous curves in the 3-cube, that generate algebraic cubature formulas on a special family of rank-1 Chebyshev lattices. These formulas are used to construct trivariate hyperinterpolation polynomials via a single 1-d Fast Chebyshev Transform (by the Chebfun package), and to compute discrete extremal sets of Fekete and Leja type for trivariate polynomial interpolation. Applications could arise in the framework of Lissajous sampling for MPI (Magnetic Particle Imaging).

4 Panel Discussions

At the closing meeting the organizers presented some slides summarizing the connections and similarities between the techniques used by the different communities gathered at the seminar. These slides are being complemented with reference material, an effort which is being continued after the seminar, and made available at the seminar's webpage or

https://www.uantwerpen.be/en/rg/cma/

as a shared document.

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Report from Dagstuhl Seminar 15261

Logics for Dependence and Independence

Edited by

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Abstract

This report documents the programme and outcomes of Dagstuhl Seminar 15261 'Logics for Dependence and Independence'. This seminar served as a follow-up seminar to the highly successful seminar 'Dependence Logic: Theory and Applications' (Dagstuhl Seminar 13071). A key objective of the seminar was to bring together researchers working in dependence logic and in the application areas so that they can communicate state-of-the-art advances and embark on a systematic interaction. The goal was especially to reach those researchers who have recently started working in this thriving area.

Seminar June 21–26, 2015 – http://www.dagstuhl.de/15261
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1 Executive Summary

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Brief Introduction to the Topic

Dependence and independence are interdisciplinary notions that are pervasive in many areas of science. They appear in domains such as mathematics, computer science, statistics, quantum physics, and game theory. The development of logical and semantical structures for these notions provides an opportunity for a systematic approach, which can expose surprising connections between different areas, and may lead to useful general results.

Dependence Logic is a new tool for modeling dependencies and interaction in dynamical scenarios. Reflecting this, it has higher expressive power and complexity than classical logics used for these purposes previously. Algorithmically, first-order dependence logic corresponds exactly to the complexity class NP and to the so-called existential fragment of second-order logic. Since the introduction of dependence logic in 2007, the framework has been generalized, e. g., to the contexts of modal, intuitionistic, and probabilistic logic. Moreover, interesting connections have been found to complexity theory, database theory, statistics, and dependence logic has been applied in areas such as linguistics, social choice theory, and physics. Although significant progress has been made in understanding the computational side of these formalisms, still many central questions remain unsolved so far.

The Dagstuhl seminar 'Dependence Logic: Theory and Applications' had a major impact to the field of dependence logic opening up connections to new application areas. The aim of this follow-up seminar was to gather together the people working in dependence logic and in the application areas, especially those researchers who have recently started working in this quickly developing area to communicate state-of-the-art advances and embark on a systematic interaction.

Organization of the Seminar and Activities

The seminar brought together 38 researchers from mathematics, statistics, database theory, natural language semantics, and theoretical computer science. The participants consisted of both senior and junior researchers, including a number of postdocs and advanced graduate students.

Participants were invited to present their work and to communicate state-of-the-art advances. Over the five days of the seminar, 27 talks of various lengths took place. Introductory and tutorial talks of 90-60 minutes were scheduled prior to seminar. Most of the remaining slots were filled, mostly with shorter talks, as the seminar commenced. The organizers considered it important to leave ample free time for discussion.

The tutorial talks were scheduled during the beginning of the week in order to establish a common background for the different communities that came together for the seminar. The presenters and topics were:

- Jouko Väänänen and Juha Kontinen, Dependence Logic
- Bernhard Thalheim, Database Constraints – A Survey
- Ilya Shpitser, Causal inference
- Lauri Hella, Modal dependence logic
- Ivanio Ciardelli, Dependency as Question Entailment
- Antti Hyttinen, Statistical Independence, Causality and Constraint Satisfaction

There were additionally two introductory talks with a more focused and technical topic:

- Alex Simpson, Sheaf semantics for independence logics
- Phokion Kolaitis, The Query Containment Problem: Set Semantics vs. Bag Semantics

Additionally, the following shorter presentations were given during the seminar:

- Åsa Hirvonen, Model theoretic independence
- Kerkko Luosto, Dimensions for Modal Dependence Logic
- Gianluca Paolini, Measure teams
- Olaf Beyersdorff, Proof Complexity of Quantified Boolean Formulas
- Antti Kuusisto, Propositional dependence logic via Kripke semantics
- Johanna Stumpf, Characterisation of the expressive power of modal logic with inclusion atoms
- Sebastian Link, Dependence-driven, non-invasive cleaning of uncertain data
- Jonni Virtema, Complexity of Propositional Inclusion and Independence Logic
- Katsuhiko Sano, Characterizing Frame Definability in Team Semantics via The Universal Modality

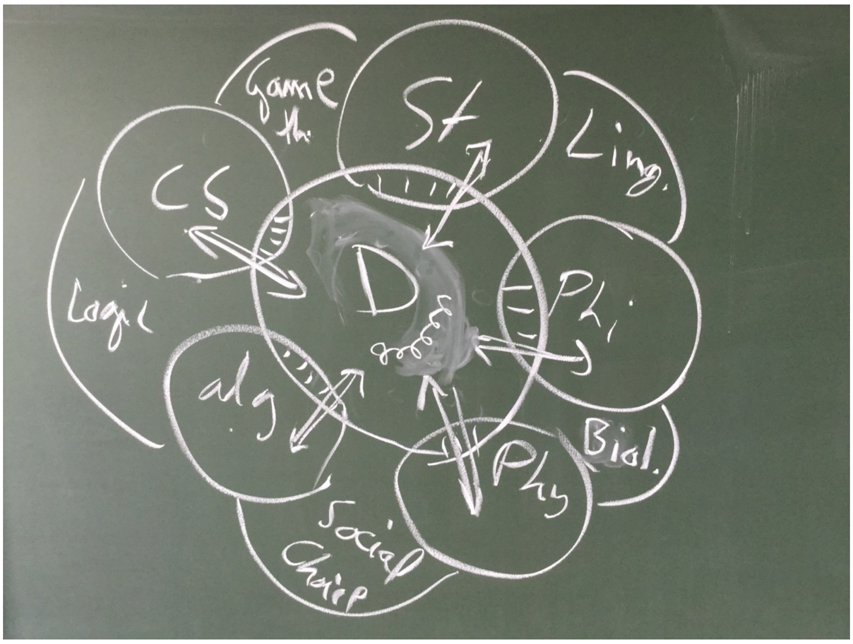

Figure 1 The blackboard after Jouko Väänänen's conclusion of the seminar.

- Raine Rönnholm, Expressing properties of teams in k-ary inclusion-exclusion logic
- Julian Bradfield, On the structure of events in Boolean games
- Fan Yang, Some proof theoretical results on propositional logics of dependence and independence
- Erich Grädel, Counting in Team Semantics
- Fredrik Engström, Generalized quantifiers and Dependence Logic
- Miika Hannula, Axiomatizing dependencies in team semantics
- Dietmar Berwanger, An NL-fragment of inclusion logic
- Nicolas de Rugy-Altherre, Tractability Frontier of Data Complexity in Team Semantics

For some of these, an abstract can be found below.

The seminar achieved its aim of bringing together researchers from various related communities to share state-of-the-art research. The organizers left ample time outside of this schedule of talks and many fruitful discussions between participants took place throughout the afternoons and evenings.

Concluding Remarks and Future Plans

The organizers regard the seminar as a great success. Bringing together researchers from different areas fostered valuable interactions and led to fruitful discussions. Feedback from the participants was very positive as well.

Finally, the organizers wish to express their gratitude toward the Scientific Directorate of the Center for its support of this seminar.

2 Table of Contents

3 Overview of Talks

3.1 Proof Complexity of Quantified Boolean Formulas

Olaf Beyersdorff (University of Leeds, GB)

Joint work of Beyersdorff, Olaf; Chew, Leroy; Janota, Mikolas
Main reference O. Beyersdorff, L. Chew, M. Janota, "Proof Complexity of Resolution-based QBF Calculi," in Proc. of the 32nd Int'l Symp. on Theoretical Aspects of Computer Science (STACS'15), LIPIcs, Vol. 30, pp. 76–89, Schloss Dagstuhl – Leibniz-Zentrum für Informatik, 2015.
URL http://dx.doi.org/10.4230/LIPIcs.STACS.2015.76

The main aim in proof complexity is to understand the complexity of theorem proving. Arguably, what is even more important is to establish techniques for lower bounds, and the recent history of computational complexity speaks volumes on how difficult it is to develop general lower bound techniques. Understanding the size of proofs is important for at least two reasons. The first is its tight relation to the separation of complexity classes: NP vs. coNP for propositional proofs, and NP vs. PSPACE in the case of proof systems for quantified boolean formulas (QBF). The second reason to study lower bounds for proofs is the analysis of SAT and QBF solvers: powerful algorithms that efficiently solve the classically hard problems of SAT and QBF for large classes of practically relevant formulas.

In this talk we give an overview of the relatively young field of QBF proof complexity. We explain the main resolution-based proof systems for QBF, modelling CDCL and expansion-based solving. In the main part of the talk we will give an overview of current lower bound techniques (and their limitations) for QBF systems. In particular, we exhibit a new and elegant proof technique for showing lower bounds in QBF proof systems based on strategy extraction. This technique provides a direct transfer of circuit lower bounds to lengths of proofs lower bounds.

Potential connections to dependence logic arise through dependencies between quantified variables. These are used frequently in QBF and QBF solving, and are systematically studied in the NEXPTIME-complete logic of dependency QBFs (DQBF).

3.2 On the structure of events in Boolean games

Julian Bradfield (University of Edinburgh, GB)

Joint work of Bradfield, Julian; Gutierrez, Julian; Wooldridge, Michael
Main reference J. Bradfield, J. Gutierrez, M. Wooldridge, "On the structure of events in boolean games ," Presentation at the 11th Conf. on Logic and the Foundations of Game and Decision Theory (LOFT'14), University of Bergen, Norway, July 27–30, 2014.

As conventionally formulated, Boolean games assume that all players make choices simultaneously, and act in complete ignorance of the choices being made by other players. For many settings, these assumptions represent gross over simplifications. In this paper, we show how Boolean games can be enriched by *dependency graphs* which explicitly represent the dependencies between choices in a game. These dependency graphs allow us to directly specify what a player knows about other choices when that player makes a choice. In addition, they capture a richer and more plausible model of concurrency than the simultaneous action model implicit in conventional Boolean games. We refer to games played with dependency graphs as *partial order Boolean games*. After motivating and presenting the partial order Boolean games model, we explore its properties. We show that while some problems associated with our new games have the same complexity as in conventional Boolean games, for others the complexity blows up dramatically. We also show that the concurrent behaviour of partial order Boolean games can be represented using a closure operator semantics, and conclude by considering the relationship of our model to IF logic.

3.3 Generalized quantifiers and Dependence Logic

Fredrik Engström (University of Göteborg, SE)

Dependence logic, proposed by Väänänen [2], is an elegant way of introducing dependencies between variables into the object language. The framework of dependence logic, so-called team semantics, has turned out to be very flexible and allows for interesting generalizations. Instead of considering satisfaction with respect to a single assignment s, team semantics considers sets of assignments X, called teams.

The semantics of Dependence logic is based on the principle that

a formula φ is satisfied by a team X if every assignment $s \in X$ satisfies φ.

The compositional semantics of dependence logic, except for the case for the dependence atom, can be derived from this one principle.

In this talk we introduce a new semantics, which is better suited for non-monotone increasing generalized quantifiers, where the above is replaced by the principle that

a formula φ is satisfied by a team X if for every assignment $s : \mathrm{dom}(X) \to M^k$, $s \in X$ iff s satisfies φ,

replacing an implication by an equivalence. When only first-order logic is considered in this new setting nothing exciting happens. It is only when we introduce atoms, like dependence atoms, or new logical operations that things start to get more exciting.

This alternative semantics will allow us to extend the logic with any generalized quantifier, not only monotone increasing ones as in [1].

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3.4 Counting in Team Semantics

Erich Grädel (RWTH Aachen, DE)

We explore different ways to add counting constructs to logics with team semantics, such as counting quantifiers and forking atoms. While on the level of existential second-order definability, counting in finite structures is available without additional constructs and without a separate numerical sort, the addition of counting quantifiers enhances the expressive power of weaker logics, such as inclusion logic.

In the context of descriptive complexity theory, fixed-point logic with counting (FPC) is of central importance and actually the logic of reference in the quest for a logic for polynomial time. We extend the equivalence of inclusion logic and positive greatest fixed-point logic, proved by Galliani and Hella on the way back from the last Dagstuhl seminar on dependence logic, to an equivalence between FPC and inclusion logic with counting. Our proof is based on a new class of games, called threshold safety games, an on interpretation arguments for such games.

This talk is partially based on joint work with Stefan Hegselmann and on discussions with Pietro Galliani and Lauri Hella.

3.5 Axiomatizing dependencies in team semantics

Miika Hannula (University of Helsinki, FI)

We consider implication problems of different database dependencies. Simulating the chase algorithm at the logical level we show how different classes of dependencies can be axiomatised in the team semantics framework. In the associated proof systems, intermediate steps of deductions are inclusion dependencies that are implicitly existentially quantified as in lax semantics.

3.6 The expressive power of modal inclusion logic

Lauri Hella (University of Tampere, FI)

Modal inclusion logic is the extension of basic modal logic with inclusion atoms, and its semantics is defined on Kripke models with teams. A team of a Kripke model is just a subset of its domain. In [1] we give a complete characterisation for the expressive power of modal inclusion logic: a class of Kripke models with teams is definable in modal inclusion logic if and only if it is closed under k-bisimulation for some integer k, it is closed under unions, and it has the empty team property. We also prove that the same expressive power can be obtained by adding a single unary nonemptiness operator to modal logic. Furthermore, we establish an exponential lower bound for the size of the translation from modal inclusion logic to modal logic with the nonemptiness operator.

References

1 Lauri Hella and Johanna Stumpf. *The expressive power of modal logic with inclusion atoms*. In Proceedings Sixth International Symposium on Games, Automata, Logics and Formal Verification (GandALF 2015), Electronic Proceedings in Theoretical Computer Science.

3.7 Model theoretic independence

Åsa Hirvonen (University of Helsinki, FI)

Independence is a crucial notion in the branch of model theory called classification theory. Classification theory aims at classifying collections of models: when are they well behaved, meaning that all models can be characterized up to isomorphism with a relatively small set of invariants, and when are they ill-behaved and exhibit the maximum number of models in each cardinality, with the models hard to distinguish from one another.

The most studied notion of independence in model theory is that of forking independence, developed by Shelah. It is designed to generalize linear independence in vector spaces and algebraic independence in algebraically closed fields. However, it gives relevant information also on more complex models. A classifiable model need not have just one dimension that characterizes it, but may have a collection of different dimensions (hence the set of invariants and not just one invariant).

With a more general independence notion, the connection between 'dependent' and 'independent' changes. The natural notion of dependence is no longer 'not independent of' but one needs a notion of generation. This is handled with various notions of primeness.

The use of model theoretic independence has extended to various contexts outside the original scope, and the properties studied vary depending on what can be achieved in different contexts as well as on different authors' different viewpoints. However, in the cases where one can define a well-behaved independence notion, it tends to be unique.

I will present the background to and some features of model theoretic independence, giving a short overview of the roles of dependence and independence in classification theory.

3.8 Statistical Independence, Causality and Constraint Satisfaction

Antti Hyttinen (University of Helsinki, FI)

A causal model and the accompanied causal graph explain how statistically modeled system behaves when interventions are applied to it. I will explain the connection between statistical dependence/independence and reachability/separation in the causal graphs, the so-called d-separation. Using this, I will present the idea of constraint-based causal discovery, in which one can deduce properties of the causal graph structure from independencies and dependencies in passively observed data. I will also go through further concepts of independence used in causality and machine learning research. I will outline connections to the implication problem of conditional independence and its possible generalizations.

3.9 Dependence logic

Juha Kontinen (University of Helsinki, FI)

We survey some recent work in the area of dependence logic and team semantics. We focus on results on the computational aspects of various logics in the team semantics framework. We also discuss axiomatizability of certain fragments and variants of dependence logic.

3.10 Propositional dependence logic via Kripke semantics

Antti Kuusisto (Stockholm University, SE)

Joint work of Kuusisto, Antti; Goranko, Valentin

We propose an alternative semantics for propositional dependence and independence logics. The semantics gives a classical interpretation for the Boolean connectives and is ultimately based on treating dependence and independence atoms as generalized modalities in a framework with a Kripke- style semantics. We argue for the naturality of the novel semantics from the point of view of natural language. We also give sound and complete axiomatizations for both propositional dependence and independence logics in the case of a global accessibility relation. Furthermore, we show that propositional dependence and independence logics in the new framework are equiexpressive with modal logic with the global modality, and thus the corresponding logics based on team semantics are strictly weaker in expressivity than the novel systems. Interestingly, however, there exists no compositional translation from standard dependence or independence logic into the novel logics.

3.11 Dependence-driven, non-invasive cleaning of uncertain data

Sebastian Link (University of Auckland, NZ)

Joint work of Koehler, Henning; Link, Sebastian

One classical approach to cleaning relational databases is to remove tuples which violate some given data dependency. To identify which of the conflicting tuples are dirty, one tries to find a minimal set of tuples to remove. For several popular classes of data dependencies, such as keys and differential dependencies, this minimization problem turns out to be equivalent to vertex cover. The classical approach ignores the uncertainty with which tuples occur and dependencies hold on the data. The classical approach is also invasive in the sense that tuples are removed from the database. In practice this is often unacceptable as some deleted tuples represent invaluable information. We propose an entirely new view on data cleaning in which both shortcomings are overcome. We depart from the classical view in which the data is considered to be dirty, and instead, view the degree of uncertainty attributed to the data as dirty. The talk will present a well-founded framework for this new view, a fixed-parameter tractable algorithmic solution using a generalization of vertex cover, and some experimental results. Fraud detection is identified as a new application of data cleaning, and several open problems are outlined for joint future research.

3.12 Dimensions for Modal Dependence Logic

Kerkko Luosto (University of Tampere, FI)

Joint work of Hella, Lauri; Luosto, Kerkko; Sano, Katsuhiko; Virtema, Jonni
Main reference L. Hella, K. Luosto, K. Sano, J. Virtema, "The Expressive Power of Modal Dependence Logic," arXiv:1406.6266v1 [cs.LO], 2014.
URL http://arxiv.org/abs/1406.6266v1

Instead of the ordinary semantics for modal logics, we use throughout this work team semantics in connection with Kripke structures. Several extensions of the basic modal logic are considered. These include the modal dependence logic and extended modal dependence logic, resp., with the added dependence atoms (dependencies between propositional symbols or basic modal formulas, resp.), and the modal logic with intuitionistic disjunction.

In the case of team semantics, the teams satisfying a given formula in a fixed Kripke structure is a family of sets whose combinatorial properties reflect the properties of the formula. It is easy to see that this family is downwards closed, implying that it is generated by its maximal elements which form a so-called Sperner family. Two dimension concepts, upper and lower one, may now be introduced to study the expressive power of formulas of these logics. The upper dimension is simply related to the number of maximal set in the family, whereas the lower dimension is the size of the largest minimal set that avoids the family, when we run over all Kripke structures.

It is proved, among other results, that even if the extended modal dependence logic and modal logic with intuitionistic disjunction have the same expressive power, the translation form the former to latter involves an exponential blow-up.

3.13 A Team Based Variant of CTL

Arne Meier (Leibniz Universität Hannover, DE)

Joint work of Meier, Arne; Krebs, Andreas; Virtema, Jonni
Main reference A. Krebs, A. Meier, J. Virtema, "A Team Based Variant of CTL," in Proc. of the 22nd Int'l Symp. on Temporal Representation and Reasoning (TIME'15), to appear; pre-print available as arXiv:1505.01964v2 [cs.LO], 2015.
URL http://arxiv.org/abs/1505.01964v2

We introduce two variants of computation tree logic CTL based on team semantics: an asynchronous one and a synchronous one. For both variants we investigate the computational complexity of the satisfiability as well as the model checking problem. The satisfiability problem is shown to be EXPTIME-complete. Here it does not matter which of the two semantics are considered. For model checking we prove a PSPACE-completeness for the synchronous case, and show P-completeness for the asynchronous case. Furthermore we prove several interesting fundamental properties of both semantics.

3.14 Measure Teams

Gianluca Paolini (University of Helsinki, FI)

Joint work of Hyttinen, Tapani; Paolini, Gianluca; Väänänen, Jouko

We give an overview of recent probabilistic developments in team semantics centered around the notion of measure team.

3.15 Expressing properties of teams in k-ary inclusion-exclusion logic

Raine Rönnholm (University of Tampere, FI)

The k-ary inclusion-exclusion logic, INEX[k], is obtained by extending first order logic with k-ary inclusion and exclusion atoms. The truth conditions for these atoms correspond to inclusion and exclusion dependencies in the database theory. We will examine the expressive power of INEX[k] on the level of formulas, by analyzing what kind properties of teams can be defined with it. By our earlier results, INEX[k] is closely related with k-ary existential second order logic, ESO[k]. We know that all ESO[k]-definable properties of k-ary relations of teams can be defined in INEX[k] and that all INEX[k]-definable properties are ESO[k]-definable.

However, when the arity of relations becomes higher than the arity of atoms, things get more exotic. We will show that for any k there are some very simple FO-definable properties of (k+1)-ary relations that cannot be defined in INEX[k]. For example, by using only unary inclusion and exclusion atoms, we cannot define the symmetry of a binary relation. But interestingly in INEX[1] we can define many properties of binary relations that are not FO-definable, such as disconnectivity of a graph.

To prove our undefinability results we will introduce a new method: Suppose that phi is an INEX[k]-formula and X is a team. Suppose also that phi is true in X, i.e. verifier has a winning strategy in the corresponding semantic game. We will then consider the reducts of this strategy for semantic games of subteams Y. We can show that if X is large enough compared to the size of phi, then the reduct strategy corresponding the team X\s is a winning strategy for any assignment s in X.

3.16 Causal Inference and Logics of Dependence and Independence

Ilya Shpitser (University of Southampton, GB)

In this talk, I introduce modern causal inference, an interdisciplinary area spanning statistics, philosophy, computer science, and the empirical sciences. Causal inference is based on the notion of counterfactual responses to *interventions*, which mathematical idealizations of randomized treatment assignment. Interventions allow clean reasoning about confounding, a phenomenon responsible for the adage ‘correlation does not imply causation.’

In the first part of my talk, I describe some recent work on techniques that allow inferring cause effect relationships from observational data, by by appropriately handling confounding [6, 7, 4]. In the second, I discuss an encoding of interventionist causality via dynamic modal logic due to [1]. Finally, I motivate a generalization of the graphoid axioms for reasoning about conditional independence to the case of generalized independence that arises in the theory of hidden variable models. [8, 5, 2, 3].

I conclude by proposing avenues for future collaboration between the causal inference community, and the community that works on logic and model theory. These included introducing quantification and other first order logic features to Halpern's logic, combining logic and probability for reasoning about uncertainty and causality together, finding connections with dependence logics, and appropriately generalizing the graphoid axioms to the hidden variable case.

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3.17 Sheaf semantics for independence logics

Alex Simpson (University of Ljubljana, SI)

Hodges' team/trump semantics provides a general semantic framework for logics based on dependence and independence, which has proven to be flexible in adapting to several varieties of such logics. However, it also gives rise to quirks such as an uneven treatment of negation (which is often restricted to atomic formulas), the failure of classical logic, and a potential proliferation of logical connectives.

I present instead an alternative semantic framework based on sheaves. In one direction, this framework generalises team semantics, in that teams arise as just one kind of possible structure for interpreting variables. In another direction, it departs from team semantics, in that the interpretation of logical connectives and quantifiers is different. In combination, these changes result in an embedding of independence primitives in ordinary classical logic, augmented by a layer of modalities, which reflect the nature of the notion of independence under consideration.

While the proposed framework arises abstractly from the interpretation of logic in certain atomic Grothendieck toposes, I instead introduce it directly from first principles. No knowledge of sheaf theory is assumed The presentation is aimed at logicians.

The material divides naturally into two parts.

Part 1: Sheaf semantics for logical independence. I introduce the framework of sheaf semantics in the case of 'logical independence', which is the form of independence usually considered in team semantics. This gives rise to classical logic augmented with independence primitives, together with two modalities (necessity and possibility).

Part 2: Sheaf semantics for probabilistic independence. I make use of the generality of the framework by giving a sheaf semantics in terms of random variables. This again gives rise to classical logic augmented with independence primitives. But now the independence primitives express probabilistic (conditional) independence, and there is a whole family of modalities capturing probability thresholds.

3.18 Characterisation of the expressive power of modal logic with inclusion atoms

Johanna Stumpf (TU Darmstadt, DE)

The syntax of modal inclusion logic and team semantics are presented. Additionally, modal logic with nonempty disjunction is introduced. It is shown that exactly those properties of teams which are closed under unions and closed under team k-bisimulation are definable in modal logic with nonempty disjunction. Then modal logic with inclusion atoms is introduced and it is proven that the same result holds there. Furthermore, a lower bound for the size of the translation from modal logic with inclusion atoms to modal logic with nonempty disjunction is established.

3.19 Complexity of Propositional Inclusion and Independence Logic

Jonni Virtema (University of Tampere, FI)

Joint work of Hannula, Miika; Kontinen, Juha; Virtema, Jonni; Vollmer, Heribert
Main reference M. Hannula, J. Kontinen, J. Virtema, H. Vollmer, "Complexity of Propositional Inclusion and Independence Logic," in Proc. of 40th Int'l Symp. on Mathematical Foundations of Computer Science (MFCS'15) – Part I, LNCS, Vol. 9234, pp. 269–280, Springer, 2015.
URL http://dx.doi.org/10.1007/978-3-662-48057-1_21

We classify the computational complexity of the satisfiability, validity and model-checking problems for propositional independence and inclusion logic and their extensions by the classical negation. Our main result shows that the satisfiability and validity problems of the extensions of propositional independence and inclusion logic by the classical negation are complete for alternating exponential time with polynomially many alternations.

3.20 Some proof theoretical results on propositional logics of dependence

Fan Yang (Utrecht University, NL)

Joint work of Yang, Fan; Iemhoff, Rosalie

In this talk, we present some proof theoretical results on propositional dependence logic and its variants, including propositional inquisitive logic. We prove the interpolation theorem for these logics. We also prove that propositional logics of dependence are structurally complete with respect to flat substitutions, that is, all admissible rules (with respect to flat substitutions) of these logics are derivable in their deductive systems.

3.21 Tractability Frontier of Data Complexity in Team Semantics

Nicolas de Rugy-Altherre (University Paris-Diderot, FR)

Joint work of de Rugy-Altherre, Nicolas; Kontinen, Juha; Durand, Arnaud; Väänänen, Jouko
Main reference A. Durand, J. Kontinen, N. de Rugy-Altherre, J. Väänänen, "Tractability Frontier of Data Complexity in Team Semantics," arXiv:1503.01144v2 [cs.LO], 2015.
URL http://arxiv.org/abs/1503.01144v2

We study the data complexity of model-checking for logics with team semantics. For dependence and independence logic, we completely characterize the tractability/intractability frontier of data complexity of both quantifier-free and quantified formulas. For inclusion logic formulas, we reduce the model-checking problem to the satisfiability problem of so-called Dual-Horn propositional formulas. While interesting in its own right, this also provides an alternative proof for the recent result of P. Galliani and L. Hella in 2013 showing that the data complexity of inclusion logic is in PTIME. In the last section we consider the data complexity of inclusion logic under so-called strict semantics.

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